

## Multi-objective optimization of multi-state reliability system using hybrid metaheuristic genetic algorithm and fuzzy function for redundancy allocation

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**ABSTRACT:** This paper proposes a methodology for optimizing the reliability of a series-parallel system on the basis of multi-objective optimization and multi-state reliability using a hybrid genetic algorithm (HGA) and fuzzy function. The considered reliability constraints include the number of selected redundant components, total cost, and total weight. First, we describe the modeling of the proposed methodology. Second, we explain the formulation of the optimization process and the solution using HGA. Most related studies have focused only on single-objective optimization of the redundancy allocation problem (RAP); multi-objective optimization has not attracted much attention thus far. This study investigates the multi-objective scenario. Specifically, multi-objective formulation is considered for maximizing system reliability and minimizing system cost and system weight simultaneously in order to solve the RAP. The objective is to determine the system configuration that achieves the optimal trade-off between reliability, cost, and weight. Finally, the obtained results show that the proposed approach can enable manufacturers to determine the number of redundant components and their reliability in a subsystem in order to develop a system that effectively satisfies the reliability, cost, and weight criteria.

**Keywords:** Multi-objective optimization; multi-state reliability; hybrid metaheuristic genetic algorithm; fuzzy function.

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### I. INTRODUCTION

Optimizing reliability in the design and operation of large- and small-scale systems is an important issue for manufacturers. The objective of this study is to optimize the reliability of a series-parallel system on the basis of a genetic algorithm (GA) by implementing solutions for the redundancy allocation problem (RAP). The problem is to set the redundancy level for each subsystem and component and to select the best redundancy strategy in order to maximize the system reliability under multiple objectives and system-level constraints, including the cost and weight at the system level.

This problem is extremely common in the theoretical design of various engineering systems. Developing robust solutions to address the issue of system reliability is important because mechanical and electrical systems and products have become increasingly complex over the years. It is crucial for systems to achieve their objectives under given circumstances and operating conditions in a certain manner. However, the level of system reliability is directly related to system cost. Thus, optimization models are required for effective decision-making and analysis. This study focuses on optimizing a combinatorial engineering design problem, i.e., maximizing the reliability and minimizing the cost and weight of a system that involves a redundant number of selected components. The main contribution of this study is that it examines the effectiveness of employing a fuzzy function along with a multi-objective genetic algorithm for solving the redundancy allocation problem.

### II. LITERATURE REVIEW

This paper focuses on multi-objective optimization and multi-state reliability of a series-parallel RAP in which the subsystems are designed in series and the components in each subsystem are organized in parallel. The series-parallel system considered (Figure 2) has M subsystems in series (see Coit *et al.* [5] and Zhao *et al.* [18]). Further, the  $i^{th}$  subsystem consists of  $N_i$  active (operating) units organized in parallel. If any subsystem

fails, the entire system fails. Each block in the diagram represents a unit. Reliability allocation is an important step in the system design because it allows for the determination of the reliability of a vector of subsystems and components in order to obtain the desired overall reliability. For a system with identified cost, reliability, weight, volume, and other system parameters, the corresponding design problem becomes a combinatorial optimization problem (see Coit *et al.* [6] and Khorshidi *et al.* [8]). The best identified reliability design problem of this type is known as the redundancy allocation problem. This paper proposes multi-objective optimization using a hybrid genetic algorithm (HGA)-based optimization methodology for the redundancy allocation problem in order to find the number of redundant components that achieve the highest possible reliability while maintaining the lowest possible cost and weight under numerous resources. The proposed methodology uses a fuzzy function in combination with HGA to find the best possible solution for the redundancy allocation problem. The redundancy allocation problem is fundamentally a nonlinear integer programming problem. In most cases, it cannot be solved by direct, indirect, or mixed search methods because it involves separate search spaces. According to Chern [4], it is often difficult to find feasible solutions for redundancy allocation problems with multiple constraints. Such redundancy allocation problems are non-deterministic polynomial-time hard (NP-hard), and they have been discussed extensively by Chambari *et al.* [3], Kuo and Prasad [9], Liang *et al.* [11], Sharifi *et al.* [14], and Tillman *et al.* [16]. The penalty function is used in constrained problem optimization (see Smith and Coit [15], Kuri-Morales and Gutiérrez-Garcia [10], and Yeniay [17]). Some researchers have investigated evolutionary algorithms using statistical analysis (see François and Lavergne [7], Mills *et al.* [12], Castillo-Valdivieso *et al.* [2], Petrovski *et al.* [13], and Abatable and Sabuncuoglu [1]). Mahaparta and Roy [22] considered a multi-objective reliability optimization problem for system reliability, in which reliability enhancement involves several mutually conflicting objectives. In this paper, a new fuzzy multi-objective optimization method is introduced, and it is used for effective decision-making with regard to the reliability optimization of series and complex systems with two objectives. Salazar *et al.* [23] demonstrated a multi-objective optimization technique for solving three types of reliability optimization problems: determining the optimal number of redundant components (redundancy allocation problem), determining the reliability of components (component reliability problem), and determining both the redundancy and the reliability of components (redundancy allocation and component reliability problem) using nondominated sorting genetic algorithm II (NSGA-II). These problems were formulated as single objective mixed-integer nonlinear programming (MINLP) problems with one or several constraints and solved using mathematical programming techniques. Azaron *et al.* [24] used a genetic algorithm to solve a multi-objective discrete reliability optimization problem involving a non-repairable cold-standby redundant system with  $k$  dissimilar units. They employed a double string using continuous relaxation based on reference solution updating. Wang *et al.* [25] proposed RAP as a multi-objective optimization problem, in which the reliability of the system and the related designing cost are considered as two different objectives. They adopted NSGA-II to solve the multi-objective redundancy allocation problem (MORAP) under a number of constraints. Sahoo *et al.* [26] formulated four different multi-objective reliability optimization problems using interval mathematics and proposed order relations of interval-valued numbers. Then, these optimization problems were solved using advanced GA and the concept of Pareto optimality. Taboada and Coit [27] proposed a GA-based multi-objective evolutionary algorithm for reliability optimization of series-parallel systems. They considered three objective functions, namely system reliability, cost, and system weight, to solve RAP; however, they did not use a fuzzy function. In the next section, we present our methodology for solving RAP using HGA and a fuzzy function.

### III. METHODOLOGY FRAMEWORK

In our experiments, to implement the proposed optimization methodology, we adopted two penalty factors that have been considered by many researchers (Abatable and Sabuncuoglu [1], Castillo-Valdivieso *et al.* [2], François and Lavergne [7], Kuri-Morales and Gutiérrez-Garcia [10], Mills *et al.* [12], Petrovski *et al.* [13], Smith and Coit [15], and Yeniay [17]). We used a full factorial design with three levels. The fuzzy function allows the optimization algorithm to identify the solution of the redundancy problem that achieves the optimal trade-off between the optimization objectives from several optimal solutions. We performed 10 simulations for every experiment and used the best result of the 10 reliability values obtained. The best configuration of each point corresponding to the largest reliability value is given with the corresponding cost and weight values. The following assumptions are made in the optimization process:

- All the components  $r_{ij}$  have different values, and every branch has a different number of components in series and parallel.
- The failure rate of the components in each subsystem is constant.
- The failure rate depends on the number of working elements.
- The components are not repairable; they are changeable only.
- The subsystems have internal linking costs.

- The failed components do not damage the system.

Figure 1 shows the flowchart of the proposed algorithm. The HGA and fuzzy function procedures developed to implement our methodology are illustrated. The proposed method involves the following steps.

Step 1: Generate a population of random individuals.

Step 2: Initialize the front counter to 1.

Step 3: Check the termination condition. If the population is not classified, then identify nondominated individuals, assign large dummy fitness values to them, and to maintain diversity in the population, share these individuals with their dummy fitness values. After sharing, ignore these nondominated individuals temporarily. Then, identify the second nondominated front in the rest of the population and assign a dummy fitness value smaller than the minimum shared dummy fitness of the previous front. Then, increment the front counter by 1.

Step 4: Continue this process until the entire population is classified into several fronts. If the termination condition is satisfied, then reproduction occurs according to the dummy fitness.

Step 5: Use the crossover and mutation genetic operations to generate a new population.

Step 6: Check the termination condition of the proposed algorithm, i.e., if the current generation number is smaller than the maximum generation number, continue the process by going back to the second step until the objectives of the problem are met and increment gen by 1. If the current generation number is not smaller than the maximum generation number, then terminate the generation process. Otherwise, go to the next generation and implement the optimal front and fuzzy function; then, select the solution with the best trade-off and stop.

The flowchart follows the same steps as classical GAs except for the classification of nondominated fronts and the sharing operation. The sharing in each front is achieved by calculating the value of the sharing function between two individuals in the same front. This method is based on several layers of classification of the individuals. Nondominated individuals are assigned a certain dummy fitness value and are then removed from the population, and the process is repeated until the entire population has been classified. To maintain the diversity of the population, the classified individuals are shared (in decision variable space) with their dummy fitness values.

The multi-objective genetic algorithm is implemented using MATLAB<sup>®</sup> Optimization Toolbox<sup>™</sup>. First, MATLAB code that represents the fitness function and calculates the values of all the objectives (reliability, cost, and weight) is generated as an M-file. Because RAP is an integer problem, the creation, mutation, and crossover functions of the GA are adapted to generate integer populations that satisfy the problem constraints. The GA is implemented in our experimental procedure to determine the initial population size considering the following parameters:

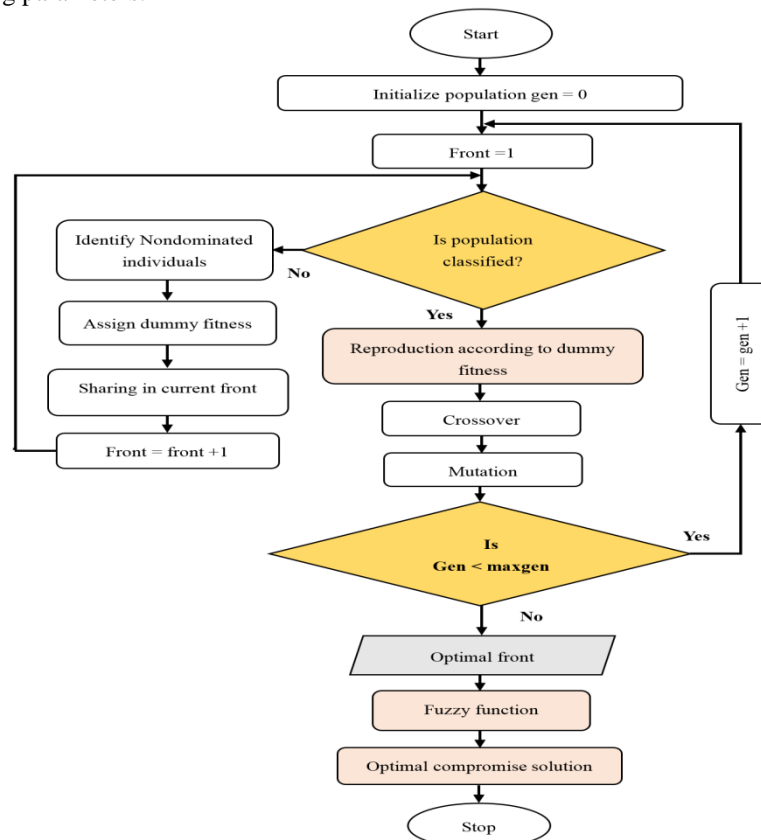


Fig. 1. Flowchart of the proposed algorithm.

- The population size in each generation is 1000, and the maximum number of iterations is 10000.
- We used 20 integers to code our chromosomes (maximum of 5 gear pairs and 4 stages).
- The value 6 from the configuration implies that this position is empty.
- We used 4 randomly generated crossover points corresponding to our 4 subsystems to improve our GA search.
- We could obtain better results by increasing the population size in order to enable the GA to search for additional points.
- However, when the population size is large, the GA will take a long time to calculate each generation.
- Finally, it is important to note that we set the population size to be at least the value of a number of variables such that the individuals in each population span the space being searched.

Optimizing the above-mentioned objective functions using a multi-objective genetic algorithm yields a set of solutions that are said to be nondominated or Pareto-optimal. Each of these solutions cannot be improved further without degrading one or more of the other objective values. The aim of the fuzzy function is to choose the optimal solution (trade-off) from the Pareto-optimal solutions. The corresponding linear fuzzy membership function value of the  $j^{th}$  objective function,  $\mu_j$ , is defined as (Brka *et al.* [21])

$$\mu_j = \begin{cases} 1 & F_j \leq F_j^{min} \\ (F_j^{max} - F_j)/(F_j^{max} - F_j^{min}) & F_j^{min} < F_j < F_j^{max} \\ 0 & F_j \geq F_j^{max} \end{cases} \quad (1)$$

Here, for the  $j^{th}$  objective functions,  $F_j$ , the minimum value is denoted as  $F_j^{min}$  and the maximum value is denoted as  $F_j^{max}$ , and  $j$  takes a value of 1, 2, or 3 because there are three objectives (reliability, cost, and weight). The normalized membership function  $\mu^k$  for each non-dominant solution is calculated as

$$\mu^k = \frac{\sum_{i=1}^{N_{obj}} \mu_j^k}{\sum_{k=1}^M \sum_{j=1}^{N_{obj}} \mu_j^k} \quad (2)$$

where  $N_{obj}$  is the number of objective functions and  $M$  is the number of non-dominated solutions.

#### IV. PROBLEM MODELING

We propose HGA-based multi-objective optimization using a fuzzy function for solving multi-state reliability and availability optimization design problems. Considering the system design, we require the simultaneous optimization of more than one objective function. In this optimization problem, there are three objectives: (1) maximizing the system reliability, (2) minimizing the system weight, and (3) minimizing the system cost while satisfying the system requirements. All the components and the system considered have a range of different states, and the fuzzy function technique is used to obtain the system availability. The notations used in our mathematical model for multi-objective optimization and multi-state reliability of RAP are summarized in Table 1.

**Table 1** Notations used in our mathematical model.

Abbreviations	Details
$R_s$	Total reliability of the series-parallel system
$C_s$	Total cost of the series-parallel system
$W_s$	Total weight of the series-parallel system
$C_{max}$	Limit of the cost constraint of the series-parallel system
$W_{max}$	Limit of the weight constraint of the series-parallel system
$s$	Number of subsystems in the system
$i$	Index of subsystem, $i \in (1, 2, \dots, s)$
$j$	Index of component type in each subsystem
$k$	Index of redundancy level
$m_i$	Total number of available component types in the $i^{th}$ subsystem
$P_i$	Minimum number of components in parallel required for the $i^{th}$ subsystem to function
$PN$	Maximum number of components in parallel that can be used in the $i^{th}$ subsystem (user-defined)
$N_i$	Set of component types, $N_i = [1, 2, \dots, m_i]$
$x_{ki}$	Number of component types assigned at position $k$ of the $i^{th}$ subsystem, $x_{ki} \in (1, 2, \dots, m_i, m_{i+1})$
$x$	System configuration matrix
$n_i(x)$	Total number of redundant components used in the $i^{th}$ subsystem
$n(x)$	Set of $n_i (n_1, n_2, \dots, n_s)$
$r_{ij}$	Reliability of the $j^{th}$ available component type in the $i^{th}$ subsystem
$c_{ij}$	Cost of the $j^{th}$ available component type in the $i^{th}$ subsystem
$w_{ij}$	Weight of the $j^{th}$ available component type in the $i^{th}$ subsystem

$R_i(x)$	Reliability components of the $i^{th}$ subsystem
$C_i(x)$	Total system cost of the $i^{th}$ subsystem
$W_i(x)$	Total weight of the $i^{th}$ subsystem

Based on the notations and basic assumptions, the following performance metrics (namely, system reliability, designing cost, and system weight) are defined.

(1) With regard to the system structure, the reliability of a series-parallel system ( $R_s$ ) can be calculated as

$$R_s(x) = \prod_{i=1}^s \left[ 1 - \prod_{k=1}^{PN} 1 - r_{ix_{ki}} \right] \quad (3)$$

where  $s$  is the number of subsystems in the system,  $PN$  is the maximum number of components that can be used in parallel in the  $i^{th}$  subsystem,  $r_i$  is the reliability of the  $j^{th}$  available component in the  $i^{th}$  subsystem, and  $x_{ki}$  is the number of component types allocated at position  $k$  of the  $i^{th}$  subsystem  $x_{ki} \in (1, 2, \dots, m_i, m_{i+1})$ .

(2) The probable total system design cost ( $C_s$ ) can be calculated as

$$C_s(x) = \sum_{i=1}^s C_i(x) = \sum_{i=1}^s \sum_{k=1}^{PN} C_{ix_{ki}} \quad (4)$$

where  $C_i$  is the cost of each available component in the  $i^{th}$  subsystem and  $x_{ki}$  is the number of component types allocated at position  $k$  of the  $i^{th}$  subsystem,  $x_{ki} \in (1, 2, \dots, m_i, m_{i+1})$ .

(3) Furthermore, we can calculate the weight of the system ( $W_s$ ) as

$$W_s(x) = \sum_{i=1}^s W_i(x) = \sum_{i=1}^s \sum_{k=1}^{PN} W_{ix_{ki}} \quad (5)$$

where  $W_i$  is the weight of each available component in the  $i^{th}$  subsystem and  $x_{ki}$  is the number of component types allocated at position  $k$  of the  $i^{th}$  subsystem,  $x_{ki} \in (1, 2, \dots, m_i, m_{i+1})$ . Multi-objective optimization refers to the solution of problems with two or more objectives to be satisfied simultaneously. Such objectives are often in conflict with each other and are expressed in different units. Because of their nature, multi-objective optimization problems usually have not one solution but a set of solutions, which are referred to as Pareto-optimal solutions or nondominated solutions (see Chankong *et al.* [19] and Hans [20]). When such solutions are represented in the objective function space, the graph obtained is called the Pareto front or the Pareto-optimal set. A general formulation of a multi-objective optimization problem consists of a number of objectives with a number of inequality and equality constraints.

The mathematical model of the problem studied herein is formulated as a multi-objective optimization problem as follows:

$$\text{Max } R_s(x) \quad (6)$$

$$\text{Min } C_s(x) \quad (7)$$

$$\text{Min } W_s(x) \quad (8)$$

Subject to

$$C_s(x) \leq C_{max} \quad (9)$$

$$W_s(x) \leq W_{max} \quad (10)$$

$$P_i \leq n_i \leq PN \quad \text{and} \quad (11)$$

$$\forall i, i = 1, 2, \dots, s \quad (12)$$

The first constraint is related to minimizing the system design cost ( $C_s$ ), while the second constraint is related to minimizing the system weight ( $W_s$ ).  $C_{max}$  and  $W_{max}$  are the upper bounds of  $C_s$  and  $W_s$ , respectively.

Figure 2 shows a typical example of a series-parallel system configuration with k-out-of-n subsystem reliabilities. The system is separated into  $s$  subsystems indicated by the index  $i$  ( $i = 1, 2, \dots, s$ ), and each subsystem consists of one or several components organized in parallel. Further,  $P_i$  is the minimum number of active components required for the  $i^{th}$  subsystem to function, i.e., the lower bound of the level of component redundancy for the  $i^{th}$  subsystem. The upper bound of the level of component redundancy for the  $i^{th}$  subsystem is denoted by  $PN$ . Thus, the system configuration can be defined as a  $PN \times s$  matrix. For this matrix, the column index  $i$  ( $i = 1, 2, \dots, s$ ) denotes the  $i^{th}$  subsystem, and the row index  $k$  ( $k = 1, 2, \dots, PN$ ) establishes the position where a component will be used in the subsystem. RAP involves defining the number of components of each type such that the total system reliability will be maximized considering the given constraints, such as cost and weight. The content of the case study is shown in Figure 3.

The objective of this test is to demonstrate the ability of the proposed algorithm in solving RAP as a gearbox reliability optimization problem, as shown by Zhao *et al.* [18], who assumed, in order to apply their method to all stages, that the minimum number of components is equal to 2 and the maximum number of

components is equal to 5. In their study, the problem is to decide how many gear pairs and what types of gear pairs are to be selected for use in each stage, which will give the maximum reliability of the gearbox while minimizing both the system cost and the system weight. Because it is assumed that all the gear pairs are active components in each stage, the gearbox is analogous to a series-parallel system with k-out-of-n G subsystems.

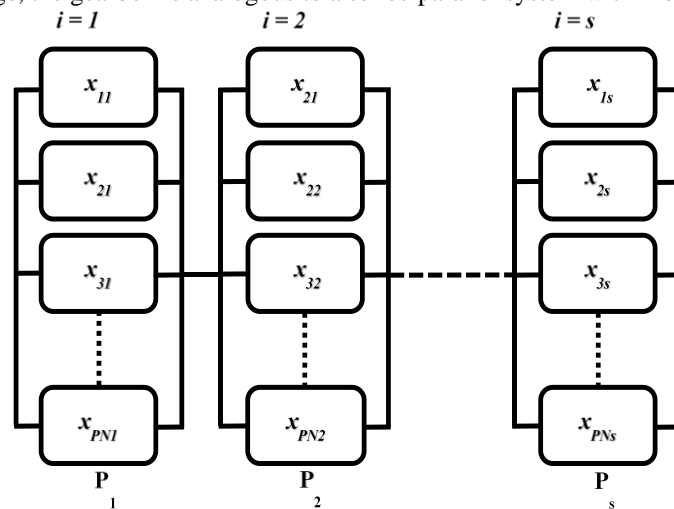


Fig. 2. Series-parallel system.

V. GEARBOX CASE STUDY

Table 2 summarizes the input data of component reliability, cost, and weight characteristics for gear pairs in each stage for reliability optimization of the series-parallel systems considered in this problem. The study is based on work conducted previously by Zhao *et al.* [18]; however, they considered only one objective. Our system consists of 4 subsystems, and each subsystem has a different design component type with similar or dissimilar characteristics, such as reliability, cost, weight, material, dimension, and transmission ratio. Here, we set  $P_i = 2$  and  $PN = 5$  in the gearbox for all stages. Each of the subsystems is represented by  $PN$  positions, with each component listed according to its reliability index. The objective is to maximize the system reliability with k-out-of-n subsystems connected in the series-parallel system under the given constraints. Table 3 lists the values of  $C_{max}$  and  $W_{max}$ . The equivalent scheme of this system is shown in Figure 4.

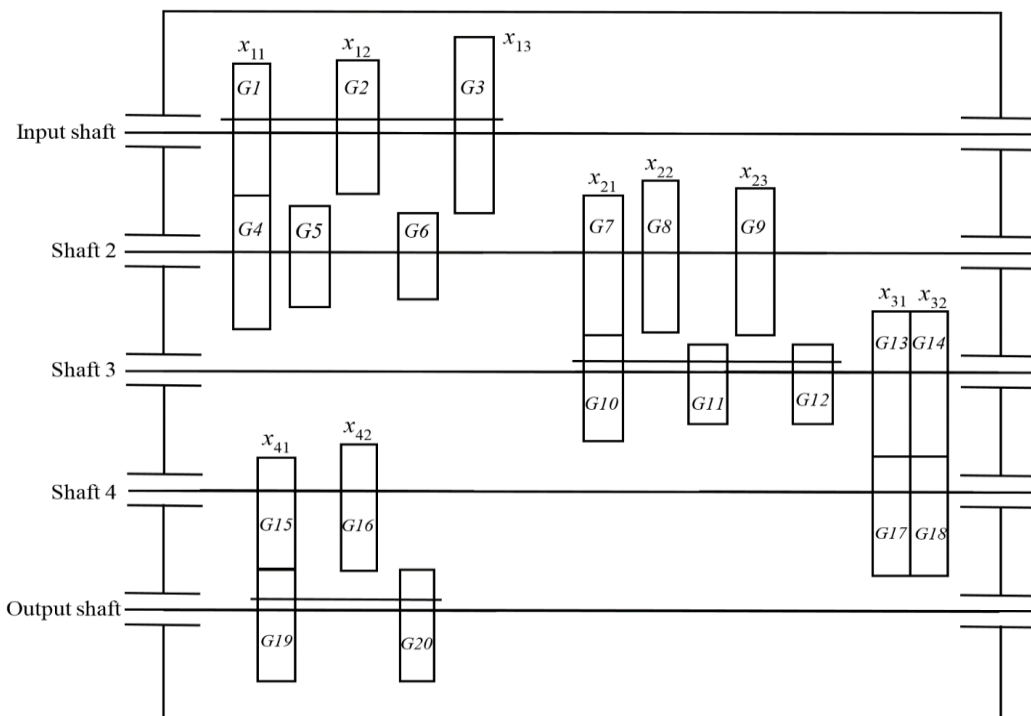


Fig. 3. Modeling of gear train system of series-parallel system.

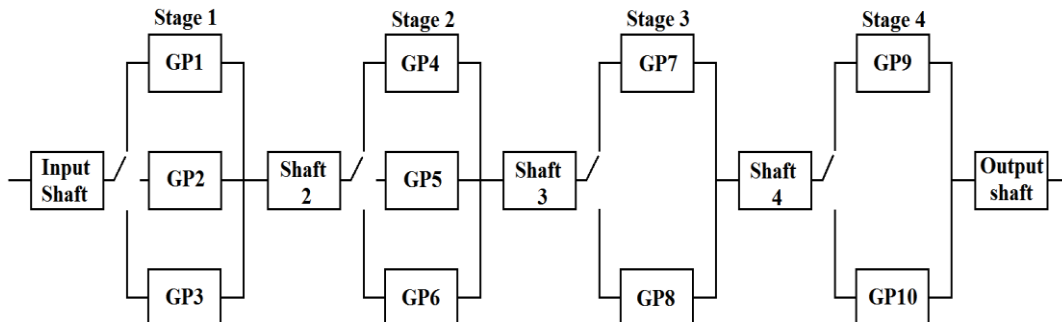


Fig. 4. Equivalent scheme for gear train system.

Table 2 Input data for RAP (Zhao et al. [18]).

Gear pair	Stage											
	Stage no 1			Stage no 2			Stage no 3			Stage no 4		
	r1	c1	w1	r2	c2	w2	r3	c3	w3	r4	c4	w4
1	0.855	3	11	0.743	5	9	0.828	9	15	0.74	6	10
2	0.706	5	12	0.882	6	11	0.842	7	14	0.922	5	10
3	0.931	5	9	0.874	2	14	0.779	7	11	0.855	11	15
4	0.737	7	11	0.783	7	11	0.911	7	12	0.864	9	13
5	0.805	6	14	0.9114	5	7	0.846	3	11	0.816	9	12

In Figure 4, let  $G1, G2, G3, G4, \dots, G20$  represent the number of teeth of each gear. For each stage, the following equations are applicable:

$G1 + G4 = G2 + G5 = G3 + G6$  (for stage 1 between input shaft 1 and shaft 2).

$G7 + G10 = G8 + G11 = G9 + G12$  (for stage 2 between shaft 2 and shaft 3).

$G13 + G17 = G14 + G18$  (for stage 3 between shaft 3 and shaft 4).

$G15 + G19 = G16 + G20$  (for stage 4 between shaft 4 and output shaft).

GP1: Gear pair [G1-G4], GP2: Gear pair [G2-G5], GP3: Gear pair [G3-G6], GP4: Gear pair [G7-G10], GP5: Gear pair [G8-G11], GP6: Gear pair [G9-G12], GP7: Gear pair [G13-G17], GP8: Gear pair [G14-G18], GP9: Gear pair [G15-G19], GP10: Gear pair [G16-G20].

Table 3 System constraint values used.

Maximum constraint limit of cost and weight					
No.	$C_{max}$	$W_{max}$	No.	$C_{max}$	$W_{max}$
1	40	115	10	65	150
2	55	125	11	70	120
3	65	130	12	70	130
4	60	120	13	70	140
5	60	130	14	70	150
6	60	140	15	75	120
7	60	150	16	75	130
8	65	120	17	75	140
9	65	140	18	75	150

## VI. RESULTS AND DISCUSSION

In this study, we perform multi-objective optimization of a combinatorial redundancy allocation problem for a series-parallel system to solve the formulated reliability optimization multi-objective genetic algorithm (ROMO GA). The reliability optimization design using a multi-objective genetic algorithm for the redundancy allocation problem is presented to determine optimal solutions, where  $k$  ( $k$ -out-of- $n$ ) influences the cost function in series-parallel systems with multiple  $k$ -out-of- $n$  subsystems. The objectives are to maximize system reliability and minimize system cost and system weight subject to cost and weight constraints. The constrained  $k$  values are considered for all subsystems; some subsystems may require more than one component to function, and the component types are also considered for each subsystem. By using a multi-objective genetic algorithm for solving optimization problems, we can obtain a number of optimal solutions constituting the Pareto-optimal set, and out of these solutions, we can evaluate the best one using an appropriate decision-making technique. The multi-objective optimization methodology is adopted to solve the RAP. Figure 5 shows the set of nondominated solutions for the last iteration of the optimization process, where  $C_{max} = 40$  and  $W_{max} = 115$ . Each point in this figure represents an individual solution that has an optimal value of one

objective function, and it cannot be improved further without deteriorating at least one of the other objectives. The fuzzy function is employed to define the solution that guarantees an optimal trade-off between the three objectives, and the result is shown in Figure 5. The employment of the fuzzy function guarantees consistency and optimality of the selected solution. Figure 6 shows the convergence between reliability, cost, and weight.

The optimal trade-off solution shown in Figure 7 is [1, 6, 6, 1, 1, 6, 3, 5, 5, 6, 6, 6, 5, 5, 5, 6, 2, 2, 6, 6], and the number of components of each stage of the series-parallel system varies from 2 to 5. Therefore, from the 20 positions, the results are illustrated as follows:

- In the first subsystem, there are 3 components of type 1.
- In the second subsystem, there are 2 components of type 5 and 1 component of type 3.
- In the third subsystem, there are 3 components of type 5.
- In the fourth subsystem, there are two components of type 2.

It can be seen that the proposed algorithm is able to obtain a set of uniformly distributed solutions along the Pareto front, as shown in Figure 8.

Thus, a new hybrid metaheuristic genetic algorithm and fuzzy function have been successfully demonstrated in this study. Table 4 lists the optimal trade-off solutions obtained when different values of the optimization constraints are chosen. From this table, it can be seen that our approach is able to find system configurations with lower cost and weight without significantly degrading the overall reliability.

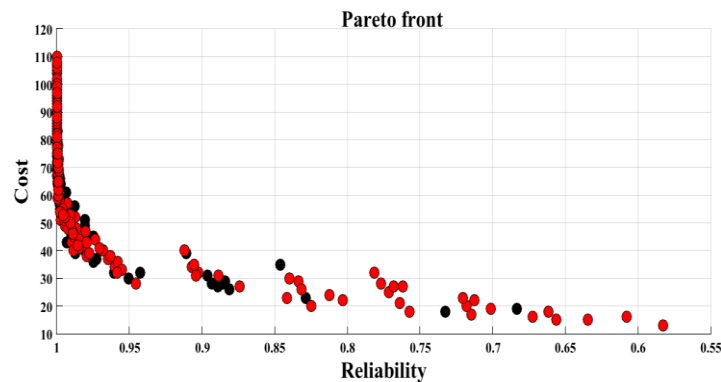


Fig. 5. Overall best Pareto front obtained by multi-objective optimization and fuzzy function: cost vs. reliability.

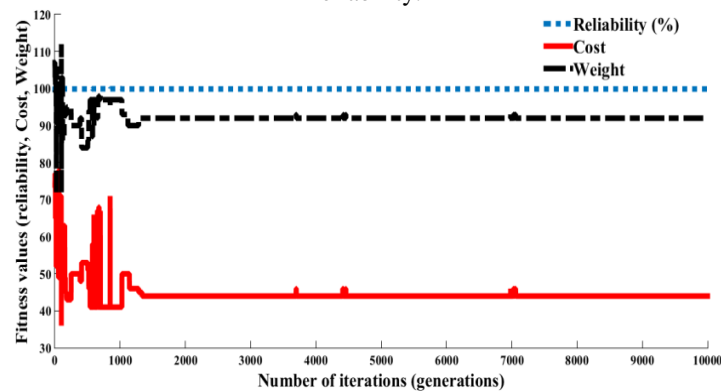


Fig. 6. Convergence of reliability, cost, and weight.

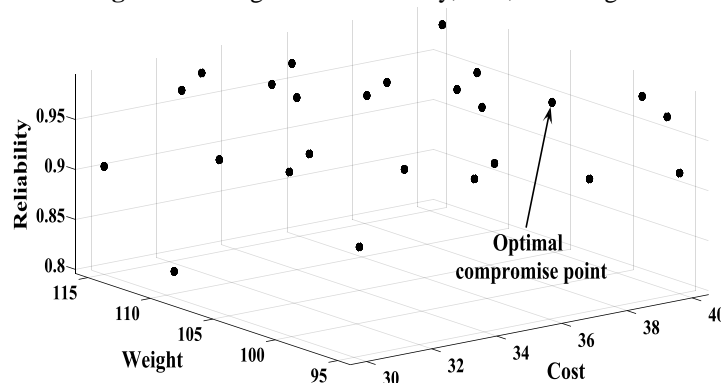


Fig. 7. Optimal trade-off point for reliability vs. weight vs. cost in 3D space.



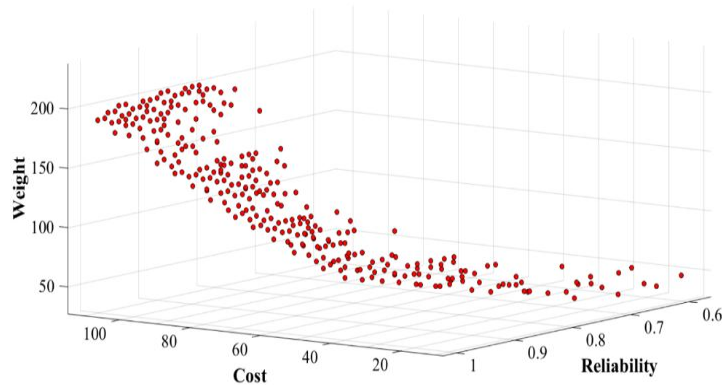


Fig. 8. Nondominated solutions obtained from the proposed algorithm for weight vs. cost vs. reliability in 3D space.

Table 4 Optimization results for different cost and weight constraints.

No.	C <sub>max</sub>	W <sub>max</sub>	Results obtained via multi-objective optimization of multi-state reliability system using HGA and fuzzy function																						
			Best configuration of 10 simulations										Reliability	Cost	Weight										
1	40	115	1	6	6	1	1	6	3	5	5	6	6	6	5	5	5	6	2	2	6	6	0.9863	40	114
2	55	125	3	1	3	6	6	5	6	5	6	5	5	6	5	5	5	2	6	6	2	2	0.9976	55	124
3	65	130	6	3	6	3	3	5	5	6	5	5	5	5	6	5	5	2	2	2	6	6	0.9986	62	129
4	60	120	3	3	6	3	6	5	6	3	6	5	6	6	4	5	4	6	2	6	2	2	0.9970	59	120
5	60	130	3	6	3	6	3	6	5	5	5	5	6	5	5	6	5	6	2	2	6	2	0.9979	57	122
6	60	140	6	3	6	3	3	5	6	3	5	5	5	6	5	5	5	2	6	2	2	6	0.9985	59	136
7	60	150	1	3	3	6	1	5	5	5	3	6	5	5	5	5	6	2	6	2	6	2	0.9988	60	149
8	65	120	1	6	3	6	3	5	6	5	5	6	6	4	4	6	4	2	6	2	2	6	0.9974	64	116
9	65	140	3	3	6	6	3	5	5	5	6	5	5	5	5	5	6	6	6	2	2	2	0.9990	65	140
10	65	150	3	3	6	6	3	5	5	5	5	6	5	5	5	5	5	6	6	2	2	2	0.9990	65	140
11	70	120	3	6	3	3	6	5	5	6	5	6	4	6	4	6	4	6	2	6	2	2	0.9978	66	114
12	70	130	6	6	3	3	3	5	5	5	6	5	5	5	5	6	4	6	2	2	2	6	0.9988	66	130
13	70	140	3	3	3	6	6	5	5	6	5	5	5	5	5	5	5	6	2	2	2	6	0.9990	65	140
14	70	150	3	6	3	3	6	5	5	5	6	5	5	5	5	5	5	6	2	2	2	2	0.9995	70	150
15	75	120	6	3	3	6	3	5	6	5	5	5	5	6	6	4	4	2	6	6	2	2	0.9979	67	120
16	75	130	3	6	3	6	3	5	6	5	5	5	5	5	6	5	4	2	2	2	6	6	0.9988	66	130
17	75	140	6	3	3	3	3	6	5	5	5	5	4	4	5	6	5	6	6	2	2	2	0.9993	75	140
18	75	150	3	3	3	6	6	5	5	5	5	6	5	5	5	5	5	6	2	2	2	2	0.9995	70	150

VII. CONCLUSION

In this study, we proposed multi-objective optimization of a multi-state reliability system for an RAP involving a series-parallel system, based on a genetic algorithm and fuzzy function. Unlike other methodologies, our methodology not only optimizes the cost, weight, and reliability of the system simultaneously but also objectively defines the system configuration that achieves the optimal trade-off between the design objectives. The results showed that our methodology can find better solutions in terms of cost and weight without significantly degrading the overall reliability. The computational results confirmed the robustness of the proposed algorithm and highlighted its potential for future application.

In the future, the proposed technique may be adopted for solving real-life decision-making problems in the form of interval-valued constrained optimization problems. In addition, it can be applied to various areas of engineering, management, and manufacturing.

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