American Journal of Engineering Research (AJER)
e-ISSN: 2320-0847 p-ISSN : 2320-0936
Volume-6, Issue-8, pp-01-07
www.ajer.org
Research Paper Open Access

Comparison on Fourier and Wavelet Transformation for an ECG Signal

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ABSTRACT: *Wavelet analysis is a new method for solving difficult problems in mathematics, physics, and engineering, with modern applications as diverse as wave propagation, data compression, signal processing, image processing, pattern recognition, computer graphics, the detection of aircraft and submarines and other medical image technology. Wavelets allow complex information such as music, speech, images and patterns to be decomposed into elementary forms at different positions and scales and subsequently reconstructed with high precision. Signal transmission is based on transmission of a series of numbers. The series representation of a function is important in all types of signal transmission. The wavelet representation of a function is a new technique. Wavelet transform of a function is the improved version of Fourier transform because Fourier transform is a powerful tool for analyzing the components of a stationary signal. But it is failed for analyzing the non stationary signal where as wavelet transform allows the components of a non-stationary signal to be analyzed. In this study our main goal is to compare an ECG signal for Fourier transformation and Wavelet transformation.*

Keywords: *ECG Signal, Fourier transformation, Wavelet transformation, Haar Wavelet transform.*

Date of Submission: 19-07-2017 Date of acceptance: 01-08-2017

I. INTRODUCTION

Historically, the concept of "Ondelettes" or "Wavelets" started to appear more frequently only in theearly 1980's.One of the main reasons for the discovery of wavelets and wavelet transforms is that the Fourier transform does not contain the local information of signals. So the Fourier transform cannot be used for analyzing signals in a joint time and frequency domain. In 1982, Jean Morlet, in collaboration with a group of French engineers, first introduced the idea of wavelets as a family of functions constructed by using translation and dilation of a single function, called the mother wavelet, for the analysis of nonstationary sgnals. However this new concept can be viwed as the synthesis of various ideas originating from different disciplines including mathematics (Calderon-Zygmund operators and Littlewood-Paley theory), physics (coherent states in quantum mechanics and the renormalization group), and engineering (quadratic mirror filters, sideband coding in signal processing, and pyramidal algorithms in image processing)[1,2].

The Wavelets conference series was started by Andrew Laine in 1993. Under the leadership of Laine, Michael Unser, and Akram Aldroubi, it grew to be the leading venue for the dissemination of research on wavelets and their applications. Manos Papadakis replaced Akram Aldroubi as a conference chair in 2005, and since then the remainder of the leadership team has turned over, with the addition of Dimitri Van De Ville and Vivek Goyal[5,6]. Wavelets are mathematical functions that cut up data into different frequency components and then study each component with a resolution matched to its scale. They have advantages over traditional Fourier methods in analyzing physical situations where the signal contains discontinuities and sharp spikes. The wavelet representation of a function is a new technique and it does not loss time information.

Keeping these things in mind, our main goal in this thesis has been to provide both a systematic exposition of the basic ideas and results of wavelet transforms and some applications in time-frequency signal analysis.

II. FOURIER TRANSFORM

The Fourier transform is probably the most widely applied signal processing tool in science and engineering. It reveals the frequency composition of a time series $x(t)$ by transforming it from the time domain into the frequency domain. In 1807, the French mathematician Joseph Fourier found that any periodic signal can be presented by a weighted sum of a series of sine and cosine functions. However, because of the uncompromising objections from some of his contemporaries such as J. L. Lagrange (Herivel 1975), his paper on this finding never got published, until some 15 years later, when Fourier wrote his own book, The Analytical Theory of Heat (Fourier 1822). In that book, Fourier extended his finding to periodic signals, stating that an periodic signal can be represented by a weighted integral of a series of sine and cosine functions [5]. Such an integral is termed the Fourier transform. Using the notation of inner product, the Fourier transform of a signal can be expressed as

$$X(f) = \langle x, e^{i2\pi ft} \rangle = \int_{-\infty}^{\infty} x(t) e^{i2\pi ft} \, dt \qquad (1)$$

Assuming that the signal has finite energy,

$$\int_{-\infty}^{\infty} \left| x(t) \right|^2 dt \langle \infty \qquad (2)$$

Accordingly, the inverse Fourier transform of the signal $x(t)$ can be expressed as

$$x(t) = \int_{-\infty}^{\infty} X(f) e^{i2\pi ft} \, df \qquad (3)$$

Signals obtained experimentally through a data acquisition system are generally sampled at discrete time intervals ΔT , instead of continuously, within a total measurement time T. Such a signal, defined as x_k , can be transformed into the frequency domain by using the discrete Fourier transform (DFT), defined as

$$DFT\ (f_n) = \frac{1}{N} \sum_{k=0}^{N=1} x^k e^{i2\pi f_n k\Delta T} \qquad (4)$$

Every signal can be written as a sum of sinusoids with different amplitudes and frequencies.

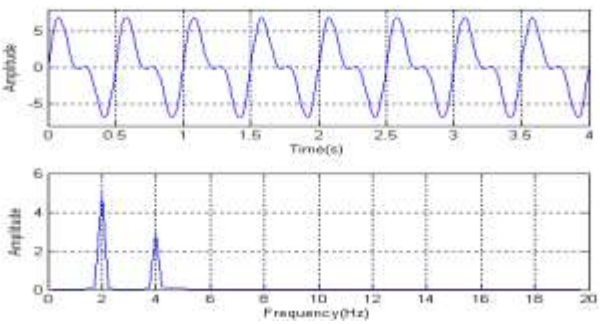

Fig.1: Discrete Fourier transforms to transform data from time into the frequency domain

III. WAVELET TRANSFORM

A wave is usually defined as an oscillating function of time or space, such as a sinusoid. Fourier analysis is wave analysis. It expands signals or functions in terms of sinusoids (or, equivalently, complex exponentials) which has proven to be extremely valuable in mathematics, science, and engineering, especially for periodic, time-invariant, or stationary phenomena [7]. A wavelet is a "small wave", which has its energy concentrated in time to give a tool for the analysis of transient, non-stationary, or time-varying phenomena. It still has the oscillating wave-like characteristic but also has the ability to allow simultaneous time and frequency analysis with a flexible mathematical foundation.

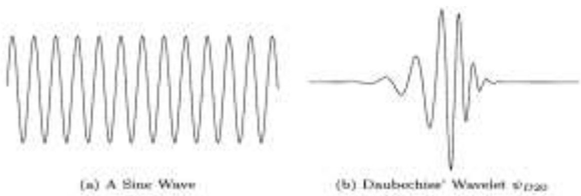

Fig.2: A Wave and A Wavelet

The wavelet means small waves and in brief, a wavelet is an oscillation that decays quickly. Equivalent mathematical conditions for wavelet are:

$$i)\ \int_R |\psi(x)|^2 dx < \infty$$

$$ii)\ \int_R \psi(x) dx = 0$$

$$iii)\ \int_R \frac{|\hat{\psi}(\omega)|^2}{|\omega|} d\omega < \infty$$

where $\hat{\psi}(\omega)$ is the Fourier Transform of $\psi(x)$.

IV. HAAR WAVELET TRANSFORM

The Hungarian mathematician Alfred Haar first introduced the Haar function in 1909 in his Ph.D. thesis.
A function defined on the real line $\Re$ as

$$\psi(t) = \begin{cases} 1 & for\ \ t \in \left[0, \frac{1}{2}\right) \\ -1 & for\ \ t \in \left[\frac{1}{2}, 1\right) \\ 0 & otherwise \end{cases} \tag{5}$$

is known as the Haar function.
The Haar function $\psi(t)$ is the simplest example of a Haar Wavelet. The Haar function $\psi(t)$ is a wavelet because it satisfies all the conditions of wavelet. This fundamental example has all the major features of the general wavelet theory. Haar wavelet is discontinuous at $t = 0, \frac{1}{2}, 1$ and it is very well localized in the time domain [12].
The Fourier transform of $\psi(t)$ is given by

$$\hat{\psi}(\omega) = i \exp\left(-\frac{i\omega}{2}\right) \frac{\sin^2\left(\frac{\omega}{4}\right)}{\frac{\omega}{4}}$$

$$\therefore\ \mathrm{Re}\{\hat{\psi}(\omega)\} = \sin\left(\frac{\omega}{2}\right) \frac{\sin^2\left(\frac{\omega}{4}\right)}{\frac{\omega}{4}}.$$

The graphs of Haar wavelet $\psi(t)$ and its Fourier transform $\hat{\psi}(\omega)$ are shown in Fig.3 (a) and Fig.3 (b).

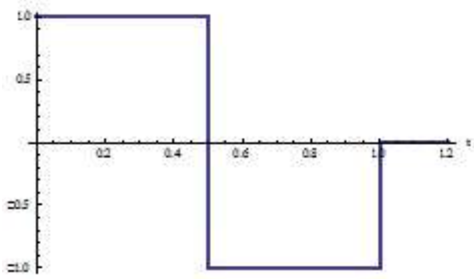

Fig.3 (a): Haar wavelet

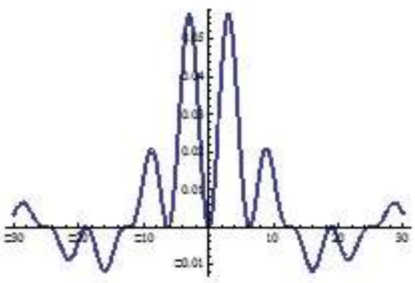

Fig.3 (b): Fourier transform of Haar wavelet

4.1 Haar Scaling Function
The Haar scaling function can be defined as

$$\varphi(t) = \aleph_{[0,1)}(t) = \begin{cases} 1 & if \quad 0 \le t < 1 \\ 0 & otherwise \end{cases}.$$

4.2 Haar Wavelet Function
Haar wavelet function ψ can be written as

$$\psi(t) = \aleph_{\left[0,\frac{1}{2}\right)}(t) - \aleph_{\left[\frac{1}{2},1\right)}(t).$$

4.3 Haar Wavelet Series and Wavelet Co-efficients
If f is defined on [0, 1], then it has an expansion in terms of Haar functions as follows. Given any integer $J \ge 0$,

$$\begin{aligned} f(t) &= \sum_{k=0}^{2^J-1} \langle f, \varphi_{J,K} \rangle \varphi_{J,K}(t) + \sum_{j=J}^{\infty} \sum_{k=0}^{2^j-1} \langle f, \varphi_{j,k} \rangle \varphi_{j,k}(t) \\ &= \sum_{k=0}^{2^J-1} c_{J,K}\, \varphi_{J,K}(t) + \sum_{j=J}^{\infty} \sum_{k=0}^{2^j-1} d_{j,k}\, \psi_{j,k}(t) \end{aligned} \tag{6}$$

The series (6) is known as the Haar wavelet series for f. $d_{j,k}$ *and* $c_{j,k}$ are known as the Haar wavelet co-efficients and the Haar scaling co-efficients respectively.

4.4 Advantages of Haar Wavelet Transform
The Haar wavelet transform has a number of advantages:
- It is conceptually simple.
- It is fast.
- It is memory efficient, since it can be calculated in place without a temporary array.
- It is exactly reversible without the edge effects that are a problem with other wavelet transforms.

4.5 Comparison on Wavelet Transform and Fourier Transform
The wavelet transform is often compared with the Fourier transform. Fourier transform is a powerful tool for analyzing the components of a stationary signal (a stationary signal is a signal where there is no change the properties of signal). For example, the Fourier transform is a powerful tool for processing signals that are composed of some combination of sine and cosine signals (sinusoids).

The Fourier transform is less useful in analyzing non-stationary signal (a non-stationary signal is a signal where there is change the properties of signal). Wavelet transforms allow the components of a non-stationary signal to be analyzed.

The main difference is that wavelets are well localized in both time and frequency domain whereas the standard Fourier transform is only localized in frequency domain. The Short-time Fourier transform (STFT) is also time and frequency localized but there are issues with the frequency time resolution and wavelets often give a better signal representation using Multiresolution analysis .Fourier transform is based on a single function $\psi(t)$ and that this function is scaled. But for the wavelet transform we can also shift the function, thus generating a two-parameter family of functions $\psi_{a,b}(t)$.

V. SIGNAL
A Signal is a discrete part of a communication. Otherworld's a signal is a Detectable transmitted energy that can be used to carry information.

There are many other definitions in electronics, telephony, information technology and many other communication technologies which are given below:

1) In **electronics**, a signal is an electric current or electromagnetic field used to convey data from one place to another. The simplest form of signal is a direct current (DC) that is switched on and off; this is the principle by which the early telegraph worked. More complex signals consist of an alternating-current (AC) or electromagnetic carrier that contains one or more data streams [11].

Data is superimposed on a carrier current or wave by means of a process called modulation. Except for DC signals such as telegraph and baseband, all signal carriers have a definable frequency or frequencies. Signals also have a property called wavelength, which is inversely proportional to the frequency.

2) In some **information technology** contexts, a signal are simply "that which is sent or received," thus including both the carrier and the data together.
3) In **telephony**, a signal is special data that is used to set up or control communication.
At last we can say that, A sign made for the purpose of giving notice to a person of some occurrence, command, or danger; also, a sign, event, or watchword, which has been agreed upon as the occasion of concerted action. The term "signal" refers to a Physical quantity that carries certain type of information and serves as a means for communication.

5.1 De-noising Methods
5.1.1 Wavelet Based De-noising
The general wavelet de-nosing procedure is as follows:
- Apply wavelet transform to the noisy signal to produce the noisy wavelet coefficients to the level which we can properly distinguish the PD occurrence. Select appropriate threshold limit at each level and threshold method (hard or soft thresholding) to best remove the noises.
- Inverse wavelet transforms of the threshold wavelet coefficients to obtain a de-noised signal.

5.2 Thresholding Methods
These methods use a threshold and determine the clean wavelet coefficients based on this threshold. There are two main ways of thresholding the wavelet coefficients, namely
(a) Soft-Thresholding Method.
(b) Hard-Thresholding Method.

5.2.1 Hard Thresholding Methods
If the absolute value of a coefficient is less than a threshold, then it is assumed to be 0, otherwise it is unchanged. Mathematically it is

$$\dot{X} = sign(Y)(Y.*(abs(Y)\rangle\lambda)$$

where Y represents the noisy coefficients is the threshold, $\dot{X}$ represents the estimated coefficients.

5.2.2 Soft Thresholding Methods
Hard thresholding is discontinuous. This causes ringing/Gibbs effect in the de-noised image. To overcome this, Donoho introduced the soft thresholding method.
If the absolute value of a coefficient is less than a threshold λ , then is assumed to be 0, otherwise its value is shrunk by λ . Mathematically it is

$$\dot{X} = sign(Y)(Y.*((abs(Y)\rangle\lambda).*(abs(Y)-\lambda)$$

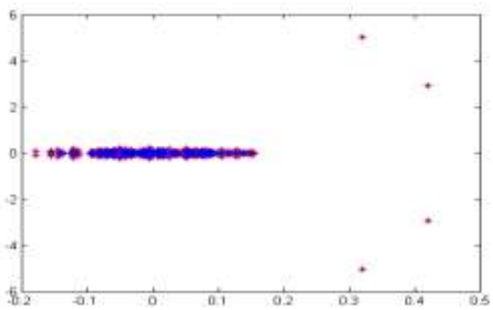
Fig.4(a): Noisy Signal

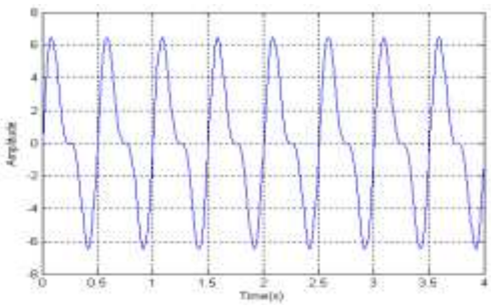
Fig.4 (b): Signal de-noising with the help of Fourier transformation

VI. ECG SIGNAL
Heart diseases, which are one of the death reasons of men/women, are among the important problems on this century. Early diagnosis and medical treatment of heart diseases can prevent sudden death of the patient. One of the ways to diagnose heart diseases is to use electrocardiogram (ECG) signals.ECG is a bioelectric signal.

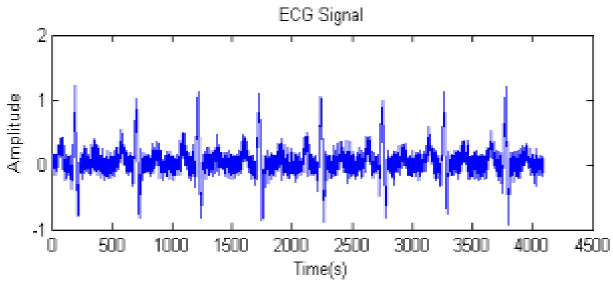

Fig.5: An ECG signal

Comparison between Fourier Transform and Wavelet Transform to de-noising ECG signal:

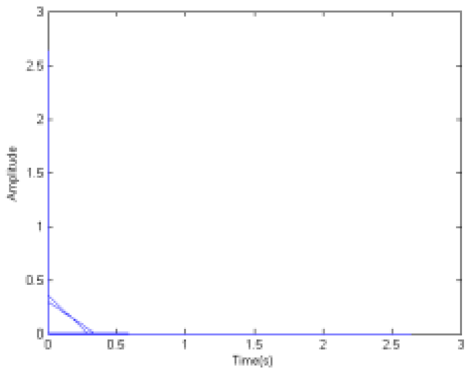

Fig.6 (a): Graph of Fourier Transform

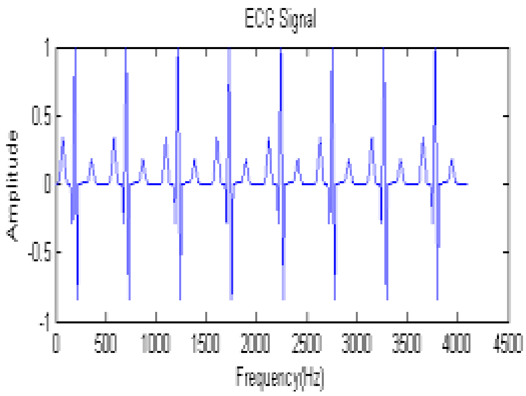

Fig.6 (b): Applying Fourier Transform to de-noising of the given ECG Signal

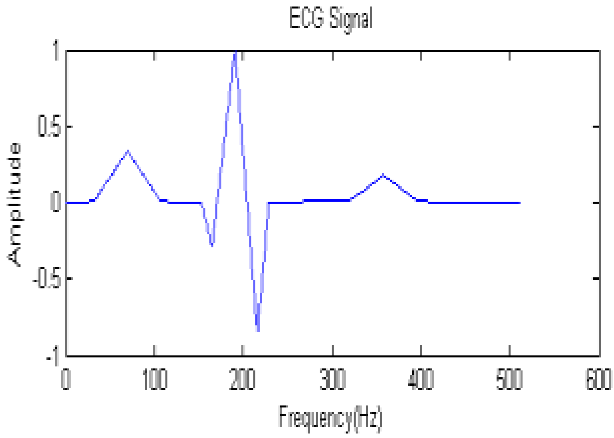

Fig.7 (a): Graph of an ECG signal using Haar Wavelet

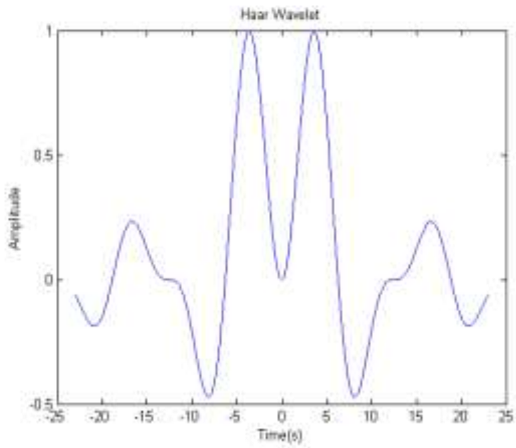

Fig.7 (b): Applying Wavelet Transform to de-noising of the given ECG Signal

VII. CONCLUSION

In our study we have tried to mention the drawback of Fourier transforms, besides this we have discussed the advantages of wavelet transform. From our above discussion it is clear that wavelet transform is much more efficient than that of Fourier transform in signal analysis. It can be concluded that the wavelet transform provides a powerful mathematical tool for the analysis and characterization of signals. The adaptive, multiresolution capability of the wavelet transform makes it well suited for decomposing signals and de-noising signals for dynamical structure. Such capability makes the wavelet transformation better in present world.

In this study a wavelet basis function applied to de-noising of an ECG signal has been carried out. The experimental results have revealed suitability of difference between Fourier transom and Wavelet basis transform for the de-noising ECG signal.

ACKNOWLEDGEMENT

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REFERENCES

[1]. Stephane Mallat. A Wavelet tour of Signal Processing with contributions from Gabriel Peyré (Third edition).
[2]. A. Akansu and Y. Liu. On signal decomposition techniques.Opt. Eng., 30:912920, 1991.
[3]. Charles K.Chu. Wavelet Analysis and its Applications:Texas A&M University, College Station, Texas.
[4]. R.K.Rao YarlagaddaAnalog and Digital Signals and Systems.
[5]. CS. Burrus, R.A. Gopinath, and H. Guo. Introduction to Wavelets and Wavelet Transforms. A Primer.Prentice-Hall, Upper Saddle River, 1997.
[6]. D.Lee Fugal. Conceptual Wavelets in Digital Signal Processing.
[7]. Shoab Ahmed Khan.Digital Design of Signal Processing Systems.
[8]. Li Tan. Digital Signal Processing Fundamentals and Applications.
[9]. C.Sidney Burrus Ramesh A.Gopinath and Haitao Guo. Introduction to Wavelets and Wavelet Transforms.
[10]. M.M Konstantinov. Advanced Textbooks in Control and Signal Processing.
[11]. A. Cohen. Numerical Analysis of wavelet methods, Elsevier, 2003.
[12]. R.R. Coifman, Y. Meyer and M.V. Wickerhauser. Size properties of wavelet packets, (1992), pp. 453-470.
[13]. A. Cohen and I. Daubechies. Non-separable bidimensional wavelet bases.
[14]. Desanka P.Radunovic, Ph.D.Wavelets from Math to Practice.
[15]. Robert X.Gao, Ruqiang Yan.Wavelets Theory and Applications for Manufacturing.
[16]. Jonathan Cohen. Wavelets and Multiscale Analysis Theory and Applications.

Mahamudul Hassan Milon. "Comparison on Fourier and Wavelet Transformation for an ECG Signal." American Journal of Engineering Research (AJER) 6.8 (2017): 01-07.