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Fuzzy Inventory Model for Deteriorating Items in a Supply Chain System with Price Dependent Demand and Without Backorder

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Abstract: This paper presents a fuzzy continuous review inventory model for deteriorating items in a supply chain management system with price dependent demand. In reality it is seen that, the cycle time of almost every supply chain system is uncertain, so we describe it as symmetric triangular fuzzy number. Signed distance method is used to defuzzify the cost function. To illustrate the proposed model a numerical example and sensitivity analysis with respect to different associated parameters has been presented.

Keywords: Supply chain management, Price dependent demand, Triangular Fuzzy number (symmetric), Signed distance method.

I. INTRODUCTION:

Inventory control plays an important role in any supply chain system. The main objective of maintaining an inventory is to provide a cushion between supply and demand for smooth and efficient running of the supply chain's operation. In any production inventory system, we face uncertainty associated with different parameters such as demand, raw materials supply, various relevant costs, rate of deterioration etc. To solve these types of problems we use fuzzy set theory. Bellman and Zadeh (1) first introduced fuzzy set theory for solving decision making problems. Thereafter, Dubois and Prade(2) developed some operations on fuzzy numbers. Park (3) presented fuzzy set theoretical interpretation of economic order quantity. Wu and Yao (4) studied fuzzy inventory with backorder for fuzzy order quantity and fuzzy shortage quantity. X. Wang, Tang, and Zhao (5) worked on fuzzy economic order quantity inventory model without backordering. Jinsong Hu et al. developed fuzzy economic quantity model with imperfect quality and service level (6). Researchers have developed inventory models assuming demand as constant, time dependent, stock dependent or price dependent. Silver and Meal, in 1973,(7) published a lot size model taking time varying demand. After that, the model with time dependent demand has been studied by several other researchers (8-12). Various authors have investigated price dependent inventory model (13-18). Other related analyses on inventory systems with stock-dependent consumption rate have been performed by Sarker, Mukherjee, and Balan (1997), Datta and Paul (2001), Goh (1992), S. Pal, Goswami, and Chaudhuri (1993), Bar-Lev, Parlar, and Perry (1994), Urban (1995), Mandal and Maiti (1999), Giri and Chaudhuri (1998) (19-26). In real world problem, deterioration is a natural and common phenomenon. There are some physical goods which deteriorate with time during their normal storage. In this area, a lot of research papers have been published by several researchers viz., Wee (1993), Liu (1999), T.-Y. Wang and Chen (2001), A. K. Pal, Bhunia, and Mukherjee (2006), Bera, Bhunia, and Maiti (2013), He, Wei, and Fuyuan (2013), Dutta and Kumar (2015), Mishra et al. (2015 (33) etc. (12, 27-34). In most models, the cycle time is considered as constant. But realistically it is seen that we cannot predict the cycle time in prior. So keeping in mind this real life situation, we consider the cycle time as uncertain and describe it as triangular fuzzy number (symmetric). The rest of this paper is organized as follows. In section 2, the assumption and notations are given. In section 3, we have developed the mathematical models. In section 4, we provided numerical examples to illustrate the results. In addition, the sensitivity analysis of the optimal solution with respect to parameters of the system is carried out in section 5. Finally, we draw the conclusions in section 6 and provided the references in section 7.

II. ASSUMPTIONS AND NOTATIONS:

Assumptions:

i) Demand is selling price dependent and is of the form $D(p) = ap^{-b}$, where a, b > 0 and p is the selling price.

ii) The rate of deterioration is constant.

iii) Replenishment is instantaneous and lead time is zero.
iv) The cycle time is uncertain and we assume it as triangular fuzzy number.
v) Shortages are not allowed.

Notations:
To develop this model the following notations have been used.
i) Demand $D(p) = ap^{-b}$, where a, b > 0 and p is the selling price.
ii) θ is the rate of deterioration.
iii) q is the initial stock level at the beginning of every inventory.
iv) D is the total deteriorated items
v) T is the length of a cycle.
vi) $\tilde{T}$ is the fuzzy cycle length.

vii) I(t) is the inventory level at any time t.
viii) h is the holding cost per unit time.
ix) A is the set up cost per cycle.
x) C is the deterioration cost per unit.
xi) TC is the total inventory cost.
xii) $\widetilde{TC}$ is the fuzzy total cost

III. MATHEMATICAL MODEL:

Let I(t) be the on hand inventory at time t ($0 \leq t \leq T$). The inventory cycle starts at t=0 with inventory level q. The inventory level decreases due to both demand and deterioration. Ultimately the inventory reaches 0 at the end of the cycle time T. Then the differential equation describing the instantaneous state of I(t) at any time t is given by :

$$\frac{dI(t)}{dt} + \theta I(t) = -ap^{-b} \quad \ldots\ldots(1)$$

With boundary conditions I(0) = q and I(T) = 0
Solving this equation we have,

$$I(t) = qe^{-\theta t} + (ap^{-b}/\theta)(e^{-\theta t} - 1) \quad \ldots\ldots(2)$$

Using I(T) = 0 we have-

$$q = (ap^{-b}/\theta)(e^{\theta T} - 1) \quad \ldots\ldots(3)$$

From (2),

$$I(t) = (ap^{-b}/\theta)(e^{\theta T} - 1).e^{-\theta t} + (ap^{-b}/\theta)(e^{-\theta t} - 1)$$
$$= ap^{-b}\left[(T - t) + \frac{(T-t)^2}{2}\theta + \frac{(T-t)^3}{6}\theta^2\right] \quad \ldots\ldots(4)$$

The inventory in a cycle is given by-

$$I_T = \int_0^T I(t)dt$$
$$= ap^{-b}\left[\frac{T^2}{2} + \frac{T^3}{6}\theta + \frac{T^4}{24}\theta^2\right] \quad \ldots\ldots(5)$$

Total deterioration in a cycle-

$$D = q - \text{total demand}$$
$$= q - \int_0^T ap^{-b}dt$$
$$= \frac{ap^{-b}}{\theta}(e^{\theta T} - 1) - ap^{-b}T \qquad \text{(Using (3))}$$
$$= \frac{ap^{-b}}{2}\theta T^2 \text{(Neglecting higher power of } \theta) \quad \ldots\ldots(6)$$

Average cost of the system-

$$TC = \frac{1}{T}(A + CD + hI_T)$$
$$= \frac{A}{T} + C\left(\frac{1}{2}ap^{-b}\theta T^2\right) + h\left(\frac{1}{2}ap^{-b}T + \frac{1}{6}ap^{-b}\theta T^2 + \frac{1}{24}ap^{-b}\theta^2 T^3\right) \quad \ldots\ldots(7)$$

Now let us describe the cycle time as triangular fuzzy number. $\tilde{T} = (T-\Delta, T, T+\Delta)$.
So, from (7) the cost function with fuzzy cycle time is-

$$\widetilde{TC} = = \frac{A}{\tilde{T}} + C\left(\frac{1}{2}ap^{-b}\theta\tilde{T}^2\right) + h\left(\frac{1}{2}ap^{-b}\tilde{T} + \frac{1}{6}ap^{-b}\theta\tilde{T}^2 + \frac{1}{24}ap^{-b}\theta^2\tilde{T}^3\right) \quad \ldots\ldots(8)$$

We know from the definition of Signed distance method that

$d(\tilde{A}, \tilde{0}) = \frac{1}{2}\int_0^1 [\, A_L(\alpha) + A_U(\alpha)\,] d\alpha$

where, $\tilde{A}=(a, b, c)$, $A_L(\alpha) = a + (b - a)\,\alpha$, $A_U(\alpha) = c - (c - b)\,\alpha$

Now, $T_L(\alpha) = (T - \Delta) + \Delta\alpha$

$T_U(\alpha) = (T + \Delta) - \Delta\alpha$

Therefore, $d(\tilde{T},\tilde{0}) = \frac{1}{2}\int_0^1 [\,(T - \Delta) + \Delta\alpha + (T + \Delta) - \Delta\alpha\,]\, d\alpha$

$= \frac{1}{2}\int_0^1 2T\, d\alpha = T$ …………………………….. …………………………… (9)

and $d(\frac{1}{\tilde{T}}, \tilde{0}) = \frac{1}{2}\int_0^1 [\,(\frac{1}{\tilde{T}})_L(\alpha) + (\frac{1}{\tilde{T}})_U(\alpha)\,] d\alpha$

$= \frac{1}{2}\int_0^1 [\,\frac{1}{T+\Delta-\Delta\alpha} + \frac{1}{T-\Delta+\Delta\alpha}\,]\, d\alpha$

$= \frac{1}{2\Delta}\ln(\frac{T+\Delta}{T-\Delta})$ ………………………………………….. ..………………………. (10)

From (8), (9) and (10) we have-

$\tilde{TC} = \frac{A}{2\Delta}\ln(\frac{T+\Delta}{T-\Delta}) + C(\frac{1}{2}ap^{-b}\theta T^2) + h(\frac{1}{2}ap^{-b}T + \frac{1}{6}ap^{-b}\theta T^2 + \frac{1}{24}ap^{-b}\theta^2T^3)$ …………..………….. (11)

Theorem: The cost function $\tilde{TC}$ given by equation (11) is strictly convex.

Proof: Here, $\frac{d(\tilde{TC})}{dT} = \frac{A}{2\Delta}\left(\frac{1}{T+\Delta} - \frac{1}{T-\Delta}\right) + C(ap^{-b}\theta T) + h(\frac{1}{2}ap^{-b} + \frac{1}{3}ap^{-b}\theta T + \frac{1}{8}ap^{-b}\theta^2T^2)$

And, $\frac{d^2(\tilde{TC})}{dT^2} = 2A\frac{T}{(T^2-\Delta^2)^2} + Cap^{-b}\theta + h(\frac{1}{3}ap^{-b}\,\theta + \frac{1}{4}ap^{-b}\,\theta^2T) > 0$

Hence $\tilde{TC}$ is strictly convex.

IV. NUMERICAL EXAMPLE

To illustrate the following model we consider the following numerical values of the parameters.

A = 200, a = 100, p =50, b = 0.05, h = 20, θ = 0.05, c = 18.

We obtain for crisp model total cost TC = 823 and cycle time T = 0.479 and for fuzzy model total cost $\tilde{TC}$= 824 and $\tilde{T}$ = 0.481

V. SENSITIVITY ANALYSIS

Table-1

Change Value		Crisp model		Fuzzy model	
		TC	T	$\tilde{TC}$	$\tilde{T}$
A	180	780	0.455	781	0.457
	190	802	0.467	803	0.469
	200	823	0.479	824	0.481
	210	843	0.491	845	0.492
	220	863	0.502	865	0.503

Table -2

Change Value		Crisp model		Fuzzy model	
		TC	T	$\tilde{TC}$	$\tilde{T}$
h	16	740	0.531	740	0.533
	18	782	0.503	783	0.505
	20	823	0.479	824	0.481
	22	862	0.458	863	0.460
	24	899	0.440	900	0.442

Table -3

Change Value		Crisp model		Fuzzy model	
		TC	T	$\tilde{TC}$	$\tilde{T}$
C	14	821	0.481	822	0.483
	16	822	0.480	823	0.482
	18	823	0.479	824	0.481
	20	824	0.478	825	0.480
	22	825	0.477	826	0479

Table -4

Change Value		Crisp model		Fuzzy model	
		TC	T	$\tilde{TC}$	$\tilde{T}$
θ	0.01	814	0.490	814	0.492
	0.03	818	0.485	819	0.487
	0.05	823	0.479	824	0.481
	0.07	828	0.474	829	0.476
	0.09	832	0.469	833	0.471

Observations:

From the above tables it is observed that:

i) The total cost (for both the models) increases as the holding cost per unit time increases.
ii) The increase in set up cost increases the total inventory cost for the two models.
iii) With the increase of the deterioration cost per unit, the total cost (for both the models) also increase.
iv) As the rate of deterioration increase, the total inventory costs for both the models also increase.

VI. CONCLUSION

In this paper I have considered a fuzzy supply chain inventory model where I have described the cycle time as a triangular fuzzy number (symmetric).The demand rate is assumed to be a function of selling price. I have tried to compare crisp model with the fuzzy model and have seen that the cycle time and the total cost obtained by crisp model is less than those obtained by fuzzy model. From the sensitivity analysis it is observed that the total cost of both the model increase as the cost associated with the model increase.

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