

Mathematical Model of Rationale for Reserve Fleet of Vehicles with Uneven Demand for Transportation

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ABSTRACT: *The problem of determining the required reserve of fleet of vehicles (cars, automobiles, etc.) based on the criterion of minimizing the lost profit from the traffic. The criterion was selected by the analysis of scientific literature on this problem, as in previous studies it was not taken into account, as well as the most appropriate for the transportation of goods with significant seasonal irregularity (cereals, some perishables etc.). The goal was put and achieved to develop a simple enough for practical use by transportation companies and adequate mathematical model for determining the reserve fleet of vehicles in view of uneven demand for transportation. The calculations based on the proposed model proved that, depending on the period of increased demand for transportation as well as other technological, operational and economic factors (model parameters), required reserve of vehicles can be from a few percent to several times of the fleet determined by traditional methods. The mathematical model uses appropriate analytical relationships linking the value of the lost profit from transportation due to shortage of vehicles with technological, operational and economic parameters of rolling stock. The described model and approach to its application can be used without fundamental changes to justify the operational reserve of containers, cars and traction rolling stock*

Keywords: *mathematical model, vehicles, fleet, reserve, transportation demand, lost profit.*

I. INTRODUCTION

Uneven offer of cargo to transportation is a known phenomenon and problem that makes it difficult to cope with for carriers, so it was always paid attention. But from this it does not lose its relevance. This irregularity has objective reasons independent of the carrier, including fluctuations in market supply and demand for certain goods the transport company is carrying. It leads to economic loss, including income from transportation at a time when demand for transportation exceeds transport capacity (availability of vehicles and / or of transport capacity). If the carrier is not able to compensate for the sudden increase in demand for transport services by some reasonable reserve of vehicles, it leads to failure in sender servicing and thus to financial and image losses to the carrier.

Reducing these losses may be due to the presence of certain reasonable reserve of vehicles through which "spikes" in demand for transportation are compensated. However, in those periods of time when the transportation service demand is reduced and supply of vehicles exceeds the demand, there arise the economic costs of another kind, which are related to the maintenance of "excess" vehicles.

1.1. Analysis of scientific literature and problem statement

Impact of irregularity, volatility, stochastic demand on transportation processes, including the size and composition of the fleet of vehicles has long been in sight of scientists from a theoretical point of view, and of transport companies – from a practical one. Only in recent years we can recall a series of articles [1, 2 – Redmer A., 2015; 2016], in which the problems and related mathematical models of optimization of the size of vehicles fleet (the fleet sizing problem – FS) and the number of types of vehicles in the fleet (the fleet composition problem – FC) are discussed, given the fact that they can be interchangeable in meeting the demand for transportation of various kind. As an optimization criterion the fleet efficiency is adopted (measured, for example, by the utilization rate). Also considered are the problems and optimization techniques of fleet renewal in case of various types of vehicles by replacing them (the fleet replacement problem – FR), taking into account the balance of the cost of their operation and disposal. The paper [3 – Gahm C. et al., 2017] discussed the impact of routes, depot locations and maintenance time constraints and transportation volume discounts on private fleet of vehicles and common carriers. The article [4–Afshar-Nadjafi Behrouz, Afshar-Nadjafi Alireza, 2017] applies

a heuristic approach and computational methods and mathematical models based on mixed integer programming for solving vehicle routing problem, taking into account hard time window constraints for the customers and limitation on maximum number of the vehicles in depots. As rightly observed in works [5–Klosterhalfen S.T., et al., 2014], [6 – Kallrath J. et al., 2016] dedicated to rail transport in chemical industry, car fleet is an important part of major working capital. To optimize fleet of cars offered mixed integrated models of non-linear or linear programming, which should derive suggestions to the questions (1) How many different rail car sizes in terms of volume are appropriate and what are their optimal volumes? and (2) For each volume, what is the optimal number of rail cars of this size? This takes into account the interchangeability of types of cars and existing uncertainties and limitations.

Similar problems are resolved with similar methods in the paper [7 – Melnikova A.N. et al., 2016] dedicated to optimizing the fleet of vehicles for the removal of solid domestic waste. At maritime transport the problem of renewal and deployment of diverse-type fleet of vessels with the uncertainty of demand for transportation and the charter costs are considered in the paper [8 – Arslan Ayşe N., Papageorgiou Dimitri J., 2017]. It is noted that unpredictable shipping cycles are rather the norm rather than the exception. Such a stochastic approach and appropriate mathematical models under uncertainty in demand are developed for tactical planning. The paper [9 – Ning Shi et al., 2014] investigated the impact on the dynamic fleet management not only of uncertain demand, but also of the customer chosen service levels, and suggested the use of structural decomposition approach to effectively address the problem. The paper [10 – Gardner L.M. et al., 2013] discussed the impact of variability in demand on travel patterns and energy consumption rates of plug-in electric vehicles fleet on the road network and the environment. Finally, the article [11 – Myronenko V.K., Gaba V.V., 2003] analyzed the problem of optimizing the fleet size of vehicles in terms of seasonal fluctuations in demand for transport and offered analytical mathematical model which, in general terms, allows for such an optimization.

In our opinion, the cited works shows, firstly a multi-criterion nature of the problem of optimization of vehicles fleet, and secondly, the fact that most of these criteria are economic (eg minimum cost of maintaining and updating the fleet), while fleet operating conditions and the need to perform certain traffic volumes and their oscillations act as limitations to appropriate mathematical models. Third, these models are mainly based on mathematical methods of linear or non-linear programming and require fairly large volume of input data and sophisticated algorithms and software with uncertain results.

Therefore, the authors of this article sought to develop a mathematical model of optimal reserve of fleet of vehicles in view of uneven demand for transportation that would be based on simple and proven analytical relationships between technological, operational and economic parameters of the model, with no special software to quickly and reliably calculate the optimal size of reserve vehicles fleet. The scope of the model should be the fleets of vehicles used for carrying of goods with considerable seasonal fluctuations in demand for transportation. For example, grain cargoes, which are to be transported in large quantities for one to two months by road or rail from agricultural regions to ports for export by sea.

1.2. The purpose and objectives of the study

To balance and minimize the losses (both in carriers' money and image) from insufficient transportation service in case of increased supply of cargo it is proposed to develop a mathematical model that represents the economic losses as a function of not only economic factors but also of technological parameters of transportation service. This model should be suitable for use in practical calculations. To achieve this it is necessary to solve the following problems:

- Justify the criterion for determining the reserve vehicles in terms of uneven seasonal demand for transportation service;
- Develop a mathematical model for determining the reserve fleet of vehicles (for example, railway rolling stock for transportation of grain);
- Check the adequacy of the model by calculation and analysis of the results and provide the basis for the practical advice to improve transportation service.

II. BASIC MATERIALS

The mathematical model must adequately describe the situation that occurs in the practice of transport. Thus, grain transportation by rail, road or sea is characterized by great unevenness in time due to the relatively short period of cultivation and harvesting, processing and transporting the cereals to elevators and ports, and from there to the importing countries. For the railways of Ukraine, for example, there is a chronic problem of shortage of grain hopper wagons during a “spike” period of grain transportation and economic costs of this are very tangible both for transporters and traders. This once again demonstrates the relevance and practical need to develop a mathematical model to describe these phenomena and help reduce their negative impact. The optimization criterion of the model should be the minimum of carrier's lost profits in case of lack of transport

capacity to cover the demand. The possible (model estimated) value of carrier's lost profits must be the basis for allocation of appropriate investment in the purchase of additional vehicles, building the reserve of fleet, thereby decreasing economic losses and improving carrier's market image. Creation of the mathematical model starts with the formalization of the function of lost profits from traffic in the period, when the demand for transportation exceeds carrying capacity. The function is represented as

$$\{LP\} = (D - S) \cdot T \cdot L \cdot (f_{t \cdot km} - c_{t \cdot km}), \quad (1)$$

where - lost profit from the carriage (accept Euro currency);

- average demand for transport services transportation for the time [tonnes per day];
- the average supply of transportation services for the time T [tonnes per day] (in this case at this time);
- duration of the period when demand exceeds supply (the period of shortage of transport capacity due to lack of vehicles, such as cars), [days];

L - average shipping distance [km];

$f_{t \cdot km}$ - tariff for carriage [Euro per tonne-km];

$c_{t \cdot km}$ - the cost of transportation [Euro per tonne-km].

Define the components of the formula (1). The values of and are independent of the carrier, so they will just vary within certain limits. Rate of transport services supply is dependent on the carrier, and this dependence has the form:

$$S = \frac{N_w \cdot q_w}{\Theta_w}, \quad (2)$$

where N_w - necessary inventory of vehicles suitable for transportation (eg, operation fleet of wagons);

q_w - weight of cargo in the wagon, depending on the characteristics of the goods and the wagon [tonnes];

Θ_w - the average duration of wagon turn-over time (the time from the beginning of one load till the beginning of the next load) [days].

Operation fleet of wagons can be defined as

$$N_w = \frac{D \cdot T \cdot \Theta_w}{365 \cdot q_w}. \quad (3)$$

The duration of wagon turn-over time can be defined as

$$\Theta_w = \frac{2}{24} \left(\frac{L}{V} + t_{od} + \frac{k_{TR} \cdot t_{TR}}{2} \right), \quad (4)$$

where - the average shipping distance [km];

- average speed of shipping [km / h.];
- the average idle time of wagons at the points of origin and destination [h.];
- the average number of transit stations, where the train stops for its turnover for wagon technical service procedures;
- the average idle time of wagons at a transit station [h].

Economic components $f_{t \cdot km}$ and $c_{t \cdot km}$ that make up the formula (1) are defined as follows.

Tariff for transportation $f_{t \cdot km}$ must be higher than its cost $c_{t \cdot km}$, that is a mandatory condition $f_{t \cdot km} > c_{t \cdot km}$ of profit earning, but it cannot exceed the solvency of the consignor or the tariff of competing carrier, such as carrier by road $a_{t \cdot km}$.

The transportation cost depends on many factors and may be defined as follows:

$$c_{t \cdot km} = (1 + \alpha_E) \frac{\Theta_w \cdot C_{YW}}{2 \cdot T \cdot L \cdot q_w}, \quad (5)$$

where - the wagon empty run ratio in relation to the loaded run;

C_{YW} - total annual costs of wagon maintenance and operation [€ a year per 1 wagon].

Total annual costs of maintenance and operation of the car are defined as

$$C_{YW} = \frac{E_Y}{N_W} + k_R \frac{P_W}{Y_W} + \frac{2TL}{\Theta_W} \cdot c_{w-km} \quad (6)$$

where C_{YW} – conditionally fixed annual cost of carrier, regardless of traffic volumes (eg, buildings, other infrastructure, administrative costs, etc.) [€ a year];

- coefficient taking into account the cost of all kinds of wagon repairs during its life cycle;
- wagon purchase price [€];
- the duration of the life cycle (service time) [years];

c_{w-km} – specific costs relating to 1 wagon-km of operation run [€].

Now we have all the necessary values to determine how many cars should be added to the operation fleet to prevent the loss of profit due to a shortage of wagons:

$$\Delta n_W = \frac{\{LP\}}{P_W} \quad (7)$$

You can also set “reserve ratio” of wagon fleet:

$$k_{RW} = \frac{\Delta n_W}{N_W} \cdot 100\% \quad (8)$$

For calculation and verification of the adequacy of the mathematical model that is presented by relations (1) – (8) the formulas and the input data are summarized for convenience in the Table 1. This bears dashes where no additional data are needed.

First, calculate the wagon turnover time. The results are shown in Table 2.

Table 2 shows the results to assess the range of Θ_W change (from 2.36 to 4.19 days) for given values of N_W and P_W , and other variables that are taken constant. With this in mind we take in further calculations of other variables the range of change of wagon turnover time Θ_W from 2 to 4 days.

Next estimated value is the necessary fleet of vehicles (wagons). It is determined at various Θ_W , and N_W . The calculation results are given in Table 3. It should be noted that the results are rounded to the whole number.

The Table 4 shows the results of calculation of transportation services supply (carrying capacity) that is obtained by a similar procedure.

As it can be seen in the Table 4, the results of calculation are reflecting the systematic lack of transport carrying capacity. Proceed to determine economic indicators that are calculated and presented on a similar form in the Tables 5 – 7 based on formulas and input data contained in the Table 1.

It is noteworthy that under certain relationships of function parameters the total annual costs may have local minima, as shown in Fig. 1 (for the period of increased demand 180 days a year the local minimum of costs is particularly marked when we have the transportation service demand of 180 tonnes per day).

The cost of 1 tonne-kilometer of transportation is defined in the Table 6.

As shown in the Table 6 and Fig. 2, the shorter is the period of increased demand for transportation and lower quantity demanded (lower traffic volumes), the higher is the cost of transportation, which is a logical result that confirms the adequacy of economic and mathematical models.

Now you can evaluate $\{LP\}$ – lost profit from transportation due to shortage of transport capacity while demand

for transportation exceeds the supply of transport services. For this value rate $f_{t-km} = 0.1$ Euro / tonne-km is assumed, which approximately corresponds to the level of competitors' tariffs – road carriers. The results are shown in Table 7 and Fig. 3.

Finally, we proceed from these economic findings to recommendations on the size and operation of the wagon fleet. These recommendations are based on the results presented in Table 8, 9 and in Fig. 4.

Fig. 4 gives graphic interpretation of reserve ratio depending on the period of increased demand for transportation for three values of daily demand (90, 180 and 270 tons) at $\Theta_W = 2$ days and $N_W = 300$ km.

III. RESULT and DISCUSSION

As shown in the Table 9 and Fig. 4, at certain ratios of transport services demand, supply and other parameters a significant increase may be required in the size or operational reserve of wagon (or any other vehicle) fleet. This reserve should be maximum in cases where existing fleet is not enough to meet existing demand that exceeds the supply of transport services (see also Fig. 3 and 1). In some cases, under severe operating conditions (high demand for transportation in a short period, a considerable transportation distance

and wagon turn-over time duration) even a many-fold increase in wagon fleet may be required to prevent the loss of profit from transportation (see the selected cell in the table 9, the number in it shows that the fleet should be increased by 132% or 2.32 times compared to existing!). This is an interesting conclusion derived from the proposed model, both of theoretical and practical importance.

IV. CONCLUSIONS

1. Analysis of scientific literature on the optimization of vehicles fleet in different conditions on different criteria and for different purposes showed that the problem is urgent, and the ways of its solution can be supplemented and expanded using the results presented in this article.
2. When determining the reserve of vehicles fleet the economic costs of their lack during the seasonal increase in demand for transport should be considered, in particular the value of lost profit from shipping should be taken into account, which is seen as the objective function to be minimized in the optimization problem.
3. The mathematical model uses appropriate analytical relationships linking the value of the lost profit from transportation due to shortage of vehicles with technological, operational and economic parameters of rolling stock utilization known in the practice of transport companies, so it can be used to improve transport services of customers under uneven demand for transportation.
4. The described model and approach to its application can be used without fundamental changes to justify the operational reserve of containers, cars and traction rolling stock (electric locomotives, diesel locomotives etc.).

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This paper is initiative work

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Table 1: Formulas and input data of the mathematical model

$\Theta_W = \frac{2}{24} \left(\frac{L}{V} + t_{ad} + \frac{k_{RN} \cdot t_{RN}}{2} \right)$	[km]		[km/h]		[h]		[h]	
	250	750	50	100	12	4	6	
$N_W = \frac{D \cdot T \cdot \Theta_W}{365 \cdot q_W}$	[t/day]		[day]		[t/wagon]		-	
	90	270	90	270	30	-	-	
$S = \frac{N_W \cdot q_W}{\Theta_W}$	-		-		-		-	
	-	-	-	-	-	-	-	
$C_{YW} = \frac{E_{Yr}}{N_W} + k_R \frac{P_r}{Y_r} + \frac{27L}{\Theta_W} \cdot c_{w-lm}$	[€]				[€]		[year]	
	70000		1,75		80000	25	0,2	
$c_{rlm} = (1 + \alpha_R) \frac{\Theta_W \cdot C_{YW}}{2 \cdot T \cdot L \cdot q_W}$	-		-		-		-	
	1,0		-		-	-	-	
$\{LP\} = (D - S) \cdot T \cdot L \cdot (f_{rlm} - c_{rlm})$	$f_{rlm}, [€/t \cdot km]$		-		-		-	
	0,1		-		-	-	-	
$\Delta H_W = \frac{\{LP\}}{P_r}$	-		-		-		-	
	-		-		-	-	-	
$k_{RW} = \frac{\Delta H_W}{N_W} \cdot 100\%$	-		-		-		-	
	-		-		-	-	-	

Table 2: Wagon turnover time [day]

[km/h]	[km]		
	350	700	1050
40	2,73	3,46	4,19
60	2,49	2,97	3,46
80	2,36	2,73	3,09

Table 3: Necessary fleet of wagons

Θ_w [days]	[t/day]	[day]		
		90	180	270
2	90	2	4	5
	180	4	7	10
	270	5	10	14
3	90	3	5	8
	180	5	10	14
	270	8	14	21
4	90	4	7	10
	180	7	13	19
	270	10	19	28

Table 4: Transportation services supply (carrying capacity) [tonnes per day]

Θ_w [days]	[t/day]	[day]		
		90	180	270
2	90	37,2	59,4	81,6
	180	59,4	103,8	148,2
	270	81,6	148,2	214,7
3	90	32,2	54,4	76,6
	180	54,4	98,8	143,2
	270	76,6	143,2	209,7
4	90	29,7	51,9	74,1
	180	51,9	96,3	140,7
	270	74,1	140,7	207,2

Table 5: Total annual costs of wagon maintenance and operation [€]

Θ_w [days]	[km]	[t/day]	[day]		
			90	180	270
2	300	90	39879	34427	34879
		180	29027	26648	28930
		270	24079	23530	26670
3	600	90	35006	33079	36443
		180	25879	27130	32070
		270	22043	24870	30473
4	900	90	31727	32048	37030
		180	23948	27249	33579
		270	20830	25479	32346

Table 6: The cost of 1 ton-km transportation [€]

Θ_w [days]	[km]	[t/day]	[day]		
			90	180	270
2	300	90	0,0985	0,0425	0,0287
		180	0,0717	0,0329	0,0238
		270	0,0595	0,0290	0,0220
3	600	90	0,0648	0,0306	0,0225
		180	0,0479	0,0251	0,0198
		270	0,0408	0,0230	0,0188
4	900	90	0,0522	0,0264	0,0203
		180	0,0394	0,0224	0,0184
		270	0,0343	0,0210	0,0177

Figure 1 –Total annual costs of wagon maintenance and operation [€]

Figure 2 – The cost of 1 ton-km transportation [€]

Table 7: Loss of profits from transportation {LP} due to shortage of transport capacity [€ a year]

Θ_w [days]	[km]	[t/day]	[day]		
			90	180	270
2	300	90	2187	95060	48650
		180	92257	276228	196553
		270	206271	466846	349443
3	600	90	109801	266841	168557
		180	353242	656931	478784
		270	618137	1054503	792765
4	900	90	233378	454613	308344
		180	628660	1052250	780012
		270	1042845	1656028	1254677

Figure 3: Loss of profits from transportation {LP} due to shortage of transport capacity [€ a year] (at =2 days and =300 km)

Table 8 – Number of wagons that can be purchased at the expense of lost profits

Θ_w [days]	[km]	[t/day]	[day]		
			90	180	270
2	300	90	0	1	1
		180	1	3	2
		270	3	6	4
3	600	90	1	3	2
		180	4	8	6
		270	8	13	10
4	900	90	3	6	4
		180	8	13	10
		270	13	21	16

Table 9: Wagon fleet reserve ratio [%] to the existing (but insufficient)

Θ_w [days]	[km]	[t/day]	[day]		
			90	180	270
2	300	90	1,1	30,0	11,2
		180	29,1	49,9	24,9
		270	47,4	59,1	30,5
3	600	90	42,6	61,3	27,5
		180	81,2	83,1	41,8
		270	100,9	92,1	47,3
4	900	90	73,7	82,1	39,0
		180	113,6	102,5	52,0
		270	132,0	110,4	56,8

Figure 4: Wagon fleet reserve ratio [%] to the existing (but insufficient)