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Voltage Stability Investigation of the Nigeria 330KV Interconnected Grid System Using Eigenvalues Method

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ABSTRACT: *Voltage stability investigation of the Nigeria 330kV grid Power Network was carried out using Eigenvalues method. The power system Jacobian matrix was derived using Newton-Raphson power flow solution iterative method to determine the eigenvalues for the voltage stability evaluation of both the existing 28-bus and improved 52-bus Nigeria 330kV power networks. Positive and negative eigenvalues quantifying the power system voltage stability were identified. For a steady state power system, positive and negative eigenvalues showed stable and unstable systems respectively. The eigenvalues voltage stability method was developed, formulated and implemented on the existing 28-bus and proposed improved 52-bus Nigeria 330kV power networks. The various eigenvalues, eigenvectors, participation factors and weak buses contributing to system voltage instability were determined using Matlab/SIMULINK Power System Analysis Toolbox (PSAT) for these power networks.*

Keywords: *Voltage Stability Investigation, Eigenvalue Voltage Stability Method, Newton-Raphson power flow solution, iterative method, Eigenvalues, Eigenvectors*

I INTRODUCTION

In the past, the electric utility industry has largely depended on conventional power-flow programs for static analysis of voltage stability. Stability is determined by computing the V-P and V-Q curves at selected load buses. Generally, such curves are generated by executing a large number of power flows using conventional models. While such procedure can be automated, they are time–consuming and do not readily provide information useful in gaining insight into causes of stability problems.

A number of methods exist, but not many of them have found widespread practical application. The recent practical approach adopted is the Eigenvalues analysis approach which gives voltage stability related information from a system wide perspective and clearly identifies areas that have potential problems. Eigenvalue voltage stability method offers information regarding the mechanism of instability. This is the principal reason for considering the use of Eigenvalues analysis method for the investigation of voltage stability for both the existing 28-bus Nigeria power System and 52-bus improved Nigeria power system in the 330kV transmission system.

Voltage stability analysis examines the ability of a power system to maintain acceptable voltage levels in response to both abrupt and gradual disturbances. Voltage stability problems mainly occur when the system is heavily stressed beyond its capability. While the disturbance leading to voltage collapse may be initiated by a variety of causes, the main problem is the inherent weakness in the power system.

Nigerian existing power network comprises 9 generating stations, 28 buses, 32 transmission lines and 11,000KM transmission lines is confronted with the following problems[1]:

- The grid system in Nigeria is almost radial single circuit lines.
- Fragile and very long nature of transmission lines.
- Poor network configuration in some regional work centers.
- Large number of overloaded transformers in the grid system.
- The use of transmission lines beyond their thermal limits.
- Inability to dispatch generated energy to meet load demand.
- Large number of uncompleted transmission line projects.
- Poor voltage profiles at most the buses, especially those of the Northern parts of the country.
- Ineffective control of the transmission line parameters and voltage and frequency controls.
- Vandalism of the 330kV transmission lines in various parts of the country.

Power Holding Company of Nigeria (PHCN) in an attempt to solve these problems resulted in its unbundling. Thus, the Nigeria 330kV integrated network intends to improve the grid stability and creates an effective interconnection. It is anticipated to increase transmission strength because of the very high demand on the existing and aging infrastructure by building more power stations and transmission lines, through the Independent Power Projects (IPPS). The 52-bus Nigerian 330kV power grid interconnection comprises 52 buses, 64 transmission lines of either dual or single circuit lines and has four control centers, one at National Control Center, Oshogbo and three supplementary control centers at Benin, Shiroro and Egbin.

As more new sources of power are added to the Nigerian electrical network, an over-riding factor in the operation of the electrical network is the desire to maintain acceptable voltage level in transmission sector within the acceptable margin of ±5% of the rated voltage. However, it is desirable that voltage at different buses in electrical network be kept equal to the normal value of 1pu (330kV) at all times.

II MATERIALS AND METHODS

Modal analysis as one of the global Voltage Stability Index (VSI) that can measure the voltage stability was applied to investigate the stability of the power systems to compute the smallest eigenvalue and the associated eigenvectors of the reduced Jacobian matrix using the steady state system model. The magnitude of the smallest eigenvalue presents a measure of how close the system is to the voltage collapse. Then, the participating factor can be used to identify the weakest node or bus in the system associated to the minimum eigenvalue.

2.1 Eigenvalues Voltage Stability Analysis Formulations

For a(nx n)square matrix **A**, left and right eigenvectors are defined as follows [2, 3, 4, 5, 6, 7, 8]:

$\mathbf{A}\mathbf{x}=\lambda\mathbf{x}$ (1)

$\mathbf{y}A=\lambda\mathbf{y}$ (2)

where $\boldsymbol{\lambda}$= eigenvalue of the matrix **A,**

$\mathbf{x}$(nx 1)=right eigenvector,

$\mathbf{y}$(1x n)=left eigenvector.

The characteristic equation of both (1) and (2) is,

det.($\mathbf{A}-\lambda\mathbf{I}$)=0 (3)

The solution of $\lambda_1,\lambda_2,\ldots..,\lambda_n$ are the eigenvalues of **A**.

For different eigenvalues λ_i,i=1,…,n;the right and left eigenvectors are defined as,

$\mathbf{x}_i$,i=1,…,n and

$\mathbf{y}_i$,i=1,…,n.

In matrix form, the right eigenvector matrix, $\mathbf{X}=[\mathbf{X}_1,\mathbf{X}_2,\ldots..,\mathbf{X}_n]$and the left eigenvector matrix,

$\mathbf{Y}=[\mathbf{y}_1^T,\mathbf{y}_2^T,\ldots\ldots.,\mathbf{y}_n^T]^T$.

It can be shown that, $\mathbf{x}_j$ and$\mathbf{y}_i$are orthogonal, such that,

$\mathbf{x}_j\cdot\mathbf{y}_i = 0, \quad \forall i\neq j$

$\neq 0, \quad \forall i = j$

In practice, eigenvectors are normalized so that $\mathbf{y}_i.\ \mathbf{x}_i = 1, \forall i = 1,\ldots,n$.

Hence, $\mathbf{Y}.\mathbf{X} = \mathbf{I}$, or, $\mathbf{Y}= \mathbf{X}^{-1}$ (4)

where **I**= Identity matrix

Now, $\mathbf{A}.\mathbf{X} = [\lambda_1\mathbf{x}_1, \lambda_2\mathbf{x}_2\ldots\ldots \lambda_n\mathbf{x}_n] = \mathbf{X}.\Lambda$ (5)

Where,

Λ = The diagonal eigenvector matrix of the J_R matrix and is given in equations (6) and (7).

$$\Lambda = \begin{bmatrix} \lambda_1 & 0 & \dots & 0 \\ 0 & \lambda_2 \dots & & 0 \\ \vdots & & \vdots & \vdots \\ 0 & 0 & \dots & \lambda_n \end{bmatrix} \quad (6)$$

or, $\mathbf{A}=\mathbf{X}\Lambda\mathbf{X}^{-1}=\mathbf{X}\Lambda\mathbf{Y}$ (7)

In the Newton-Raphson Power flow, there is a linear system model or equation, written in matrix form used to represent the injected power in the buses and this model is given in equation (8).

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_{p\delta} & J_{pv} \\ J_{q\delta} & J_{qv} \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta V \end{bmatrix} \quad (8)$$

Where $\Delta\mathbf{P}$ and $\Delta\mathbf{Q}$ are the changes in the real and reactive powers respectively,
$\Delta\boldsymbol{\delta}$ and $\Delta\mathbf{v}$ are the deviations in bus voltage angles and bus voltage magnitude respectively.
For calculating V-Q sensitivities, assume $\Delta\mathbf{P} = 0$

$$\begin{bmatrix} 0 \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_{p\delta} & J_{pv} \\ J_{q\delta} & J_{qv} \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta V \end{bmatrix} \quad (9)$$

Hence, $\mathbf{J}_{p\delta}\cdot\Delta\boldsymbol{\delta} + \mathbf{J}_{pv}\cdot\Delta\mathbf{P} = \mathbf{0}$
Or,$\Delta\boldsymbol{\delta} = \mathbf{J}_{p\delta}^{-1}.\mathbf{J}_{pv}.\Delta\mathbf{V}$ (10)

The Reduced Jacobian Matrix can be found by solving the equation
$\Delta\mathbf{Q} = \mathbf{J}_{q\delta}\cdot\Delta\boldsymbol{\delta} + \mathbf{J}_{qv}\cdot\Delta\mathbf{V} = \mathbf{J}_{q\delta}[-\mathbf{J}_{p\delta}^{-1}.\mathbf{J}_{pv}]\cdot\Delta\mathbf{V} + \mathbf{J}_{qv}\cdot\Delta\mathbf{V} = \mathbf{J}_{R}\cdot\Delta\mathbf{V}$ (11)

Where, $\mathbf{J}_R = [\mathbf{J}_{qv} - \mathbf{J}_{q\delta}.\mathbf{J}_{p\delta}^{-1}.\mathbf{J}_{pv}]$ (12)

$\mathbf{J}_R$=The Reduced Jacobian Matrix
Hence, $\Delta\mathbf{V} = \mathbf{J}_R^{-1}.\Delta\mathbf{Q}$ (13)

And $\Delta\mathbf{Q} = \mathbf{J}_R.\Delta\mathbf{V}$ (14)

Also $\frac{\Delta V}{\Delta Q} = J_R^{-1}$ (15)

Now, assuming $\mathbf{J}_R = \mathbf{A}$; and using equation (7) gives
$\mathbf{J}_R = \mathbf{X}.\boldsymbol{\Lambda}\mathbf{Y}$ (16)

Or,$\mathbf{J}_R^{-1} = \mathbf{Y}^{-1}.\boldsymbol{\Lambda}^{-1}.\mathbf{X}^{-1} = \mathbf{X}\boldsymbol{\Lambda}^{-1}.\mathbf{Y}$

$\mathbf{J}_R^{-1}$ = the reduced V-Q Jacobian of the system. It is also known as the sensitivity of V-Q.

By substituting (15) into (14) resulted to:

$\Delta\mathbf{V} = \mathbf{X}\boldsymbol{\Lambda}^{-1}\mathbf{Y}.\Delta\mathbf{Q}$
Or, $\mathbf{Y}\Delta\mathbf{V} = \boldsymbol{\Lambda}^{-1}\mathbf{Y}.\Delta\mathbf{Q}$;[since $\mathbf{X}=\mathbf{Y}^{-1}$]

Hence, $\mathbf{V}_m = \boldsymbol{\Lambda}^{-1}.\mathbf{q}_m$ (17)

Where $\mathbf{V}_m$= vector of modal voltage variation
$\mathbf{q}_m$= vector of modal reactive power variation

$$= \begin{matrix} 0 \dots & 0 \\ \lambda_2^{-1} \dots & 0 \\ \vdots & \vdots \\ 0 \dots & \lambda_n^{-1} \end{matrix}\Bigg] \quad (18)$$

Thus, $\mathbf{V}_{mi} = \lambda_i^{-1}.\mathbf{q}_{mi}, \forall i = 1, 2, \dots n$ (19)

$$\Delta \mathbf{V}|_{bus} = \lambda_i^{-1}.\Delta \mathbf{Q} \quad , \forall i = 1, 2, ...n \tag{20}$$

It can be seen that for any i, if $\lambda_i>0$,thenthevariationof$\mathbf{v}_{mi}$and $\mathbf{q}_{mi}$are in the same direction and the system is voltage stable. When $\lambda_i<0$ for any *i*, the system is voltage unstable. Then, if $\lambda_i=0$, the voltage experiences collapse when mode *I* reactive power has a small variation. Other modes are not affected, thus it can be said that voltage collapse is actually the collapse of modal voltage. The contribution of each physical voltage to the modal voltage can be used to determine the bus that is most prone to voltage collapse, that is, the bus that would cause the system to tend to instability. For the same amount of physical reactive power change at a given bus, the bus with a larger eigenvector entry would yield more significant power change. The voltage stability can be defined by the mode of the eigenvalue λ_i. The minimum eigenvalue in the power system is the global VSI value. The larger values of $\lambda_i$will give smaller changes in voltages when a small disturbance happens. When the system is weaker, the voltage becomes weaker. A power system network is voltage stable when the eigenvalues of the J_R matrix are all positive [3].

2.2 Identification of Weak Buses

The minimum eigenvalues, which causes the network to become close to instability, needs to be observed more closely. The relationship between system voltage stability and eigenvalues of the reduced Jacobian matrix J_R is best understood by relating the eigenvalues with the V-Q sensitivities of each bus (which must be positive for stability). J_R can be taken as a symmetric matrix and therefore the eigenvalues of J_R are close to being purely real. If all the eigenvalues are positive, J_R is positive definite and the V-Q sensitivities [3,4,5,6,7,8]. Once the minimum eigenvalue has been established, the participation factor is calculated and used to identify the weakest node or bus in the system. The bus with the highest value of participation factor contributes most to the instability of the system.

The modes of the power system are determined by inverting J_R matrix to J_R^{-1} from its eigenvalues and eigenvectors of equation (16).

$$\mathbf{J}_R = \mathbf{X\Lambda Y} \tag{21}$$

$$\mathbf{J}_R^{-1} = \mathbf{X\Lambda^{-1}.Y} \tag{22}$$

If **X** and **Y** represent the right and left eigenvectors respectively for the eigenvalue λ_i of the matrix $\mathbf{J}_R$, the participation factor measuring the participation of the kth bus in ith mode is defined as:

$$\mathbf{P}_{ki} = \mathbf{X}_{ki}\mathbf{Y}_{ki} \tag{23}$$

The left and right eigenvectors corresponding to the critical modes are used to identify the buses participating in the critical modes. Thus, the nodes or elements with large participation factors are identified as weak areas of the power grid from the perspective of voltage stability.

Computing of the eigenvalue of the transmission network would appear cumbersome to handle. Thus, Matlab/SIMULINK Power System Analysis Toolbox (PSAT) program was deployed for the dynamic simulation and calculations of the eigenvalues and eigenvectors of the state matrix, and the participation factors identifying the weak buses for the 28-bus and 52-bus Nigeria 330kV power networks. The algorithm/flow chart showing the sequential procedures for the voltage stability investigation of these Nigeria transmission grid networks using eigenvalues method is illustrated in figure 1

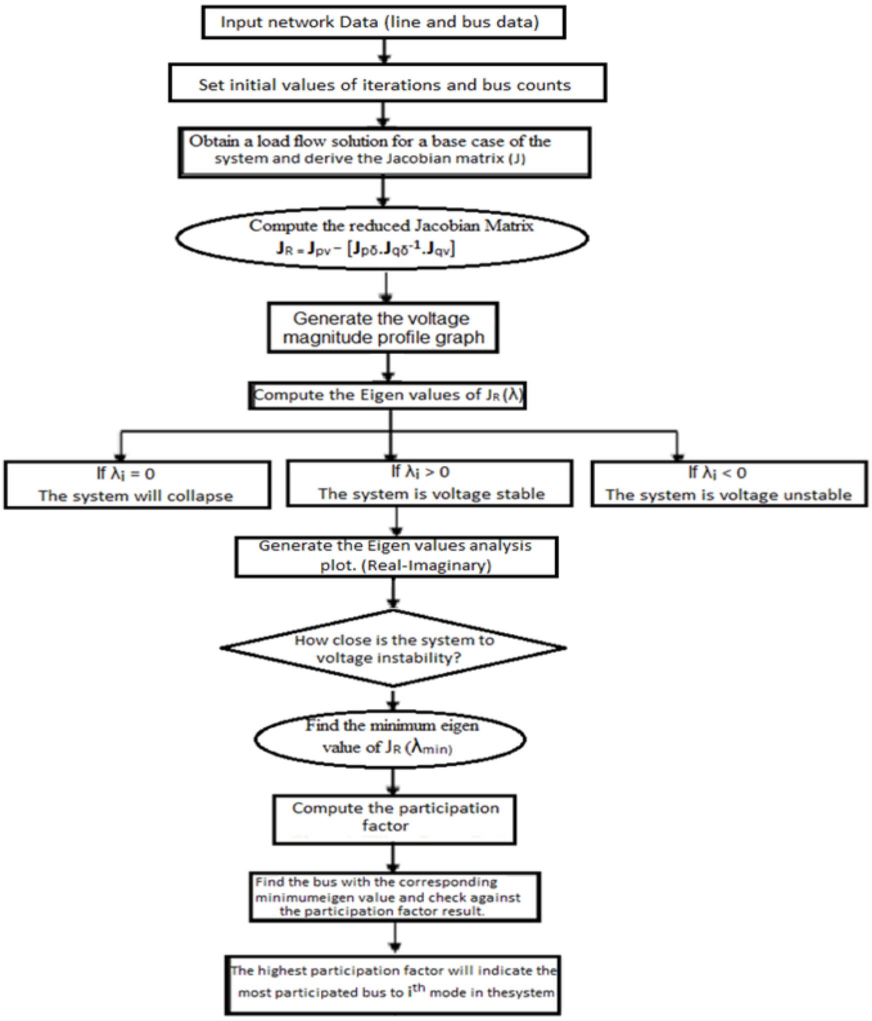

Fig. 1 Flow chart for the Voltage stability investigation of the Nigeria transmission grid using Eigenvalues method.

2.3 Data Collection

The input data for Eigenvalue analysis of voltage stability consist of bus data, transmission line data and transformer data, as shown in tables 1 to 4 respectively. The one – line diagrams of the existing 28-bus and improved 52-bus 330kV Nigeria power networks, as obtained from Power Holding Company of Nigeria(PHCN) are presented figures 2 and 3 respectively [9,10,11]. These parameters were modeled and simulated in MATLAB/SIMULINK Power System Analysis Toolbox (PSAT) program environment using Eigenvalue computational algorithm as shown in figures 4 and 5 respectively.

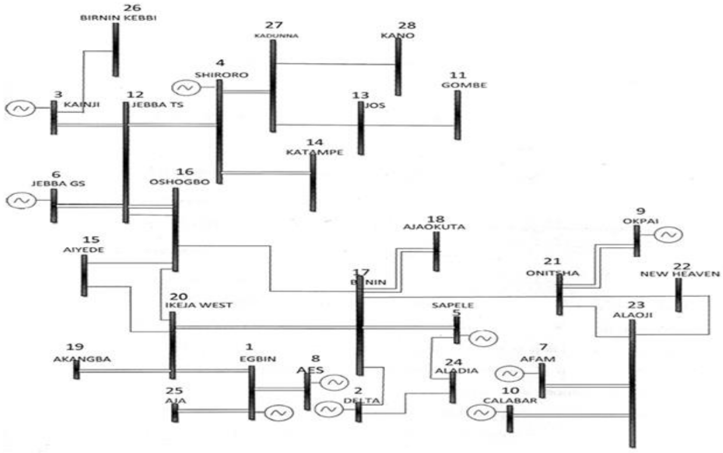

Fig.2 One-line diagram of the existing 28 bus 330kV Nigerian transmission grid.

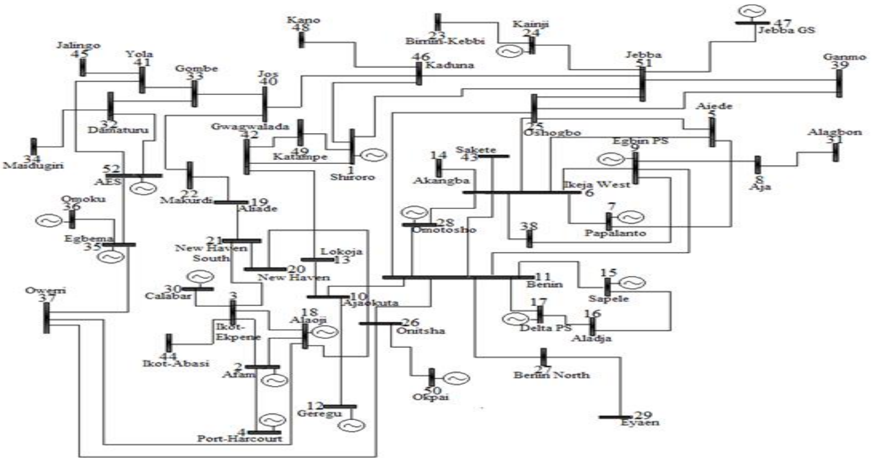

Fig. 3 One-line diagram of the proposed improved 52-bus 330kV integrated Nigerian transmission grid.

Table 1: Bus Data of the 28-bus networks used as input for the Simulation.

Bus No.	Bus Code	Bus voltage (V_{sp})	Phase Voltage (θ^o)	Load		Generation				Tap Setting
				P_{Li}	Q_{Li}	P_{Gi}	Q_{Gi}	Q_{min}	Q_{max}	
1	1	1	0	150	105.62	0	0	-200	200	1
2	2	1	0	200	300	882	0	-300	320	1
3	2	1	0	0	0	760	0	-210	222	1
4	2	1	0	0	0	600	0	-120	140	1
5	2	1	0	0	0	1020	0	-250	260	1
6	2	1	0	0	0	578	0	-200	210	1
7	2	1	0	0	0	931.6	0	-290	300	1
8	2	1	0	0	0	302	0	-100	110	1
9	2	1	0	0	0	480	0	-200	210	1
10	2	1	0	0	0	600	0	-120	140	1
11	3	1	0	0	0	0	0	0	0	1
12	3	1	0	130	80	0	0	0	0	1
13	3	1	0	220	154.8	0	0	0	0	1
14	3	1	0	114	90	0	0	0	0	1
15	3	1	0	110	80	0	0	0	0	1
16	3	1	0	104	70	0	0	0	0	1
17	3	1	0	36	25	0	0	0	0	1
18	3	1	0	72	45	0	0	0	0	1
19	3	1	0	136	84	0	0	0	0	1
20	3	1	0	72	45	0	0	0	0	1
21	3	1	0	39	27.8	0	0	0	0	1
22	3	1	0	84	50	0	0	0	0	1
23	3	1	0	146	84.5	0	0	0	0	1
24	3	1	0	32	17.8	0	0	0	0	1
25	3	1	0	110	80	0	0	0	0	1
26	3	1	0	100	58.4	0	0	0	0	1
27	3	1	0	80	49.6	0	0	0	0	1
28	3	1	0	26	15.3	0	0	0	0	1

Table 2: Bus Data of the 52-bus networks used as input for the Simulation.

Bus No.	Bus Code	Bus voltage (V_{sp})	Phase Voltage (θ^o)	Load		Generation				Tap Setting
				P_{Li}	Q_{Li}	P_{Gi}	Q_{Gi}	Q_{min}	Q_{max}	
1	1	1	0	0	0	0	0	-200	200	1
2	2	1	0	315	157.5	760	428	-210	222	1
3	2	1	0	321	160.5	578	207	-200	210	1
4	2	1	0	316	158	600	298	-120	140	1
5	2	1	0	70.5	35.11	414	207	-100	110	1
6	2	1	0	60.5	30.11	335	167.5	-90	100	1
7	2	1	0	700	350	1020	510	-250	260	1
8	2	1	0	300	150	882	441	-150	160	1
9	2	1	0	110	55	252	126	-200	210	1
10	2	1	0	230	115	480	240	-200	210	1
11	2	1	0	360	80	931.6	465.8	-290	300	1

12	2	1	0	75.1	37.5	300	150	-80	100	1
13	2	1	0	300	150	500	250	-200	210	1
14	2	1	0	200	100	253	126.5	-70	80	1
15	2	1	0	179	89.5	600	298.8	-180	200	1
16	2	1	0	315	157.5	730	365	-279	280	1
17	2	1	0	107.4	53.49	500	250	-100	120	1
18	3	1	0	65	33	0	0	0	0	1
19	3	1	0	136	84	0	0	0	0	1
20	3	1	0	72	45	0	0	0	0	1
21	3	1	0	39	27.8	0	0	0	0	1
22	3	1	0	84	50	0	0	0	0	1
23	3	1	0	146	84.5	0	0	0	0	1
24	3	1	0	32	17.8	0	0	0	0	1
25	3	1	0	110	80	0	0	0	0	1
26	3	1	0	100	58.4	0	0	0	0	1
27	3	1	0	80	49.6	0	0	0	0	1
28	3	1	0	26	15.3	0	0	0	0	1
29	3	1	0	440	220	0	0	0	0	1
30	3	1	0	400	200	0	0	0	0	1
31	3	1	0	400	200	0	0	0	0	1
32	3	1	0	450	225	0	0	0	0	1
33	3	1	0	400	200	0	0	0	0	1
34	3	1	0	440	220	0	0	0	0	1
35	3	1	0	400	200	0	0	0	0	1
36	3	1	0	450	225	0	0	0	0	1
37	3	1	0	400	200	0	0	0	0	1
38	3	1	0	440	220	0	0	0	0	1
39	3	1	0	400	200	0	0	0	0	1
40	3	1	0	450	225	0	0	0	0	1
41	3	1	0	440	220	0	0	0	0	1
42	3	1	0	400	200	0	0	0	0	1
43	3	1	0	450	225	0	0	0	0	1
44	3	1	0	430	215	0	0	0	0	1
45	3	1	0	450	225	0	0	0	0	1
46	3	1	0	460	230	0	0	0	0	1
47	3	1	0	450	225	0	0	0	0	1
48	3	1	0	460	230	0	0	0	0	1
49	3	1	0	480	240	0	0	0	0	1
50	3	1	0	400	200	0	0	0	0	1
51	3	1	0	450	225	0	0	0	0	1
52	3	1	0	440	220	0	0	0	0	1

Table 3: Line Data of the 28-bus networks used as input for the Simulation.

S/N	Transmission line		Line Impedance		B/2 (pu)
	From Bus	To Bus	R (pu)	X (pu)	
1	1	8	0.0001	0.0004	0.0996
2	1	20	0.0004	0.0029	0.0771
3	1	25	0.0007	0.0057	0.0771
4	3	17	0.0008	0.0063	0.3585
5	2	24	0.0008	0.0063	0.3585
6	3	26	0.0041	0.0304	1.8135
7	3	12	0.001	0.0082	0924
8	4	27	0.0011	0.0097	0.546
9	4	12	0.0022	0.0234	1.3905
10	4	14	0.009	0.0067	1.7933
11	5	17	0.0002	0.0015	0.936
12	5	24	0.0008	0.0063	0.3585
13	6	12	0.0001	0.0004	0.0996
14	7	23	0.0015	0.0012	0.312
15	9	21	0.0008	0.0063	0.3585
16	10	23	0.0163	0.014	0.786
17	11	13	0.0032	0.0027	1.515
18	12	16	0.0019	0.0159	0.8955
19	13	27	0.0027	0.0202	1.2114
20	15	16	0.0013	0.01	0.5999
21	15	20	0.0016	0.0134	0.8057
22	16	17	0.003	0.0254	1.431
23	16	20	0.0033	0.0227	1.4819
24	17	18	0.0023	0.0198	1.1117

25	17	20	0.0034	0.0016	1.7015
26	17	21	0.0016	0.0139	0.781
27	19	20	0.0007	0.0057	0.3855
28	21	22	0.0011	0.0097	0.5475
29	21	23	0.0163	0.014	0.786
30	22	23	0.0023	0.0171	1.3905
31	27	28	0.0027	0.0202	1.2114

Table 4: Line Data of the 52-bus networks used as input for the Simulation.

S/N	Transmission line		Line Impedance		B (pu)
	From Bus	To Bus	R (pu)	X (pu)	
1	49	1	0.0029	0.0205	0.308
2	3	18	0.009	0.007	0.104
3	3	3	0.0155	0.0172	0.104
4	3	4	0.006	0.007	0.104
5	19	25	0.0291	0.0349	0.437
6	19	6	0.0341	0.0416	0.521
7	19	7	0.0291	0.0349	0.437
8	8	9	0.0155	0.0172	0.257
9	8	31	0.006	0.007	0.257
10	10	11	0.0126	0.0139	0.208
11	10	12	0.0155	0.0172	0.257
12	10	13	0.0155	0.0172	0.257
13	14	6	0.0155	0.0172	0.065
14	16	15	0.016	0.019	0.239
15	18	37	0.006	0.007	0.308
16	16	17	0.016	0.019	0.239
17	16	26	0.035	0.0419	0.524
18	16	3	0.0155	0.0172	0.257
19	19	21	0.006	0.0007	0.308
20	19	22	0.0205	0.0246	0.308
21	23	24	0.0786	0.0942	1.178
22	11	6	0.0705	0.0779	1.162
23	11	15	0.0126	0.0139	0.208
24	11	17	0.016	0.019	0.239
25	11	25	0.0636	0.0763	0.954
26	11	26	0.0347	0.0416	0.521
27	11	27	0.049	0.056	0.208
28	11	9	0.016	0.019	0.239
29	11	28	0.016	0.019	0.365
30	27	29	0.0126	0.0139	0.208
31	30	3	0.0126	0.0139	0.208
32	32	33	0.0786	0.0942	1.178
33	32	34	0.0786	0.0942	1.178
34	35	36	0.0126	0.0139	0.208
35	35	37	0.0126	0.0139	0.208
36	9	6	0.0155	0.0172	0.257
37	9	38	0.016	0.019	0.239
38	38	6	0.016	0.019	0.239
39	33	25	0.016	0.019	0.239
40	33	51	0.0341	0.0416	0.239
41	33	40	0.067	0.081	1.01
42	33	41	0.0245	0.0292	1.01
43	42	13	0.0156	0.0172	0.257
44	42	1	0.0155	0.0172	0.257
45	6	25	0.0341	0.0416	0.521
46	6	28	0.024	0.0292	0.365
47	6	7	0.0398	0.0477	0.597
48	6	43	0.0398	0.0477	0.521
49	44	3	0.0155	0.0172	0.257
50	51	25	0.0398	0.0477	0.597
51	45	41	0.0126	0.0139	0.208
52	51	47	0.002	0.0022	0.033
53	51	24	0.0205	0.0246	0.308
54	51	1	0.062	0.0702	0.927
55	40	46	0.049	0.0599	0.927
56	40	22	0.002	0.0022	0.308
57	46	48	0.058	0.0699	0.874

58	46	1	0.0249	0.0292	0.364
59	46	1	0.0205	0.0246	0.308
60	20	26	0.024	0.0292	0.365
61	20	21	0.0205	0.0246	0.308
62	50	26	0.006	0.007	0.104
63	26	37	0.006	0.007	0.104
64	3	21	0.0205	0.0246	0.257

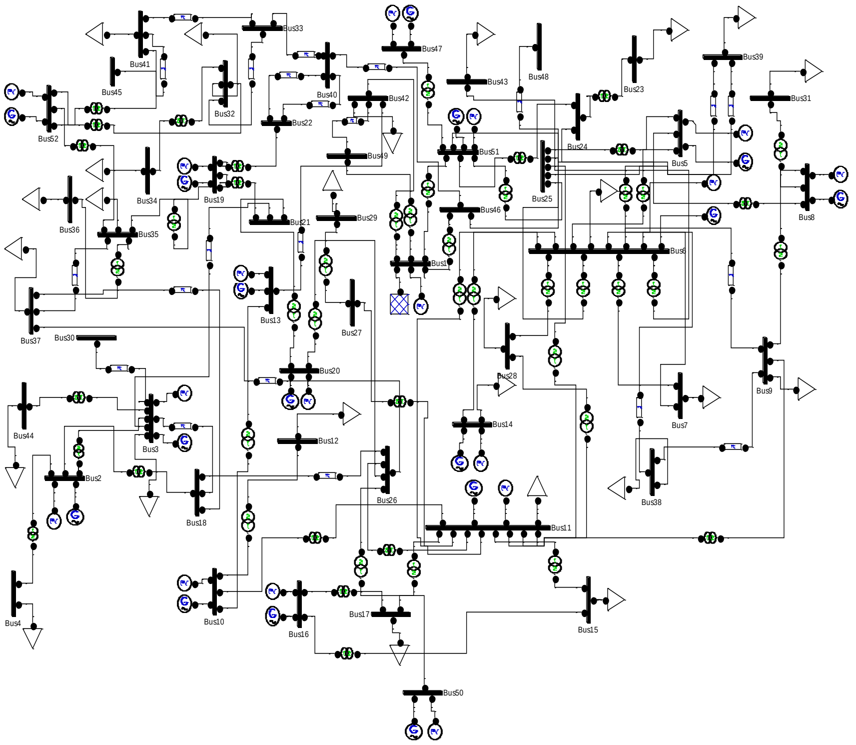

Fig. 4: MATLAB SIMULINK model/diagram of voltage stability investigation of 28 – bus grid network using Eigenvalues method

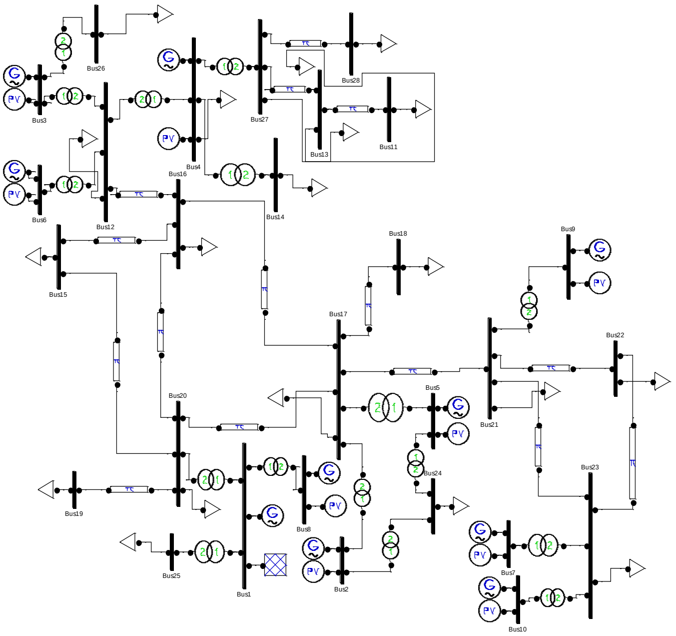

Fig. 5: MATLAB SIMULINK model / diagram of voltage stability investigation of 52– bus grid network using Eigenvalues method.

The modal analysis method was applied to 28-bus and 52-bus network systems and the voltage profile of the buses are presented from the load flow simulation. The eigenvalues of the reduced Jacobian matrix was calculated. After that, the weakest buses, which would cause the system to tend to instability, were identified by computing the participating factors. Eigenvalues analysis was implemented in analyzing both 28-bus and 52-bus networks as illustrated in figures 4 and 5 and the results of the analysis are presented.

III RESULTS AND DISCUSSION

3.1 Eigenvalues Result for 28-bus network

Table 5 gives the simulated results of eigenvalues of power Jacobian matrix of the 28-bus network which comprises the eigenvalues numbered serially, the most assosiated bus, the real and imaginary parts of the eigenvalues.

Table 5: Eigenvalues computational analysis result for 28 bus network.

S/N	Most Associated Bus	Eigen values	
		Real part P (MW)	Imaginary part Q (MVAr)
1	Bus1	51969.4946	0
2	Bus17	15535.2174	0
3	Bus23	16351.1257	0
4	Bus20	6176.703	0
5	Bus24	4453.0512	0
6	Bus4	4321.244	0
7	Bus12	3727.1173	0
8	Bus5	3420.1079	0
9	Bus21	3244.0056	0
10	Bus25	2334.634	0
11	Bus27	1810.9941	0
12	Bus13	1240.9641	0
13	Bus3	1300.772	0
14	Bus10	1021.7758	0
15	Bus28	579.0141	0
16	Bus16	452.4502	0
17	Bus25	414.1856	0
18	Bus22	323.3636	0
19	Bus26	302.2654	0
20	Bus19	273.0337	0
21	Bus10	-1.5857	0
22	Bus11	4.923	0
23	Bus10	21.1973	0
24	Bus11	42.3206	0
25	Bus18	80.3031	0
26	Bus26	141.7944	0
27	Bus26	133.1816	0
28	Bus15	123.8346	0

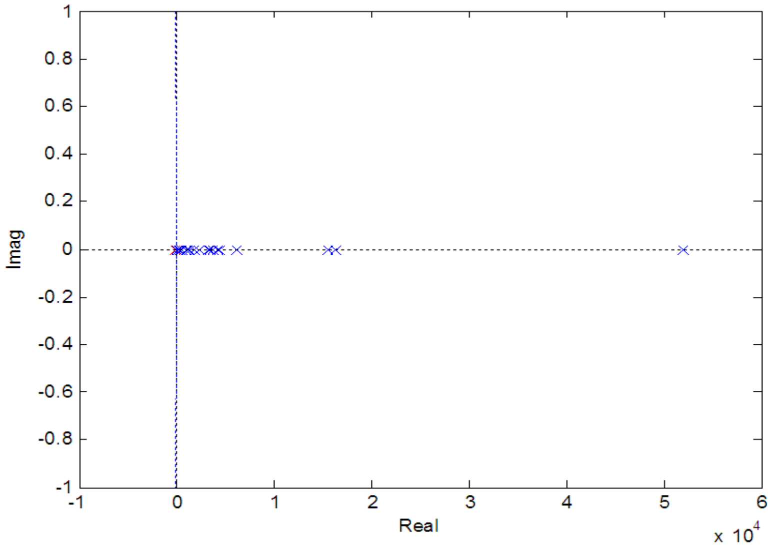

Fig.6 Eigenvalues computational analysis plot for 28-bus network.

The existing 28-bus Nigeria 330kV power network presents positive and negative eigenvalue computation results in 27 buses and one bus for stability and instability respectively, as shown in table 5 and figure 6. The most critical mode occurred at bus 10(Calabar) with minimum eigenvalue of -1.5857MW, whereas, the maximum eigenvalue of 51,969.4946MW occurs at bus 1 (Egbin), as shown in table 5 and figure 6. The computed participation factors results used to determine the weak buses or load buses that contribute to the voltage collapse instability for 28-bus power network are displayed in tables 6 to 9. The largest participation factor value of 0.82795 recorded at bus 18 indicates the highest contribution of this bus to the voltage collapse as shown in table 7.The weakest bus on the 28- bus system is bus 18(Ajaokuta). The statistics of eigenvalue computational analysis results for 28-bus power network is shown in table 9.

Table 6: Computed Participation Factors (Euclidean Norm) for Bus 9 - Bus 16 of the 28 bus network.

Eigenvalues No.	Bus 9	Bus 10	Bus 11	Bus 12	Bus 13	Bus 14	Bus 15	Bus 16
#1	0	0	0	0	0	0	0	0
#2	0	0	0	0	0	0	0	0
#3	0	0.00099	0	0	0	0	0	0
#4	0	0	0	0	0	0	0.00026	0
#5	2e-005	0	0	0	0	0	0	0
#6	0	0	0	0.28695	3e-005	0.11264	0	0.00017
#7	0	0	0	0.4234	4e-005	0.09046	0	0.00035
#8	0.05197	0	0	0	0	0	0	1e-005
#9	0.3882	0	0	0	0	0	0	0
#10	5e-005	0	0	0	0	0	0.00055	0
#11	0	0	0.0013	5e-005	0.00543	0.21508	0	0
#12	0	0	0.46574	0	0.52738	0.00189	0	0
#13	0	0	0	0.00439	0	0.00038	0	4e-005
#14	5e-005	0.62319	0	0	0	0	0	0
#15	0	0	0	0.01004	0	0.37848	0.00052	0.00203
#16	0.00069	0	1e-005	0.00414	0	0.00114	0.35072	0.35791
#17	0.00397	0	1e-005	0.00347	0	0.00038	0.08409	0.15571
#18	0.18756	0.00609	0	0.00017	0	0	8e-005	0.00033
#19	6e-005	1e-005	0.00108	0.13803	0.00025	0.00103	0.05996	0.00356
#20	0.00015	0.00111	0	2e-005	0	0	0.00065	0.0001
#21	0.03493	0.03456	0.037	0.03496	0.03688	0.0354	0.0382	0.03776
#22	0.03731	0.05264	0.09167	0.04299	0.08932	0.05539	0.00035	0.00257
#23	0.03009	0.13828	0.02088	0.00029	0.01922	0.00204	0.02678	0.01293
#24	0	0.00132	0.35882	0.04314	0.30712	0.01893	0.00389	0.01462
#25	0.00043	0.00095	0.00172	0.0005	0.0013	0.00184	0.03685	0.02206
#26	0.02926	0.01379	0.00642	0.00434	0.00369	0.02778	0.13036	0.1168
#27	0.15321	0.07835	0.00812	0.00267	0.00482	0.03234	0.02286	0.03372
#28	0.08164	0.04871	0.00723	0.00044	0.00451	0.02479	0.24387	0.2393

Table 7: Computed Participation Factors (Euclidean Norm) for Bus 17 - Bus 24 of the 28 bus network.

Eigenvalues No.	*Bus 17*	*Bus 18*	*Bus 19*	*Bus 20*	*Bus 21*	*Bus 22*	*Bus 23*	*Bus 24*
#1	0	0	0	0.00286	0	0	0	0
#2	0.5326	1e-005	0	0.00434	4e-005	0	0	0.00934
#3	0	0	0	0	2e-005	2e-005	0.52713	0
#4	0.00389	0	0.0015	0.66704	0	0	0	0.00621
#5	0.01639	1e-005	1e-005	0.00126	7e-005	0	0	0.50599
#6	0	0	0	0	0	0	0	0
#7	0	0	0	0	0	0	0	0
#8	0.21386	0.00012	0	0.00042	0.07442	0.0002	1e-005	0.14114
#9	0.03064	2e-005	0	0	0.48342	0.00139	5e-005	0.02425
#10	0.00603	1e-005	0.00316	0.17007	1e-005	0	0	0.01916
#11	0	0	0	0	0	0	0	0
#12	0	0	0	0	0	0	0	0
#13	0	0	0	0	0	0	0	0
#14	0	0	0	0	5e-005	0.00209	0.16146	0
#15	0	0	0	0	0	0	0	0
#16	0.00962	0.00044	0.03059	0.01148	0.00035	0.00026	0	0.02328
#17	0.03401	0.00189	0.13324	0.03092	0.00214	0.00238	0	0.07557
#18	0.00098	9e-005	0.02498	0.00082	0.11663	0.64669	0.00172	0.0018
#19	0.00022	3e-005	0.00319	3e-005	4e-005	0.00057	0	0.00039
#20	0.05761	0.00919	0.63956	0.00024	0.0001	0.0363	0.00037	.09461
#21	0.03539	0.03894	0.03589	0.0359	0.035	0.03517	0.03463	0.03517
#22	0.01389	0.01851	0.01248	0.01188	0.03706	0.04478	0.05163	0.01395
#23	0.02908	0.06665	0.04365	0.03675	0.02925	0.06997	0.12918	0.03003

#24	0.00208	0.01321	0.00233	0.00166	0	0.00025	0.00116	0.00223
#25	0.00232	0.82795	0.02531	0.01298	0.00038	1e-005	0.00073	0.00264
#26	0.00348	0.00465	0.01788	0.00423	0.02416	0.02205	0.00866	0.00444
#27	0.00138	0.00225	0.01999	0.00524	0.12775	0.09782	0.05056	0.00174
#28	0.00652	0.01604	0.00623	0.00188	0.0691	0.04005	0.0327	0.00805

Table 8: Computed Participation Factors (Euclidean Norm) for Bus 25 - Bus 28 of the 28 bus network.

Eigenvalues No.	Bus 25	Bus 26	Bus 27	Bus 28
#1	0.00065	0	0	0
#2	0	0	0	0
#3	0	0	0	0
#4	0.01521	0	0	0
#5	0.00042	0	0	0
#6	0	0.0001	0.06776	0.00166
#7	0	0.00032	0.05906	0.00204
#8	0.0151	0	0	0
#9	0.00282	0	0	0
#10	0.61103	0	0	0
#11	0	1e-005	0.62201	0.14235
#12	0	0	0.00269	0.00224
#13	0	0.0162	0	0
#14	0	0	0	0
#15	0	0.00426	3e-005	0.41255
#16	0.07089	0.00281	0.00036	0.0063
#17	0.15592	0.00285	0.00033	0.0041
#18	0.00275	0.00027	3e-005	0.00015
#19	9e-005	0.28415	0.02967	0.1293
#20	0.00062	7e-005	1e-005	2e-005
#21	0.0362	0.03463	0.03561	0.03573
#22	0.01223	0.04681	0.05874	0.06032
#23	0.03971	0.00041	0.00289	0.00314
#24	0.00191	0.08518	0.00829	0.0097
#25	0.01672	0.0021	0.00173	0.00233
#26	0.00675	0.38484	0.03797	0.06658
#27	0.00815	0.12485	0.04233	0.0719
#28	0.00283	0.01015	0.0305	0.04957

Table 9: Statistics of Eigenvalues computational analysis results for the 28 bus network.

S/N	DESCRIPTION	VALUE
1	Number of Buses	28
2	Number of Eigenvalues with Real part < 0	1
3	Number of Eigenvalues with Real part > 0	27
4	Number of Real Eigenvalues	28
5	Number of Complex pairs	0
6	Number of Zero Eigenvalues	0

3.2 Eigenvalues Result for 52-bus network

Table 10 gives the simulated results of eigenvalues of the power Jacobian matrix of the 52-bus network which comprises the eigenvalues numbered serially, the most associated bus, the real and imaginary parts of the eigenvalues.

Table 10: Eigenvalues computational analysis result for 52 bus network.

S/N	Most Associated Bus	Eigen values	
		Real part P (MW)	Imaginary part Q (MVAr)
1	32	1464.9761	0
2	11	479.8526	0
3	51	419.5145	0
4	6	387.5623	0
5	35	230.0423	0
6	25	220.9453	0
7	8	209.4432	0
8	2	206.6919	0
9	26	198.9949	0
10	52	181.0412	0
11	10	170.902	0
12	16	151.554	0
13	3	147.843	0

14	24	142.2904	0
15	24	136.4007	0
16	35	132.0797	0
17	22	115.2072	0
18	27	112.183	0
19	9	91.7813	0
20	46	89.7678	0
21	42	87.0869	0
22	28	76.5811	0
23	18	74.1289	0
24	46	71.6405	0
25	41	61.3271	0
26	49	57.2954	0
27	47	57.5992	0
28	45	0.67785	0
29	45	2.5769	0
30	30	4.3892	0
31	14	50.6219	0
32	33	49.1877	0
33	43	45.1424	0
34	42	45.3042	0
35	50	41.2736	0
36	37	38.9977	0
37	48	10.7597	0
38	23	11.4015	0
39	45	12.3235	0
40	31	29.7276	0
41	44	30.5156	0
42	38	28.1254	0
43	39	21.8864	0
44	38	22.6408	0
45	50	24.8205	0
46	40	24.1581	0
47	4	18.2338	0
48	29	16.7164	0
49	30	15.3974	0
50	12	15.8207	0
51	17	90.44	0
52	1	1998	0

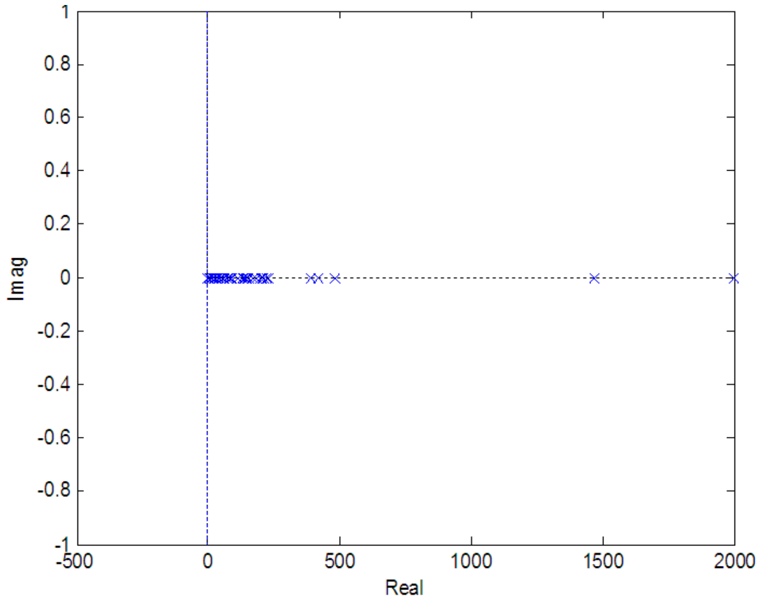

Fig. 7: Eigenvalues computational analysis plot for 52 bus network.

The 52-bus 330kV Nigerian power network presents positive eigenvalues computational results in all the buses, which quantify voltage stability in all these buses as shown in table 10 and figure 7 respectively. The result shows that, the highest computed participation factors at bus 45 are 0.43087 and 0.12395 with Eigenvalues numbered 39 and 37 respectively corresponding to bus 45 (Jalingo bus). The largest participation factor value 0.43087 at bus 45 indicates the highest contribution of this bus to the voltage collapse instability.

The weakest bus on the 52 bus power system is identified as bus 45 (Jalingo bus).The part of computed participation factor used in determining the weak buses is given in table 11. The statistics of eigenvalue computational analysis results for 52-bus power network is shown in table 12.

Table 11: Computed Participation Factors (Euclidean Norm) for bus 41 - bus 48 of the 52-bus network.

Eigenvalues No.	Bus 41	Bus 42	Bus 43	Bus 44	Bus 45	Bus 46	Bus 47	Bus 48
#1	0	0	0	0	0	0	0	0
#2	0	0		0	0	0	0.00075	0
#3	0	0	0.00112	0	0	0	0.21345	0
#4	0	0	0.01251	0	0	0	0.02881	0
#5	0.01376	0	0	0.00037	6e-005	1e-005	0.00012	0
#6	0.00026	0	0.00051	1e-005	0	0	0.01595	0
#7	0.00215	0	4e-005	0.00446	1e-005	0	0.0118	0
#8	0.00195	1e-005	9e-005	0.01311	1e-005	0	0.01284	0
#9	0.00211	3e-005	2e-005	0.01147	1e-005	0	0.00223	0
#10	0.06035	4e-005	0	0.00016	0.00047	9e-005	3e-005	0
#11	0.00054	0.00309	0	5e-005	0	0	0.00039	0
#12	0.00049	0.00026	7e-005	0	1e-005	0	0.00124	0
#13	0.01616	0	0	0.06399	0.00019	9e-005	1e-005	0
#14	1e-005	0	0.00029	1e-005	0	0	0.06867	0
#15	0.00489	0.00019	0.00082	0.00161	7e-005	0	0.00069	0
#16	0.09892	4e-005	3e-005	0.01607	0.00153	2e-005	5e-005	0
#17	0.02592	1e-005	0	0.00637	0.00055	0.01288	0	0.00028
#18	0.00205	0.00047	4e-005	0.00136	5e-005	0.0028	0.00022	7e-005
#19	9e-005	0.00104	0.0004	0	0	0.00209	0.01613	8e-005
#20	0.018	0.0001	5e-005	0.00088	0.00068	0.41987	3e-005	0.01655
#21	0	0.67383	9e-005	0	0	0	0	0
#22	3e-005	0.00517	0.00818	0.00026	0	0.01129	0.00388	0.00066
#23	0	0	0	0.06087	0	0.05445	1e-005	0.00343
#24	0.05757	5e-005	8e-005	0.00661	0.00379	0.39166	0.00014	0.02688
#25	0.42439	0	0	0.00186	0.04163	0.00562	1e-005	0.00058
#26	3e-005	0.03192	1e-005	0	0	1e-005	0.0043	0
#27	0	0.00017	0.02265	0	0	0	0.44404	0
#28	0.03893	0.00028	0.01236	0.03874	0.04281	0.00129	0.00381	0.00141
#29	0.04544	0.00066	0.0317	0.00128	0.06696	0.00091	0.01117	0.00133
#30	0.03597	8e-005	0.00446	0.11822	0.07344	0.00047	0.002	0.00095
#31	5e-005	0.00033	0.11624	0	1e-005	2e-005	0.08952	0
#32	0.10223	0	2e-005	0.00773	0.01831	0.01904	0	0.00357
#33	0	0.00021	0.54457	0	0	0	0.00036	0
#34	1e-005	0.2648	0.00038	0	0	0	2e-005	0
#35	0.01695	0	0.0001	0.01638	0.00511	0.0014	0	0.00045
#36	0.00615	0	0	0.0073	0.00221	0.00269	0	0.00102
#37	0.00866	3e-005	5e-005	0.00176	0.12395	0.0538	0.00052	0.70404
#38	0.00109	0.00186	5e-005	0.00304	0.02245	0.00366	0.0189	0.06734
#39	0.01068	0.00054	0.00058	0.00349	0.43087	0.00402	0.00174	0.13702
#40	0	0	0.01074	0	0	0	0.00345	0
#41	3e-005	0	0	0.57028	2e-005	0.00073	0	0.00066
#42	0	0	0.10214	0	0	0	0.00017	0
#43	0.00016	0.00057	0.00166	0.00112	0.00066	0.00022	0.0323	0.00097
#44	0.00025	0.00058	0.02187	0.00321	0.00083	0.00048	0.01013	0.00177
#45	0.00011	6e-005	0.00073	0.00475	0.00023	9e-005	0	0.00021
#46	0.00013	3e-005	0.00061	0.0125	0.0003	0.01008	0	0.02585
#47	0.00315	4e-005	0.00217	0.0196	0.05176	0.00021	3e-005	0.00402
#48	0	0.00101	0.09698	0.00013	1e-005	0	0	0
#49	0.0003	0.00102	6e-005	0.00082	0.10936	0	0	0.00086
#50	1e-005	0.01144	0.0045	0.0001	0.00164	0	5e-005	0
#51	0	0	0	0	0	0	0	0
#52	0	0	0	0	0	0	0	0

Table 12: Statistics of Eigenvalues computational analysis result for the 52 bus network.

S/N	DESCRIPTION	VALUE
1	Number of Buses	52
2	Number of Eigenvalues with Real part < 0	0
3	Number of Eigenvalues with Real part > 0	52
4	Number of Real Eigenvalues	52
5	Number of Complex pairs	0
6	Number of Zero Eigenvalues	0

IV CONCLUSION

Eigenvalue voltage stability analysis was conducted for both the existing 28-bus network and proposed 52-bus network with MATLAB SIMULINK Power System Analysis Toolbox (PSAT) program. A comparative analysis was carried out on the eigenvalue computational results obtained from the two networks to determine the voltage instability and those weakest buses that contribute to system's instability. The Eigenvalue computational analysis results showed that the existing 28-bus power system is unstable at only bus 10 (Calabar bus) with a negative Eigenvalue of -1.5857MW, and stable in 27 buses with positive eigenvalues. The weakest bus identified in 28-bus power network is bus 18 (Ajaokuta bus), and the highest computed participation factors, which contribute to this instability are 0.82795MW and 0.06665MW eigenvalues numbered 25 and 23 respectively.

The bus voltage profile of 52-bus power network showed that all the buses are stable for static load without any bus violated. The 52-bus power network presents positive eigenvalues computational analysis result in all the buses, which proved the stability of this network. Bus 45 (Jalingo bus) with the least eigenvalue computational analysis result of 0.67785MW was considered as the weakest mode in this power network.

The computed participation factor for bus 45(Jalingo bus) was read to ascertain the weakest bus. The highest computed participation factor values are 0.43087 and 0.12395 obtained for eigenvalues numbered 39 and 37 respectively corresponding to bus 45 (Jalingo bus),which is the weakest bus, that contributes most to the system's instability.

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