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Vehicle Routing Problem Using Savings-Insertion and Reactive Tabu with a Variable Threshold

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ABSTRACT: *This paper focuses on how the competitiveness of companies is impacted by products distribution and transportation costs, especially in the context of exports. This paper uses our two-steps approach, consisting of building a good initial solution and then improving it to solve vehicle routing problem. To that end, first, our mathematical model is adapted; secondly, our Savings-insertion builds a good initial solution, and thirdly, our Reactive tabu with a variable threshold improves the initial solution. The objective is the minimization of the total distance of transport by respecting the specified time window and the demand of all customers, which are important for some transportation companies'. Finally, the experimental results obtained with our methodology for Solomon 100 customers 56 vehicle routing problems are provided and discussed. Our methodology provides the best solutions for problems types R2 and RC2. Therefore, using our methodology reduces the total distance for long-haul vehicle routing problems.*

Keywords: *Insertion, reactive tabu, savings, time windows, vehicle routing.*

I. INTRODUCTION

Product distribution is a process by which products are moved from sources to customers. According to Toth and Vigo [1], transportation costs typically represent 10-20% of the final price of goods on the market. Over the past 20 years, the Vehicle Routing Problem (VRP) was mainly solved through the use of meta-heuristics. Eksioglu et al. [2] carried out a taxonomic review of VRP characterizing this research field and conducted a detailed classification of variants with many examples. Following the review of previous classifications and taxonomies, major journals having published articles on the subject issue are listed, and a taxonomy is proposed. They conducted a classification by type of study, scenario characteristics, physical characteristics of the problem, and by characteristics of information and data used. Potvin [3] in turn conducted a review of biologically-inspired algorithms used to handle the VRP. He highlighted the different variants of the problem and the different methodologies used to solve them. These include evolutionary algorithms, ant colonies, particle swarm optimization, neural networks, artificial immune systems and hybrid algorithms. Gendreau and Tarantilis [4] for their part conducted a review of the state of the art of large scale VRP, indicating the difficulty of solving the problems of more than 100 customers with exact methods. They criticized the major works on large scale VRP by highlighting the techniques used. The review compared the performance of different algorithms and conducted an analysis based on key attributes such as effectiveness, efficiency, simplicity, and flexibility. Karakatič and Podgorelec [5] in turn made a survey of genetic algorithms for solving multi-depot VRP. They highlighted the different variants of the problem and the different genetic algorithms approaches used to solve them. This review focuses on the comparison of different selection, crossover and mutation operators used by the researchers.

Thangiah et al. [6] developed a heuristic model with pickups, deliveries and time windows. Each route starts with a first customer and the other are inserted one by one until the capacity or time limit constraint is not reached. Then, this initial solution is improved using local search heuristics specific to routing problems, namely, the exchange of customers between routes and crossover of parts of routes. Berger and Barkaoui [7] developed for the VRP a hybrid genetic algorithm based on the simultaneous evolution of two populations of solutions for different purposes and a time constraint relaxation. The first population evolved individuals to minimize the total distance traveled while the second minimize the time constraint violation to generate a feasible solution. Flisberg et al. [8] proposed a multi-depot forest transportation model. The resolution method they used involved the generation of transport nodes by solving the linear programming problem of flow distribution and routing of these nodes using a tabu search. Li et al. [9] proposed a multi-depot VRP with a simultaneous delivery and pick-up model. The resolution method used was the iterated local search embedded adaptive neighborhood selection approach. McNabb et al. [10] tested local search move operators on the VRP with split deliveries and time windows. To that end, they used eight local search operators, in combinations of up to three of them, paired with a max-min ant system. According to Dhaenens et al. [11], the VRP is assimilated to an extension of the traveling salesman problem. According to Ismail et al. [12], this problem is known as an NP-complete problem. Therefore, the VRP is NP-complete.

The goal of this paper is to apply our methodology based on Savings-insertion, followed by the Reactive tabu with a variable threshold (SI&RTVT), to solve every type of VRP minimizing the total distance of transport. The minimization of the total distance of transport is the most important objective for some transportation companies'. Our main contribution is to generalize our VRP solving methodology. This methodology provides a good solution for every type of VRP.

In the next section, we remember our methodology: first, we describe the problem and adapt our mathematical model to the minimization of the total distance of transport; secondly, we remember our global methodology; thirdly, we remember our Savings-insertion heuristic, and fourthly, we remember our Reactive tabu with a variable threshold meta-heuristic. In the third section, we present our results, followed by a discussion, and finally, we end with a conclusion in the fourth section.

II. METHODOLOGY

2.1 Problem description and mathematical model

To perform VRP optimization, we remember our optimization based on Savings-insertion, followed by the Reactive tabu with a variable threshold. This allows the minimization of the total distance of transport, including hard capacity and hard time windows. Below, we present our mathematical model (see Bagayoko et al. [13], Bagayoko et al. [14], Bagayoko et al. [15] and Bagayoko et al. [16]), adapted to this objective.

Let us assume that m vehicles, with a load capacity of Q, are needed. There are Lcustomers and one depot, which takes the index 1 at the start of the route and the index $L+2$ at the route end. The fleet is homogeneous, and every customer demand must be satisfied within his time window. We split every customer having a demand upper than the vehicles' load capacity to get each customer demand lower than or equal to the vehicles' load capacity. The following assumptions are made in modeling the problem:

Assumptions

i. Each customer location (x_j, y_j), demand q_j, and time window (t^s_j = start time, t^e_j = end time) are known;
ii. Each customer is served only by one vehicle at a time;
iii. Each vehicle leaves the depot (index 1) and returns to the depot (index $L+2$);
iv. All vehicles needed are immediately available;
v. The average vehicle moving speed VS is known;
vi. Each customer demand q_j is lower than or equal to vehicles' load capacity Q.

Notation

x_j, y_j	customer j location
t^s_j	customer j start time
t^e_j	customer j end time
q_j	customer j demand
D_t	total distance of transport
m	number of vehicles used
VS	average moving speed of vehicles
Q	vehicles' load capacity
L	number of customers
d_{ij}	distance between two locations i and j
x_{ijk}	indicates if vehicle k goes from i to j
t_{jk}	arrival time of vehicle k to customer j
w_{jk}	waiting time for vehicle k at customer j
s_j	customer j service time
t_{ij}	time spent from i to j
y_{jk}	indicates if customer j is served by vehicle k
T_k	end of vehicle k time

The studied problem is modeled and the mathematical model objective is givenin (1):

$$\min D_t = \sum_{i=1}^{L+1}\sum_{j=2}^{L+2}\sum_{k=1}^{m} d_{ij} x_{ijk} \quad (1)$$

This objective function is the total distance of transport, which is the summation of all distances traveled. The time spent going from *i* to *j* is:

$$t_{ij} = d_{ij} / VS \quad i \in [1, L+1],\ j \in [2, L+2] \quad (2)$$

The waiting time for vehicle *k* at customer *j*is:

$$w_{jk} = \max(t_j^s - t_{jk}, 0) \quad j \in [2, L+2], k \in [1, m] \quad (3)$$

There are eleven constraints restrictions:
The (4) is the first constraint, and imposes the condition that the variabley_{jk} be binary. The (5) is the second constraint, and imposes the condition that the variable x_{ijk} be binary.

$$y_{jk} = \begin{cases} 1, \text{ the customer } j \text{ is served by the vehicle } k \\ 0, \text{ otherwise} \end{cases} \quad (4)$$

$$x_{ijk} = \begin{cases} 1, \text{ the vehicle } k \text{ goes from } i \text{ to } j \\ 0, \text{ otherwise} \end{cases} \quad (5)$$

The (6) is the third constraint, and imposes the condition that every vehicle leaves the depot (index *1*). The (7) is the fourth constraint, and imposes the condition that every vehicle returns to the depot (index *L+2*).

$$\sum_{k=1}^{m} y_{1k} = m \quad (6)$$

$$\sum_{k=1}^{m} y_{\{L+2\}k} = m \quad (7)$$

The (8) is the fifth constraint, and imposes the condition that every customer is served only by one vehicle. The (9) is the sixth constraint, and for each customer j, it means that the customer is served only by one vehicle passing through only one othercustomer. The (10) is the seventh constraint, and indicates that the total load for vehicle *k* cannot exceed the vehicles' load capacity *Q*.

$$\sum_{k=1}^{m} y_{jk} = 1 \quad j \in [2, L+1] \quad (8)$$

$$\sum_{i=1}^{L+1} x_{ijk} = y_{jk} \quad j \in [2, L+1],\ k \in [1, m] \quad (9)$$

$$\sum_{j=2}^{L+1} q_j y_{jk} \le Q \quad k \in [1, m] \quad (10)$$

The (11) is the eighth constraint, and gives the relation between vehicle *k* arrival time to the customer *i* and its arrival time to customer *j*. The (12) is the ninth constraint, and indicates that the service at customer *j* must begin before T_k, the end of the vehicle *k* time.

$$t_{jk} = x_{ijk}(t_{ik} + w_{ik} + s_i + t_{ij}) \quad i \in [1, L+1], j \in [2, L+2], k \in [1, m] \quad (11)$$

$$t_{jk} + w_{jk} \le T_k \quad j \in [2, L+2], k \in [1, m] \quad (12)$$

The (13) is the tenth constraint, and indicates that no customer can be served before his start time. The (14) is the eleventh constraint, and indicates that no customer can be served after his end time.

$$t_{jk} + w_{jk} \ge t_j^s \quad j \in [2, L+2], k \in [1, m] \quad (13)$$

$$t_{jk} + w_{jk} \le t_j^e \quad j \in [2, L+2], k \in [1, m] \quad (14)$$

These eleven constraints restrictions allow the realization of the objective of minimizing the total distance of transport by obtaining a feasible solution directly. In the next subsection, we remember our global methodology proposed to solve this mixed-integer linear programming problem.

2.2 Global methodology

The proposed approach to solve the VRP consists of two-steps. The initial solution technique for the VRP based on our Savings-insertion technique for generating the initial solution is developed (subsection 2.3.) in order to serve as the starting point of our improvement technique. Our improvement technique for VRP is based on our Reactive tabu with a variable threshold, and is used to improve the initial solution (subsection

2.4.). This allows us to find a good solution of the problem. The different steps of the global methodology (see the flow chart in Bagayoko et al. [16]) are presented in the following pseudo-code:

- Import VRP data
- Initialize Savings-insertion parameters
- Generate an initial solution S using Savings-insertion
- Initialize Reactive tabu with a variable threshold parameters
- Set best solution Sbest = Initial solution S
- Improve the initial solution using Reactive tabu with a variable threshold
- Export two best VRP solutions

In the next subsection, we remember our initial Savings-insertion solution technique.

2.3 Initial solution by Savings-insertion heuristic

Based on the Clarke and Wright [17] saving algorithm, our first algorithm (see the flow chart in Bagayoko et al. [16]) is remembered in the following pseudo-code.

i. Import VRP data
ii. Initialize Savings-insertion parameters
iii. For each customer j, create a full route if $q_j = Q$
iv. Calculate number of smallest and biggest customers to insert according to the values of the percentage of smallest customers and the percentage of biggest customers
v. Order customers in ascending $q_j(t_{1j}+t_{j1})(t^s_j+t^e_j)$
vi. Remove insertion customers, remaining customers are savings customers
vii. Calculate the saving matrix of pairs of customers
viii. Construct routes using savings technique, each time applying local-shift and local-swap to the affected route
ix. Order insertion customers in descending $q_j(t_{1j}+t_{j1})(t^s_j+t^e_j)$
x. Insert insertion customers one by one using the maximum saving technique, each time inserting the best customer in the best position of the best route; if the maximum saving is not positive, create a new route using the first customer
xi. For all routes with more than one customer, apply local-shift and for all routes with more than two clients, apply the local swap to minimize the objective
xii. For 0 to the maximum percentage of smallest customers and for 0 to the maximum percentage of biggest clients, do ii to xi
xiii. Export best Savings-insertion solution

This Savings-insertion technique extends the Clarke and Wright [17] savings heuristic by adding the insertion of smallest customers (using a percentage of smallest customers) and biggest customers (using a percentage of biggest customers) according to the value of $q_j(t_{1j}+t_{j1})(t^s_j+t^e_j)$. In the next subsection, we remember our Reactive tabu with a variable threshold.

2.4 Improvement by Reactive tabu with a variable threshold meta-heuristic

The tabu search is a meta-heuristic aims to avoid the weakness of neighborhood search algorithms which is its possible trap into local optima, by allowing non-improving moves. Let us remember our second main algorithm (see the flow chart in Bagayoko et al. [16]). The optimization technique for the Reactive tabu with a variable threshold aimed at improving the initial solution (improvement) in order to find a good solution of the problem. It can quickly check the feasibility of the movement suggested, and then react to the repetition to guide the search. This algorithm is performed via a tabu list size (tls) update mechanism elaborated in five steps. The different steps of the technique are remembered in the following pseudo-code:

i. Import VRP data and initial solution
ii. Initialize *Reactive tabu with a variable threshold* parameters, Set Best solution *Sbest* = Initial solution *S*
iii. If repetition of solution *S*, then:
CurTimeRept = NbIt
GapRept = CurTimeRept- LasTimeRept (S)
LasTimeRept (S) = NbIt
Repetition(S) = Repetition (S) +1
Else
LasTimeRept (S) = NbIt
Store the solution *S*
Go to vii.
iv. If *repetition of solution S > REP*, then:

Chaotic = Chaotic+1
Else go to vi.
v. If *Chaotic > Chaos*, then:
Clear and rebuild *Reactive tabu with a variable threshold* data structures, Apply Escape
vi. If *GapRept<GapMax*, then:
*tls = tls*Increase, LastChange = 0*
*MovAvg = 0.1*GapRept + 0.9*MovAvg*
Go to viii
Else go to viii.
vii. If *LastChange>MovAvg*, then:
*tls = max (tls*Decrease, Threshold)*
LastChange = 0
viii. If the termination criterion is not satisfied, then:
Solution *S'* = Best solution not tabu or new best solution overall, exploring the neighborhood of *S* using *λ-interchange*
Solution *S''* = Application of *local-shift* and *local-swap* to the two affected routes, set *S = S''* as the current solution
Tabu = max (Tabu-1, 0)
NbIt = NbIt+1, Tabu (S) = tls
If *Cost (S) <Cost (Sbest)* then set *Sbest = S*
Go to iii.
Else
Threshold = Threshold+1
Go to ii.
ix. For 1 to maximum *Threshold*, do ii to viii
x. Export two best *Reactive tabu with a variable threshold* solutions
This Reactive tabu with a variable threshold algorithm is an extension of the Reactive tabu search developed by Battiti and Tecchiolli [18]. The counters and parameters used in Reactive tabu with a variable threshold are defined in accordance with Wassan, Wassan and Nagy [19]as follows, and initialized to the following values.
Minimum of tabu list size (*tls*) value: *Threshold= 1 to 10*
Counter for often-repeated sets of solutions: *Chaotic = 0*
Moving average for the detected repetitions: *MovAvg= 0*
Gap between two consecutive repetitions: *GapRept= 0*
Number of iterations since the last change in *tls*value: *LastChange= 0*
Iteration number when an identical solution was last noticed: *LasTimeRept= 0*
Iteration number of the most recent repetition: *CurTimeRept = 0*
Maximum limit for often-repeated solutions: *REP = 5*
Maximum limit for sets of often-repeated solutions: *Chaos = 5*
Increase factor for the tabu tenure value: *Increase = 2*
Decrease factor for the tabu tenure value: *Decrease = 0.5*
The constant used for comparison with GapRept to get the moving average: *GapMax= 100.*

According to Thangiah et al. [6], "The *λ-interchange* method is based on the interchange of customers between sets of routes. This technique generation mechanism can be described as follows. Given a solution to the problem represented by the set of routes $S = \{R_1,\ldots,R_p,\ldots,R_q,\ldots,R_k\}$, where each route is the set of customers serviced on this route, a *λ -interchange* between R_p and R_q is the replacement of a subset of customers $S_1 \subseteq Rp$ of size $|S_1| \leq \lambda$ by another subset $S_2 \subseteq Rq$ of size $|S_2| \leq \lambda$, and vice-versa, to get two new routes $R'_p= (R_p - S_1) + S_2$ and $R'_q= (R_q - S_2) + S_1$ and a new neighboring solution $S' = \{R_1,\ldots,R'_p,\ldots,R'_q,\ldots,R_k\}$". In this work, we limited ourselves to sequences of consecutive customers. The neighborhood $N_\lambda(S)$ of a given solution S is the set of all neighbors S' generated in this manner for a given value of λ. We established our *1-interchange+* by adding the operators (2, 1) and (1, 2) to *1-interchange*. Thus, we can more explore the search space than the *1-interchange* in less time than the *2-interchange.*

Below, we remember details of our Reactive tabu with a variable threshold, which updates the value of the Tabu list size (*tls*) during the search, according to repetitions(see Bagayoko et al. [16]). First, our Reactive tabu with a variable threshold extends the Reactive tabu by adding a parameter for setting a minimum value of the tabu list size (*tls*) called the threshold. Secondly, in accordance with Wassan, Wassan and Nagy [19], we use local-shift, which is an intra-route move that relocates a customer to a different position within the route, if doing so improves the solution quality. Thirdly, we similarly use local-swap, which is an intra-route move that exchanges the positions of two customers within the route, if doing so improves the solution quality. In the next section, the experimental results of our initial Savings-insertion solution and Reactive tabu with a variable threshold improved solution results are presented.

Table I: Savings-insertion & Reactive tabu with a variable threshold total distance minimization results

Problem	*Savings-insertion*		*Reactive tabu with a variable threshold*		*Best-knownresult*		*Source*
	NV	*TD*	*NV*	*TD*	*NV*	*TD*	
C1-01	10	829,70	10	828,94	10	827,30	Desrochers et al. [22]
C1-02	11	910,18	10	828,94	10	827,30	Desrochers et al. [22]
C1-03	11	992,03	10	**828,06**	10	828,06	Rochat and Taillard[23]
C1-04	11	939,75	10	828,07	10	824,78	Rochat and Taillard[23]
C1-05	10	829,70	10	**828,94**	10	828,94	Rochat and Taillard[23]
C1-06	10	829,70	10	828,94	10	827,30	Desrochers et al. [22]
C1-07	11	891,11	10	828,94	10	827,30	Desrochers et al. [22]
C1-08	10	828,94	10	828,94	10	827,30	Desrochers et al. [22]
C1-09	10	**828,94**	10	**828,94**	10	828,94	Rochat and Taillard[23]
Average	**10,44**	**875,56**	**10,00**	**828,75**	**10,00**	**827,47**	
C2-01	3	**591,56**	3	**591,56**	3	591,56	Rochat and Taillard[23]
C2-02	3	**591,56**	3	**591,56**	3	591,56	Rochat and Taillard[23]
C2-03	4	652,10	3	600,21	3	591,17	Rochat and Taillard[23]
C2-04	4	668,74	3	604,31	3	590,60	Potvin and Bengio [24]
C2-05	3	588,88	3	588,88	3	588,16	Tan et al. [25]
C2-06	3	602,81	3	590,25	3	588,49	Potvin and Bengio [24]
C2-07	3	589,20	3	**588,29**	3	588,29	Rochat and Taillard[23]
C2-08	3	607,12	3	588,32	3	588,03	Tan et al. [25]
Average	**3,25**	**611,50**	**3,00**	**592,92**	**3,00**	**589,73**	
R1-01	21	1789,50	20	1645,84	18	1607,70	Desrochers et al. [22]
R1-02	19	1580,04	19	1500,47	17	1434,00	Desrochers et al. [22]
R1-03	16	1344,84	14	1242,15	13	1175,67	Lauet al. [26]
R1-04	13	1052,49	12	1026,35	10	974,24	Tan et al. [25]
R1-05	17	1504,95	16	1383,23	15	1346,12	Kallehauge et al. [27]
R1-06	16	1349,00	14	1272,91	12	1251,98	Mester et al. [28]
R1-07	14	1199,31	12	1105,37	11	1051,84	Kallehauge et al. [27]
R1-08	12	1024,57	11	970,70	10	954,03	Tan et al. [25]
R1-09	16	1282,50	13	1162,50	12	1013,16	Chiang and Russell [29]
R1-10	14	1149,73	13	1104,59	10	1080,36	Rochat and Taillard[23]
R1-11	13	1186,46	12	1073,06	11	1070,90	Rochat and Taillard[23]
R1-12	11	1052,03	10	968,40	10	953,63	Rochat and Taillard[23]
Average	**15,17**	**1292,95**	**13,83**	**1204,63**	**12,42**	**1159,47**	
R2-01	7	1252,92	6	**1188,89**	7	1197,09	Jawarneh et al. [30]
R2-02	7	1119,50	7	**1062,59**	6	1077,66	Tan et al. [31]
R2-03	7	**915,91**	7	**905,15**	5	933,29	Tan et al. [31]
R2-04	6	776,20	6	**770,29**	4	771,47	Yu et al. [32]
R2-05	5	1018,29	5	**982,69**	3	994,42	Rousseau et al. [33]
R2-06	5	944,36	5	920,76	3	833,00	Thangiah et al. [34]
R2-07	5	874,98	5	840,75	3	814,78	Rochat and Taillard[23]
R2-08	5	734,74	4	**724,57**	2	725,75	Mester et al. [28]
R2-09	7	937,17	6	896,14	3	855,00	Thangiah et al. [34]
R2-10	7	969,67	7	939,39	3	939,34	Mester et al. [28]
R2-11	4	821,33	4	**768,68**	4	811,59	Tan et al. [25]
Average	**5,91**	**942,28**	**5,64**	**909,08**	**3,91**	**904,85**	
RC1-01	17	1795,09	16	1669,33	15	1619,80	Kohl et al. [35]
RC1-02	16	1656,85	15	1493,55	13	1470,26	Tan et al. [25]
RC1-03	13	1444,68	12	1339,24	11	1110,00	Thangiah et al. [34]
RC1-04	12	1225,77	11	1168,76	10	1135,48	Cordeau et al. [36]
RC1-05	17	1725,40	16	1591,27	15	1535,80	Jawarneh et al. [30]
RC1-06	13	1558,48	13	1403,98	13	1371,69	Tan et al. [25]
RC1-07	13	1352,61	12	1256,89	11	1222,16	Tan et al. [25]
RC1-08	12	1199,89	11	1139,07	11	1133,90	Tan et al. [25]
Average	**14,13**	**1494,85**	**13,25**	**1382,76**	**12,38**	**1324,89**	
RC2-01	9	1388,15	9	1284,08	6	1134,91	Tan et al. [25]
RC2-02	9	1158,29	9	**1121,54**	5	1130,53	Tan et al. [25]
RC2-03	6	**1009,30**	5	**964,64**	4	1010,74	Jawarneh et al. [30]
RC2-04	5	845,89	5	821,26	3	798,41	Mester et al. [28]
RC2-05	9	1245,00	9	**1178,74**	7	1221,28	Jawarneh et al. [30]
RC2-06	7	1154,61	6	1098,38	6	1097,65	Jawarneh et al. [30]
RC2-07	7	1032,16	7	**999,89**	5	1024,17	Jawarneh et al. [30]
RC2-08	5	**808,43**	5	**798,93**	3	828,14	Ibaraki et al. [37]
Average	**7,13**	**1080,23**	**6,88**	**1033,43**	**4,88**	**1030,73**	

NV: number of vehicles; **TD:** total distance

III. RESULTS AND DISCUSSION

Our data are Solomon 100 customers 56 problems, see Solomon [20]. The central depot takes the index *1* at the start of the route and the index *L+2* at the route end and all customers are served from this depot. We used 50 as themaximum percentage of smallest customers and 49 as themaximum percentage of biggest customers. Thus, at one end, we fall on savings and the other on insertion. The distance between two locations *i* and *j*is calculated using a symmetric problem formula:

$$d_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \quad i \in [1, L+1],\ j \in [2, L+2] \quad (15)$$

Each route duration is calculated according to Kay [21], as follows: departure at the depot starttime (t^s_1) and forward scan to determine earliest finish time; reverse scan from earliest finish time to determine the latest start time for this earliest finish time (t_{1k}); departure at the determined latest start time and second forward scan to delay waits as much as possible to the end of the route.

Table II: Comparison between total distance minimization best-known results methodologies versus Savings-insertion & Reactive tabu with a variable threshold (SI&RTVT)

Source	*C1*		*C2*		*R1*		*R2*		*RC1*		*RC2*	
	NV	*TD*	*NV*	*TD*	*NV*	*TD*	*NV*	*TD*	*NV*	*TD*	*NV*	*TD*
Thangiah et al. [34]	10,00	832,00	3,00	650,00	12,30	1238,00	3,00	1005,00	12,00	**1284,00**	3,40	1229,00
Rochat and Taillard[23]	10,00	828,45	3,00	590,32	12,58	1197,42	3,09	954,36	12,38	1369,48	3,62	1139,79
Potvin and Bengio[24]	10,00	838,00	3,00	589,90	12,60	1296,80	3,00	1117,70	12,10	1446,20	3,40	1360,60
Chiang and Russell [29]	10,00	**828,38**	3,00	591,42	12,17	1203,03	2,73	985,68	11,88	1397,44	3,25	1231,57
Cordeau et al. [36]	10,00	**828,38**	3,00	**589,86**	12,08	1210,14	2,73	969,57	11,50	1389,78	3,25	1134,52
Lau et al. [26]	10,00	832,13	3,00	612,26	13,75	1224,04	3,55	1010,96	13,63	1405,31	4,25	1228,10
Tan et al. [31]	10,00	842,21	3,38	612,71	13,33	1197,28	4,64	952,15	13,38	1366,69	5,25	1108,64
Rousseau et al. [33]	10,00	**828,38**	3,00	**589,86**	12,08	1210,21	3,00	941,08	11,63	1382,78	3,38	1105,22
Ibaraki et al. [37]	10,00	**828,38**	3,00	**589,86**	11,92	1214,26	2,73	959,11	11,63	1378,72	3,25	1122,79
Tan et al. [25]	10,00	828,74	3,00	590,69	12,92	**1187,35**	3,55	951,74	12,38	1355,37	4,25	1068,26
Mester et al. [28]	10,00	**828,38**	3,00	**589,86**	12,00	1208,18	2,73	954,09	11,50	1387,12	3,25	1119,70
Yu et al. [32]	10,00	829,01	3,00	590,78	13,00	1196,96	4,18	951,36	12,25	1380,55	4,75	1095,84
Jawarneh et al. [30]	10,00	837,54	3,00	593,09	13,25	1205,91	4,64	955,01	12,75	1369,82	5,25	1074,25
SI&RTVT	10,00	828,75	3,00	592,92	13,83	1204,63	5,64	**909,08**	13,25	1382,76	6,88	**1033,43**

NV: number of vehicles; **TD:** total distance

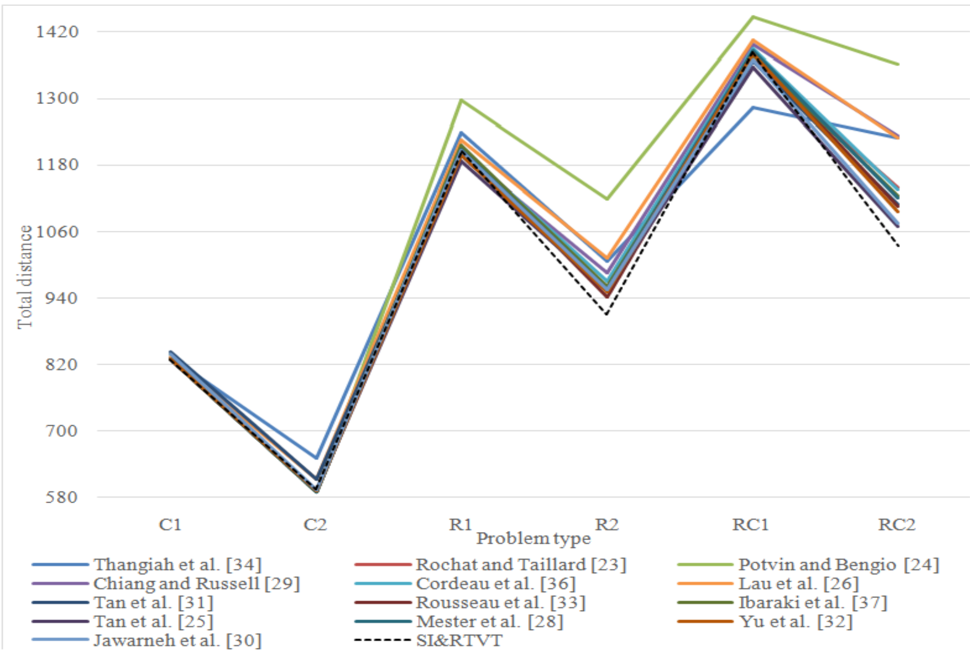

Figure 1: comparison between total distance minimization best-known results methodologies versus SI&RTVT

TABLE I shows the results of applying our Savings-insertion followed by the Reactive tabu with a variable threshold to Solomon 100 customers 56 problems. Firstly, our Savings-insertion results shows that we reached 3 best-known results and found 3 new best results for these problems. Secondly, our Reactive tabu with a variable threshold results shows that we reached 6 best-known results and found 12 new best results for the same problems. On TABLEII is showed the comparison between our results and the results of methodologies which found total distance minimization best-known results. The average total distances of each type of problems are compared. The comparison shows that our methodology provides the best solutions for problems types R2 and RC2. Fig. 1 shows that first for problems types C1, C2, R1 and RC1 our results and other methodologies results are comparable and secondly, for problems types R2 and RC2 our results are the best. On TABLE III is showed our 12 new best results, the corresponding old best known results and the corresponding total distance decrease value. Fig. 2 shows the total distance decrease value of 12 best known solutions obtained by applying our Savings-insertion followed by Reactive tabu with a variable threshold and allows us to know the significance of each improvement obtained.

TABLE III: Comparison between total distance minimization old best-known results and our new best results

Problem	*Our new best*		*Old best-known*			*Total distance decrease*
	NV	*TD*	*NV*	*TD*	*Source*	
R2-01	6	1188,89	7	1197,09	Jawarneh et al. [30]	8,20
R2-02	7	1062,59	6	1077,66	Tan et al. [31]	15,07
R2-03	7	905,15	5	933,29	Tan et al. [31]	28,14
R2-04	6	770,29	4	771,47	Yu et al. [32]	1,18
R2-05	5	982,69	3	994,42	Rousseau et al. [33]	11,73
R2-08	4	724,57	2	725,75	Mester et al. [28]	1,18
R2-11	4	768,68	4	811,59	Tan et al. [25]	42,91
RC2-02	9	1121,54	5	1130,53	Tan et al. [25]	8,99
RC2-03	5	964,64	4	1010,74	Jawarneh et al. [30]	46,10
RC2-05	9	1178,74	7	1221,28	Jawarneh et al. [30]	42,54
RC2-07	7	999,89	5	1024,17	Jawarneh et al. [30]	24,28
RC2-08	5	798,93	3	828,14	Ibaraki et al. [37]	29,21

NV: number of vehicles; **TD:** total distance

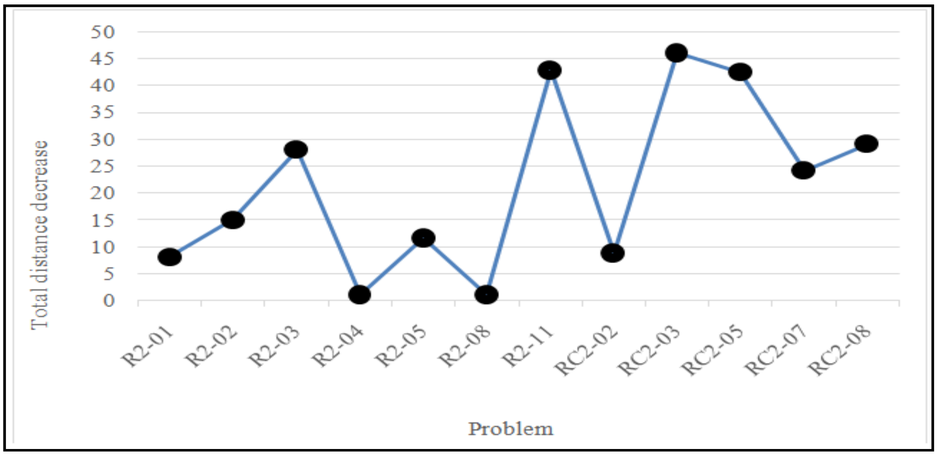

Figure 2: total distance decrease values of our 12 new best results

IV. CONCLUSION

In our paper we resolved the total distance minimization of Solomon 100 customers 56 VRP using our approach based on Savings-insertion followed by the Reactive tabu with a variable threshold. For this purpose, we adapted our mathematical model to minimize the total distance of transport and including hard capacity and hard time windows constraints.Our results show that our methodology provides good solutions of the concerned problems. The comparison shows that our methodology provides the best solutions for problems types R2 and RC2. We reached 6 best-known results and found 12 new best results for Solomon 100 customers 56 problems. Our Reactive tabu with a variable threshold is better than the Reactive tabu, but with one more parameter to control. We then conclude that Savings-insertion followed by the Reactive tabu with a variable threshold is a promising approach and will be used to explore more in our future searches in VRP.

REFERENCES

[1] Toth, P., and D. Vigo. 2001. “The vehicle routing problem”,monographs on discrete mathematics and applications. Philadelphia: SIAM.

[2] Eksioglu, B., A.V. Vural and A. Reisman. 2009. “The vehicle routing problem: A taxonomic review”. *Computers & Industrial Engineering*, vol. 57, No. 4, pp. 1472-1483.

[3] Potvin, J.Y. 2009. “A review of bio-inspired algorithms for vehicle routing”. *Bio-inspired Algorithms for the Vehicle Routing Problem*, pp. 1-34.

[4] Gendreau, M., and C.D. Tarantilis. 2010. "Solving large-scale vehicle routing problems with time windows: The state-of-the-art". CIRRELT.
[5] Karakatič, Sašo, and Vili Podgorelec. 2015. "A survey of genetic algorithms for solving multi depot vehicle routing problem". *Applied Soft Computing,* vol. 27, p. 519-532.
[6] Thangiah, S.R., J.Y. Potvin and T. Sun. 1996. "Heuristic approaches to vehicle routing with backhauls and time windows". *Computers & Operations Research,* vol. 23, No. 11, pp. 1043-1057.
[7] Berger, J., and M. Barkaoui. 2004. "A parallel hybrid genetic algorithm for the vehicle routing problem with time window". *Computers & Operations Research,* vol. 31, n° 12, p. 2037-2053.
[8] Flisberg, P., B. Lidén and M. Rönnqvist. 2009. "A hybrid method based on linear programming and tabu search for routing of logging trucks". *Computers & Operations Research,* vol. 36, No. 4, pp. 1122-1144.
[9] Li, Jian, Panos M Pardalos, Hao Sun, Jun Pei and Yong Zhang. 2015. "Iterated local search embedded adaptive neighborhood selection approach for the multi-depot vehicle routing problem with simultaneous deliveries and pickups". *Expert Systems with Applications,* vol. 42, No. 7, pp. 3551-3561.
[10] McNabb, Marcus E, Jeffery D Weir, Raymond R Hill and Shane N Hall. 2015. "Testing local search move operators on the vehicle routing problem with split deliveries and time windows". *Computers & Operations Research,* vol. 56, pp. 93-109.
[11] Dhaenens, C., M.L. Espinouse and B. Penz. 2008. "Classical combinatorial problems and solution techniques". *Operations Research and networks,* pp. 71-103.
[12] Ismail, S.B., F. Legras and G. Coppin. 2011. "Synthèse du problème de routage de véhicules".
[13] Bagayoko, M., Dao, T. M., &Ateme-Nguema, B. H. (2013, October). "Optimization of forest vehicle routing using the metaheuristics: reactive tabu search and extended great deluge". In *Industrial Engineering and Systems Management (IESM), Proceedings of 2013 International Conference on* (pp. 1-7). IEEE.
[14] Bagayoko, M., Dao, T. M., &Ateme-Nguema, B. H. (2013, December). "Optimization of forest vehicle routing using reactive tabu search metaheuristic". In *2013 IEEE International Conference on Industrial Engineering and Engineering Management* (pp. 181-185). IEEE.
[15] Bagayoko, M., Dao, T. M., &Ateme-Nguema, B. H. (2015, March). "Forest vehicle routing problem solved by New Insertion and meta-heuristics". In *Industrial Engineering and Operations Management (IEOM), 2015 International Conference on* (pp. 1-8). IEEE.
[16] Bagayoko, M., Dao, T.-M., and Ateme-Nguema, B.-H. 2016. "Multi-Objective Forest Vehicle Routing Using Savings-Insertion and Reactive Tabu with a Variable Threshold". In press in *International Journal of Modern Engineering Research.*
[17] Clarke, G. U., and John W. Wright. 1964. "Scheduling of vehicles from a central depot to a number of delivery points". *Operations research,* vol. 12, No. 4, pp. 568-581.
[18] Battiti, Roberto and GiampietroTecchiolli. 1994. "The reactive tabu search". *ORSA journal on computing,* vol. 6, No. 2, pp. 126-140.
[19] Wassan, Niaz A, A Hameed Wassan and Gábor Nagy. 2008. "A reactive tabu search algorithm for the vehicle routing problem with simultaneous pickups and deliveries". *Journal of combinatorial optimization,* vol. 15, No. 4, pp. 368-386.
[20] Solomon, Marius M. 1987. "Algorithms for the vehicle routing and scheduling problems with time window constraints". *Operations research,* vol. 35, n° 2, p. 254-265.
[21] Kay, M. (2016), "Matlog: Logistics Engineering Matlab Toolbox", http://www.ise.ncsu.edu/kay/matlog (last accessed on July 29, 2016).
[22] Desrochers, Martin, Jacques Desrosiers and Marius Solomon. 1992. "A new optimization algorithm for the vehicle routing problem with time windows". *Operations research,* vol. 40, n° 2, p. 342-354.
[23] Rochat, Yves, and Éric D Taillard. 1995. "Probabilistic diversification and intensification in local search for vehicle routing". *Journal of heuristics,* vol. 1, n° 1, p. 147-167.
[24] Potvin, Jean-Yves, and SamyBengio. 1996. "The vehicle routing problem with time windows part II: genetic search". *INFORMS Journal on Computing,* vol. 8, n° 2, p. 165-172.
[25] Tan, Kay Chen, YH Chew and Loo Hay Lee. 2006. "A hybrid multi-objective evolutionary algorithm for solving truck and trailer vehicle routing problems". *European Journal of Operational Research,* vol. 172, n° 3, p. 855-885.
[26] Lau, H. C., Y. F. Lim, Q. Liu. 2001. "Diversification of neighborhood via constraint-based local search and its application to VRPTW". Proc. CP-AI-OR 2001 Workshop, Kent, UK.
[27] Kallehauge, Brian, Jesper Larsen and Oli BG Madsen. 2006. "Lagrangian duality applied to the vehicle routing problem with time windows". *Computers & Operations Research,* vol. 33, n° 5, p. 1464-1487.
[28] Mester, David, Olli Bräysy and WoutDullaert. 2007. "A multi-parametric evolution strategies algorithm for vehicle routing problems". *Expert Systems with Applications,* vol. 32, n° 2, p. 508-517.
[29] Chiang, Wen-Chyuan, and Robert A Russell. 1997. "A reactive tabu search metaheuristic for the vehicle routing problem with time windows". *INFORMS Journal on Computing,* vol. 9, n° 4, p. 417-430.
[30] Jawarneh, Sana, and Salwani Abdullah. 2015. "Sequential insertion heuristic with adaptive bee colony optimisation algorithm for vehicle routing problem with time windows". *PloS one,* vol. 10, n° 7, p. e0130224.
[31] Tan, KC, LH Lee and K Ou. 2001. "Artificial intelligence heuristics in solving vehicle routing problems with time window constraints". *Engineering Applications of Artificial Intelligence,* vol. 14, n° 6, p. 825-837.
[32] Yu, Bin, ZZ Yang and BZ Yao. 2011. "A hybrid algorithm for vehicle routing problem with time windows". *Expert Systems with Applications,* vol. 38, n° 1, p. 435-441.
[33] Rousseau, Louis-Martin, Michel Gendreau and Gilles Pesant. 2002. "Using constraint-based operators to solve the vehicle routing problem with time windows". *Journal of heuristics,* vol. 8, n° 1, p. 43-58.
[34] Thangiah, Sam R, Ibrahim H Osman and Tong Sun. 1994. "Hybrid genetic algorithm, simulated annealing and tabu search methods for vehicle routing problems with time windows". *Computer Science Department, Slippery Rock University, Technical Report SRU CpSc-TR-94-27,* vol. 69.
[35] Kohl, Niklas, Jacques Desrosiers, Oli BG Madsen, Marius M Solomon and Francois Soumis. 1999. "2-path cuts for the vehicle routing problem with time windows". *Transportation Science,* vol. 33, n° 1, p. 101-116.
[36] Cordeau, Jean-François, Gilbert Laporte and Anne Mercier. 2001. "A unified tabu search heuristic for vehicle routing problems with time windows". *Journal of the Operational Research Society,* vol. 52, n° 8, p. 928-936.
[37] Ibaraki, Toshihide, Shinji Imahori, Mikio Kubo, Tomoyasu Masuda, Takeaki Uno and MutsunoriYagiura. 2005. "Effective local search algorithms for routing and scheduling problems with general time-window constraints". *Transportation Science,* vol. 39, n° 2, p. 206-232.