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New Multi-Criteria Group Decision-Making Method Based on Vague Set Theory

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ABSTRACT: *In light of the deficiencies and limitations for existing score functions, Lin has proposed a more effective and reasonable new score function for measuring vague values. By using Lin's score function and a new weighted aggregation score function, an algorithm for multi-criteria group decision-making method was proposed to solve vague set based group decision-making problems under vague environments. Finally, a numerical example was illustrated to show the effectiveness of the proposed multi-criteria group decision-making method.*

Keywords: *Multi-criteria group decision-making, Score function, Vague sets, Weighted aggregation function*

I. INTRODUCTION

Vague sets are very suitable to describe imprecise decision information and deal with theoretical models, algorithms and practical applications under fuzzy decision making environments. In the past, many researchers have presented some score functions for handling fuzzy multi-criteria decision making problems [1-7]. This is due to: (a) the effectiveness in tackling the subjectiveness and imprecision, (b) the simplicity for the decision makers to assign their subjective assessments in the form of a membership degree and a non-membership degree, and (c) the efficiency in aggregating the decision makers' assessments [8]. However, several deficiencies remain evident when using these vague based score functions to handle multi-criteria decision making problems. These include ignorance of the unknown part that can cause information loss, inefficient calculation of the score value and unreasonable comparison results for ranking the vague values. In light of these deficiencies and limitations, Lin [9] proposed a novel score function for a more effective and reasonable method for measuring the degree of suitability to which an alternative satisfies the decision maker's requirement. Therefore, the main purpose of this study was to propose a vague based multi-criteria group decision making method for ranking alternatives under vague and uncertain environment. In order to achieve this purpose, the goal of this study has been tri-fold. The first objective was to propose a new weighted aggregation function for aggregating vague set based weightings and ratings for each alternative. The second objective was to apply Lin's score function to transfer the weighted aggregated vague values into comparable crisp scores and rank the alternatives. By using the weighted aggregation function and Lin's score function, the third objective was to propose a computational algorithm for the multi-criteria group decision making method under uncertain environments.

II. PRELIMINARIES OF VAGUE SET THEORY

Atanassov [10] extended the concept of fuzzy sets by involving the degree of hesitation and introduced the concept of intuitionistic fuzzy sets (IFSs), comprising by a membership function and a non-membership function. Gauand Buehrer [11] proposed the concept of vague set, where the grade of membership is bounded to a subinterval $[t_A(x_i), 1\text{-}f_A(x_i)]$ of [0, 1]. Bustince and Burillo [12] and Deschrijver & Kerre[13] showed that vague sets are equivalent to intuitionistic fuzzy sets. A vague set, as well as being an intuitionistic fuzzy set, is a further generalization of fuzzy set [10-12]. Instead of using point-based membership as in fuzzy set, interval-based membership is used in vague set. Relevant definitions and operations of vague sets, which are in [11, 14-17], are briefly reviewed as follows.

Definition 2.1. Vague sets

A vague set A in the universe of discourse X is given by$\boldsymbol{A}= \{(x, [t_A(x), 1\text{-}f_A(x)]) \,|x\in X\}$, where t_A is called a truth membership function, t_A: $X\rightarrow [0, 1]$, and f_A is called a false membership function, f_A: $X\rightarrow [0, 1]$. $t_A(x)$ is a lower bound of the grade of membership of x derived from the "evidence for x", $f_A(x)$ is a lower bound on the negation of x derived from the "evidence against x",and$0 \leqq t_A(x)+f_A(x) \leq 1$. These lower bounds are used to create a subinterval $[t_A(x),1\text{-}f_A(x)]$ on [0, 1] to generalize the $\mu_A(x)$ of fuzzy sets, where $t_A(x) \leqq \mu_A(x) \leqq 1\text{-}f_A(x)$. $t_A(x)$ and $f_A(x)$ define the he degree of membership and degree of non-membership of the element $x \in X$ to the set $\boldsymbol{A}$, which is a subset of X. The value of $\pi_A(x)=1\text{-}f_A(x)\text{-}t_A(x)$ represents the degree of hesitation (degree of uncertainty or indeterminacy).

Let X be a universe of discourse, $X= \{x_1, x_2 \ldots x_n\}$, with a generic element of X denoted by x_i. When the universe of discourse X is discrete, a vague set $\boldsymbol{A}$of the universe of discourse Xcan be written as: $\boldsymbol{A}=\sum^{n}_{i=1} [t_A(x_i), 1\text{-}f_A(x_i)]/x_i$, $x_i \in X$. When the universe of discourse X is continuous, a vague set $\boldsymbol{A}$ of the universe of discourse X can be written as: $\boldsymbol{A}=\int_x [t_A(x),1\text{-}f_A(x)]/x,\ x \in X$. Where, the interval $[t_A(x_i), 1\text{-}f_A(x_i)]$ is the vague membership number (also called vague value) of the object x_i in vague set $\boldsymbol{A}$. Vague value is an interval value. The vague value $[t_A(x_i), 1\text{-}f_A(x_i)]$ indicates that the exact grade of membership $\mu_A(x_i)$ of x_i may be unknown but it is bounded by $t_A(x_i) \leq \mu_A(x_i) \leq 1\text{-} f_A(x_i)$.

Definition 2.2. Maximum operation of two vague values

Let $x_A= [t_A(x), 1\text{-}f_A(x)]$ be the vague value of x in the vague set $\boldsymbol{A}$, and $x_B= [t_B(x),1\text{-} f_B(x)]$ be the vague value of x in the vague set $\boldsymbol{B}$, where $t_A(x),t_B(x), f_A(x), f_B(x) \in [0,1]$. The result of the maximum operation of the vague values x_A and x_B is a vague value x_C, written as $x_C= x_A \vee x_B =[t_C(x),1\text{-}f_C(x)]= [Max(t_A(x), t_B(x)), Max(1\text{-}f_A(x),1\text{-}f_B(x))]= [Max(t_A(x), t_B(x)),1\text{-}Min(t_A(x), t_B(x))]$.

Definition2.3. Minimum operation of two vague values

Let $x_A= [t_A(x),1\text{-}f_A(x)]$ be the vague value of x_i in the vague set A, and $x_B= [t_B(x),1\text{-}f_B(x)]$be the vague value of x in the vague set B, where $t_A(x), t_B(x), f_A(x), f_B(x) \in [0,1]$. The result of the minimum operation of the vague values x_A and x_B is a vague value x_C, written as $x_C= x_A \wedge x_B= [t_C(x),1\text{-}f_C(x)]= [Min(t_A(x), t_B(x)), Min(1\text{-}f_A(x),1\text{-}f_B(x))]= [Min(t_A(x), t_B(x)),1\text{-}Max(t_A(x), t_B(x))]$.

Definition 2.4. Intersection of two vague sets

The intersection of two vague sets A and B is a vague set Z, written as $Z= A \wedge B$, whose truth membership function and false-membership function are t_Z and f_Z, respectively, where $\forall\ x \in X$, $t_Z(x)= Min(t_A(x), t_B(x))$, $1\text{-}f_Z(x))= Min(1\text{-}f_A(x),1\text{-}f_B(x))= 1\text{-}Max(f_A(x), f_B(x))$. That is, $[t_Z(x), 1\text{-}f_Z(x)]= [Min(t_A(x), t_B(x)), Min(1\text{-}f_A(x),1\text{-}f_B(x))]= [Min(t_A(x), t_B(x)), 1\text{-}Max(f_A(x), f_B(x))]$.

Definition 2.5. Union of two vague sets

The union of two vague sets A and B is a vague set Z, written as $Z = A \vee B$, whose truth membership function and false-membership function are t_Z and f_Z, respectively, where $\forall\ x \in X$, $t_Z(x)= Max(t_A(x), t_B(x))$, $1\text{-}f_Z(x) = Max(1\text{-}f_A(x),1\text{-}f_B(x))=1\text{-}Min(f_A(x), f_B(x))$. That is, $[t_Z(x), 1\text{-}f_Z(x)]=[Max(t_A(x), t_B(x)), Max(1\text{-}f_A(x),1\text{-}f_B(x))]= [Max(t_A(x), t_B(x)), 1\text{-}Min(f_A(x), f_B(x))]$.

III. LIN'S SCORE FUNCTION FOR MEASURING VAGUE VALUES

A score function can be used to measure the degree of suitability of each alternative for ranking and selection in decision-making process based on vague sets. Several familiar research works on score functions of vague set are reviewed and examined [8]. The main problems of the results drawn by the defined score functions are as follows: insufficient information of the unknown part to cause information loss, inefficient for calculating the score value, and undesirable or unreasonable for ranking the vague values. To overcome the above problematic deficiencies of score functions for decision-making, Lin [9] proposed a novel score function to measure the vague value to which an alternative is satisfied the decision maker's requirements.

As shown in Fig. 1, Lin [9] presented a new 3D representation for visualizing a vague value $[t_A(x), 1\text{-}f_A(x)]$. The vague set Ain the universe of discourse X, $\boldsymbol{A}=\{(x, [t_A(x), 1\text{-}f_A(x)])|\ x \in X\}$, and where the interval $[t_A(x), 1\text{-}f_A(x)]$ of [0, 1] is a vague value to the object x in vague set $\boldsymbol{A}$. In the third dimension, a corresponding second membership function $\mu_A(x, \mu_A(x))$ maps the membership degree of the elements in the interval $[t_A(x), 1\text{-}f_A(x)]$. The value $\mu_A(x, \mu_A(x))$ is a random value from the interval [0,1]. It means that the second membership function $\mu_A(x, \mu_A(x))$ indicates to what degree of support an element on the interval $[t_A(x), 1\text{-}f_A(x)]$ falls under "the concept x_i is true". In the interval $[t_A(x), 1\text{-}f_A(x)]$, if an element has a grade of second membership function $\mu_A(x, \mu_A(x))$ equal to 1, this reflects a complete fitness between the element and "the concept x is true"; if an element has a grade of support membership function $\mu_A(x, \mu_A(x))$ equal to 0, then the element does not belong to that "the concept x is true". For the element $t_A(x)$, the property of "being true" is fully satisfied. Hence the membership degree under "being true" is equal to 1. For the element $(1\text{-}f_A(x))$, the property of "being true" is equal to zero. For the elements $\mu_A(x)$ less than $t_A(x)$ or more than $(1\text{-}f_A(x))$, the property of "being true" is completely excluded from this set. For the element $\mu_A(x)$ between $t_A(x)$ and $(1\text{-}f_A(x))$, the property of "being true" is partially satisfied.

The second membership value of each element in the interval [$t_A(x)$, 1-$f_A(x)$] can be read as follows: the second membership function $\mu_A(x, \mu_A(x))$ takes numerical values "Equal to 1 and is continuous and strictly decreasing to 0 as the $\mu_A(x)$ value increases between $t_A(x)$ and (1-$f_A(x)$)". Therefore, the second membership function $\mu_A(x, \mu_A(x))$ are plausibly to be strictly decreasing on the interval [$t_A(x)$, 1-$f_A(x)$]. Using this function, the second membership value $\mu_A(x, \mu_A(x))$ on the interval [$t_A(x)$, 1-$f_A(x)$] is linearly mapped to a value in range [1, 0].

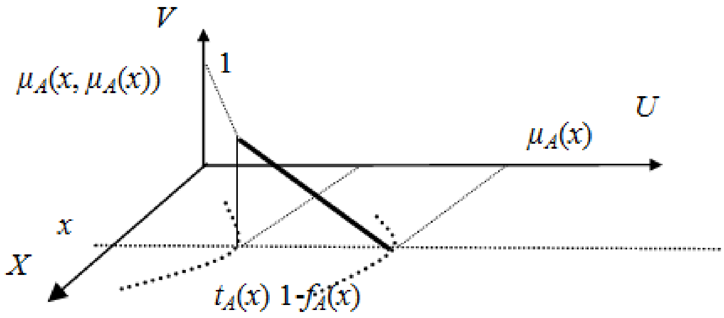

Figure 1. Secondary membership function of vague value

Definition 3.1.Vague value and its secondary membership function **[9]**

If X is a collection of objects denoted generically by x , then $\boldsymbol{A}$ is defined to be a vague set of the universe of discourse X, written as $\boldsymbol{A}=\{(x,[t_A(x), 1\text{-}f_A(x)])|x\in X\}$. The vague value [$t_A(x)$, 1-$f_A(x)$] indicates that the exact grade of membership $\mu_A(x)$ of x may be unknown but it is bounded by $t_A(x) \le \mu_A(x) \le 1\text{-}f_A(x)$. Therefore, the vague set $\boldsymbol{A}$ and its secondary membership function can be presented as $\mu_A(x, \mu_A(x))$. This implies that the value of the primary membership of x, $\mu_A(x)$, is also referred to as the secondary domain of the secondary membership function $\mu_A(x, \mu_A(x))$. In this case, X is referred to as the primary domain; U is referred to as the secondary domain, as well as the value of the primary membership of x; V is referred to as the secondary membership value of x. As shown in Fig.1, $\mu_A(x)$ of [0, 1] is the primary membership function, whose primary domain is the universe of discourse X; $\mu_A(x, \mu_A(x)))$ of [0, 1] is the secondary membership function, whose secondary domain is the vague value U.

Definition 3.2. Lin's New Score Function **[9]**

As aforementioned, the secondary membership value corresponding to each primary membership value in the closed interval [$t_A(x)$, 1-$f_A(x)$] can be represented as follows.

$$\mu_A(x, \mu_A(x)) = \begin{cases} 0, & \text{for } \mu_A(x)<t_A(x); \\ (1-f_A(x)-\mu_A(x))/(1-f_A(x)-t_A(x)), & \text{for } t_A(x)\le\mu_A(x)\le 1-f_A(x); \\ 0, & \text{for } 1-f_A(x)<\mu_A(x) , \end{cases}$$

where $\mu_A(x, \mu_A(x))$: $X \times [0,1] \rightarrow [0,1]$.

By above definition, the interval [$t_A(x)$, 1-$f_A(x)$] and the secondary membership value, which is both "normal" and "convex", define a right triangular fuzzy number denoted as TFN($t_A(x)$, $t_A(x)$, 1-$f_A(x)$). Each data object in the interval [$t_A(x)$, 1-$f_A(x)$] is characterized by its degree of secondary membership function $\mu_A(x, \mu_A(x))$, linearly decreasing from 1 to 0. As shown in Fig.2, by applying Yager's centroid method [18], the transformed numerical score of the defined triangular fuzzy number TFN($t_A(x)$, $t_A(x)$, 1-$f_A(x)$) can be regarded as the centroid index of the right triangular fuzzy number TFN($t_A(x)$, $t_A(x)$, 1-$f_A(x)$). Thus, the numerical score of the vague value $S_L(E(A))$ can be calculated by the following transforming score function:

$$S_L(E(A)) = \int_{t_A(x)}^{1-f_A(x)} \frac{x\cdot(1-f_A(x)-t_A(x))}{1-f_A(x)-t_A(x)}dx \Big/ \int_{t_A(x)}^{1-f_A(x)} \frac{(1-f_A(x)-t_A(x))}{1-f_A(x)-t_A(x)}dx$$

$$= t_A(x) + (1-f_A(x) - t_A(x)) / 3$$

$$=2\times t_A(x)/3+(1-f_A(x))/3 \quad (1)$$

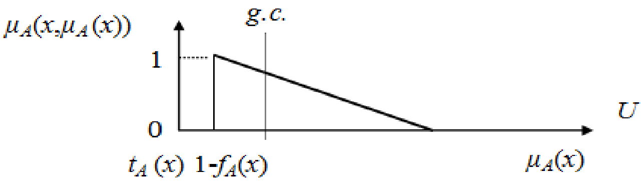

Figure2. Membership function of a vague value $V(x)$= [$t_A(x)$, 1-$f_A(x)$]

To illustrate, at x, the primary membership values are in the vague interval [$t_A(x)$, 1-$f_A(x)$]= [0.45, 0.6]. The primary membership function $\mu_A(x)$ and the secondary membership function $\mu_A(x, \mu_A(x))$ define a right triangular fuzzy number TFN($t(x)$, $t_A(x)$, 1-$f_A(x)$)= TFN(0.45, 0.45, 0.6) and can be transformed as: $S_L(E(\mathbf{A}))$= 2×0.6/3 + 1×0.9/3= 0. 7.

This transforming score function differs from existing score functions in that the proposed score function is a linear function to transform the vague values rather than non-linear functions proposed by other authors. In [9], the capability of the proposed new score function was illustrated using the data set which is slightly modified from an example presented earlier in [16]. The proposed new score function was demonstrated to draw comparisons of ranking orders with some existing score functions. The comparison result suggest that Lin's novel score function provides more distinguishable, easily computable and reasonable way than other score functions for discriminating vague values. Therefore, all alternatives can be ranked reasonable and efficiently through this novel score function.

IV. PROPOSED MULTI-CRITERIA GROUP DECISION MAKING METHOD

As in real situation, the decision-making group rarely comes to a unanimous agreement to an alternative. The decision-making group should take into account all three voting situations: the proportion of decision-makings who vote for, the proportion of those who vote against, and the proportion of those who vote abstain. In this study, the author proposed a vague set based multi-criteria group decision-making method to rank alternatives with vague voting information in a more precise way. The proposed multi-criteria group decision-making method composed of a new weighted aggregation function, a new score function and a computational algorithm.

Definition4.1. Vague set based multi-criteria group decision making problem

A multi-criteria group decision-making problem involves a group of individuals to rank alternatives and select the most preferred alternative from a finite set of feasible alternatives assessed on multi-criteria. Consider a multi-criteria group decision-making problem with m alternatives, n criteria and p decision makers. Let $\mathbf{A}$ be a finite set of m alternatives, $\mathbf{C}$ be a set of n independent criteria and let $\mathbf{P}$ be a set of k decision makers, where $\mathbf{A}$={A_1, A_2…A_m}, $\mathbf{C}$ ={C_1, C_2…C_n} and $\mathbf{P}$={P_1, P_2…P_k} respectively.

Suppose the performance rating against a criterion C_j of alternative A_i can be characterized by a vague value r_{ij}=[t_{rij},1-f_{rij}]. Then the performance of A_i can be represented by the vague set shown as following criterion-rating vector: $\mathbf{R}_i$={(C_1, r_{i1}), (C_2, r_{i2})…(C_n, r_{in})}={(C_1,[t_{ri1}, 1-f_{ri1}]), (C_2,[t_{ri2},1-f_{ri2}])…(C_n, [t_{rin} ,1-f_{rin}])}. Let 1- f_{rij}= t^*_{rij}, then r_{ij} can be rewritten as r_{ij}= [t_{rij}, t^*_{rij}] and $\mathbf{R}_i$ can be rewritten as $\mathbf{R}_i$={(C_1, [t_{ri1}, t^*_{ri1}], (C_2, [t_{ri2}, t^*_{ri2}])...(C_n , [t_{rin} , t^*_{rin}])}.

Suppose the importance weighting against a criterion C_j of alternatives A_i (i=1…m) can be characterized by a vague value ω_j=[t_{wj}, 1-f_{wj}]. Then the importance of each alternative A_i can be represented by the vague set shown as following criterion-weighting vector: $\mathbf{W}$={(C_1, ω_1), (C_2, ω_2)…(C_n, ω_n)}= {(C_1, [t_{w1}, 1-f_{w1}]),(C_2, [t_{w2}, 1-f_{w2}])…(C_n, [t_{wn},1-f_{wn}])}. Let 1-f_{wj}=t^*_{wj}, then ω_j can be rewritten as ω_j= [t_{wj}, t^*_{wj}] and $\mathbf{W}$ can be rewritten as $\mathbf{W}$={(C_1, [t_{r1}, t^*_{w1}], (C_2, [[t_{w2}, t^*_{w2}])...(C_n , [t_{rn}, t^*_{wn}])}.

4.1. New weighted aggregation function for performing weighted aggregation on vague values

Assume that there is a decision making team who wants to evaluate an alternative which satisfies the criteria C_1, C_2,…,and C_n or which satisfies the criteria C_s. This decision making group's requirement is represented by the following expression: C_1 AND C_2 AND...AND C_n OR C_s.

The performance rating r_{ij} of alternative A_i against C_j can be characterized by a vague value. Thus, the degrees that the alternative A_i satisfies and does not satisfy the decision making group's requirement can be determined to obtain a weighted aggregated vague value. The aggregated vague value can be obtained by the evaluation function $E(\mathbf{R}_i)$as follows:

$$E(\mathbf{R}_i)= [t_{ri1}, t^*_{ri1}]\wedge[t_{ri2}, t^*_{ri2}]\ldots\wedge[t_{rin}, t^*_{rin}]\vee[t_{ris}, t^*_{ris}]= [min(t_{ri1}...t_{rin}),min(t^*_{ri1}...t^*_{ri2})]\vee[t_{ris}, t^*_{ris}]$$
$$= [max(min(t_{ri1}...t_{rin}), t_{ris}), max(min(t^*_{ri1}...t^*_{ri2}), t^*_{ris})]=[t_{ri}, t^*_{ri}], \quad (2)$$

where ∧ denotes the minimum operator and ∨ stands for maximum operator of the vague values.

There are often different importance weightings that need to be considered to define relative importance of criteria. Therefore, assume that the importance weightings against C_j can be derived and represented by a weighting vector and are represented by the following vague vector: $\boldsymbol{W}=\{(\omega_1, \omega_2 \ldots \omega_n\}=\{[t_{w1}, t^*_{w1}], [t_{w2}, t^*_{w2}] \ldots [t_{wi}, t^*_{wi}] \ldots [t_{wn}, t^*_{wn}]\}$.By using intersection operation and union operation of vague sets (Definition 4 and 5), the weighted aggregation on vague ratings that the alternative A_i satisfies and does not satisfy the decision-making group's requirement can be determined to obtain a weighted aggregated vague value. The weighted aggregated vague value can be obtained by the weighted aggregation function $W(E(\boldsymbol{R}_i))$ as follows:

$$W(E(\boldsymbol{R}_i))= [t_{ri1}, t^*_{ri1}]\wedge[t_{w1}, t^*_{w1}]\vee[t_{ri2}, t^*_{ri2}]\wedge[t_{w2}, t^*_{w2}]\vee\ldots\vee[t_{rij}, t^*_{rij}]\wedge[t_{wj}, t^*_{wj}]\ldots[t_{rin}, t^*_{rin}]\wedge[t_{wn}, t^*_{wn}]\vee[t_{ris},t^*_{ris}]$$
$$=[(t_{ri1}\wedge t_{w1})\vee(t_{ri2}\wedge t_{w2})\vee\ldots\vee(t_{rin}\wedge t_{wn}), (t^*_{ri1}\wedge t^*_{w1})\vee(t^*_{ri2}\wedge t^*_{w2})\vee\ldots\vee(t^*_{rin}\wedge t^*_{wn})]$$
$$=[max(min(t_{ri1},t_{w1}), min(t_{ri2},t_{w2})\ldots min(t_{rin}, t_{wn}), t_{ris}), max(min(t^*_{ri1}, t^*_{w1}), min(t^*_{ri2}, t^*_{w2})\ldots min(t^*_{rin}, t^*_{wn})), t^*_{ris})]$$
$$=[t_{ri}, t^*_{ri}] \tag{3}$$

4.2. Lin's score function for performing score transformation on weighted aggregated vague values

In vague set based MCDM, the score function is the widely used approach to transform the vague values into comparable crisp values. However, several deficiencies remain evident when using these vague based score functions for ranking the vague values to handle multi-criteria decision-making problems. Thus, Lin's score function [9] is proposed to measure the degree of suitability of each alternative, with respect to a set of criteria characterized by vague values. The weighted aggregated vague values $W(E(\boldsymbol{R}_i))$ (i=1…m) derived in the last step can be transformed into comparable crisp scores $S(W(E(\boldsymbol{R}_i))$ (i=1…m) by applying Lin's [9]score function(Eq.2). The greater the value of $S(W(E(\boldsymbol{R}_i))$ (i=1…m), the higher the degree of appropriateness that alternatively satisfies some criteria.

4.3. The algorithm for the multi-criteria group decision making method

A multi-criteria group decision-making problem with m alternatives $\boldsymbol{A}_i$ (i = 1...m), n criteria $\boldsymbol{C}_j$ (j=1...n) and p decision makers $\boldsymbol{P}_k$ (k=1...p) can be formulated and expressed by a group decision matrix, which includes a group rating matrix:$\boldsymbol{R}= (r_{ij})_{m\times n}$ and a group weighting vector: $\boldsymbol{W}= \{\omega_j\}(j=1...n)$. A decision-making group can use the decision rating matrix to compare alternatives with respect to multiple criteria of different levels of importance.The computational algorithm for the multi-criteria group decision-making method is presented as follows:

Step 1: Constructing a group decision matrix

In a decision making process, votingmodel is a popularmechanism as the decision of human voters can be divided into three groups: those who "vote for", those who "vote against", and those who "vote abstain". The voting model can be precisely and efficiently interpreted by vague set theory. Let N_{rij} be the total number of decision makers who are responsible for conducting suitability evaluation by casting a vote. For aggregating suitability ratings on criterion C_j for alternative A_i, if N^t_{rij} is the number of decision makers who voted "agree", N^f_{rij} is the number of decision makers who voted "disagree", and N^π_{rij} is the number of decision makers who remained "undecided", then $t_{rij}=N^t_{rij}/N_{rij}$, $f_{rij}= N^f_{rij}/N_{rij}$, $\pi_{rij}= N^\pi_{rij}/N_{rij}$ can be derived. The aggregated suitability rating on criterion C_j for alternative A_i can be expressed as $r_{ij}= [t_{rij}, 1\text{-}f_{rij}]= [t_{rij}, t^*_{rij}]$, where r_{ij} is the rating of the alternative A_i to the criterion C_j . As expressed in Table 1, if the criteria rating vector for alternative i, $\boldsymbol{R}_i$= $\{r_{ij}\}(i=1\ldots m$ and $j=1\ldots n)$, then $\boldsymbol{R}= (r_{ij})_{m\times n}= ([t_{rij}, t^*_{rij}])_{m\times n}$ is known as a group rating matrix. Similarly, for aggregating importance weightings on criterion C_j, if N^t_{wj} is the number of decision makers who filled "agree", N^f_{wj} is the number of decision makers who filled "disagree", and N^π_{wj} is the number of decision makers who remained "undecided", then $t_w= N^t_{wj}/N_{wj}$, $f_{wj}= N^f_{wj}/N_{wj}$, $\pi_{rj}= N^t_{wj}/N_{wj}$ can also be derived. The aggregated importance weighting on criterion C_j can be expressed as $\omega_j=[t_{wj}, 1\text{-}f_{wj}]= [t_{wj}, t^*_{wj}]$. Then $\boldsymbol{W}=\{\omega_1, \omega_2,...,\omega_n\}=\{[t_{w1}, t^*_{w1}],[t_{w2}, t^*_{w2}]\ldots[t_{wj}, t^*_{wj}]\ldots[t_{wn}, t^*_{wn}]\}$ is known as the group weighting vector.

Table1. The vague set based group decision matrix

	C_1	...	C_j	...	C_n
W	ω_1		ω_j		ω_n
$\boldsymbol{A_1}$	$[t_{r11}, t^*_{r11}]$	...	$[t_{r1j}, t^*_{r1j}]$	...	$[t_{r1n},t^*_{r1n}]$
$\boldsymbol{A_2}$	$[t_{r21},t^*_{r21}]$	...	$[t_{r2j}, t^*_{r2j}]$	...	$[t_{r2n}, t^*_{r2n}]$
⋮	⋮	...	⋮	...	⋮
$\boldsymbol{A_i}$	$[t_{ri1}, t^*_{ri1}]$	...	$[t_{rij}, t^*_{rij}]$	...	$[t_{rin}, t^*_{rin}]$
⋮	⋮	...	⋮	...	⋮
$\boldsymbol{A_m}$	$[t_{rm1}, t^*_{rm1}]$	...	$[t_{rmj}, t^*_{rm1}]$	...	$[t_{rmn}, t^*_{rmn}]$

Step 2: Performing weighted aggregation on vague values

By using weighted aggregation function (Eq.3), weighted aggregated vague value for each alternative can be derived by multiplying the vague weighting vector with vague rating vector.

Step 3: Performing score transformation on weighted aggregated vague values

By applying Lin's score function (Eq.1), comparable crisp scores for the previous weighted aggregated vague values of alternatives can be transformed. The larger the score represent the better alternative.

Step 4: Ranking all the alternatives

Then, by utilizing the transformed scores, ranking of all the alternatives can be obtained in accordance with the crisp scores of the vague values.

V. ILLUSTRATIVE NUMERICAL EXAMPLE

For a vague set based multi-criteria group decision-making problem, let $\boldsymbol{A}= \{A_1, A_2, A_3, A_4, A_5, A_6, A_7\}$ be a set of alternatives, let $\boldsymbol{C}= \{C_1, C_2, C_3, C_4\}$be a set of criteria and let $\boldsymbol{P} = \{P_1, P_2...P_{20}\}$be a set of decision makers. This decision making group's requirement is represented by the following expression: C_1AND C_2 AND C_3 OR C_4.A vague set based multi-criteria group decision-making problem can be concisely formulated and expressed by a numerical group decision matrix, which includes a group rating matrix $\boldsymbol{R}= (r_{ij})_{4\times4}= ([t_{rij}, t^*_{rij}])_{4\times4}$and a group weighting vector $\boldsymbol{W}= \{(\omega_1, \omega_2...\omega_4\}$, as shown in Table 2. The aggregated performance ratings with respect to the criteria C_j for alternative A_i can be solicited and expressed as $r_{ij}= [t_{rij}, 1\text{-}f_{rij}]= [t_{rij}, t^*_{rij}]$, i=1…m, j=1…n. Let $1\text{-}f_{rij}= t^*_{rij}$, then r_{ij} can be rewritten as $r_{ij}= [t_{rij}, t^*_{rij}]$. The aggregated weightings with respect to the criteria C_j for all alternatives can be solicited and expressed as $\omega_j= [t_{wj}, 1\text{-}f_{wj}]= [t_{wj}, t^*_{wj}]$, j=1…n.

Table 2. The numerical group decision matrix

	C_1	C_2	C_3	C_4
$\boldsymbol{W}$	[0.7,0.8]	[0.6, 0.8]	[0.6, 0.9]	[1, 1]
$\boldsymbol{A_1}$	[0.7, 0.9]	[0.8, 0.9]	[0.7, 0.9]	[0.7, 0.9]
$\boldsymbol{A_2}$	[0.5, 0.6]	[0.5, 0.7]	[0.5, 0.6]	[0.5, 0.6]
$\boldsymbol{A_3}$	[0.6, 0.7]	[0.6, 0.7]	[0.7, 0.7]	[0.6, 0.8]
$\boldsymbol{A_4}$	[0.4, 0.4]	[0.4, 0.4]	[0.4, 0.4]	[0.4, 0.4]

The rating vectors for alternatives $\boldsymbol{A_i}$ (i=1…m) against criteria $\boldsymbol{C_j}$ (j=1…n) are solicited and aggregated as follows:

$\boldsymbol{R_1} = \{(C_1, [0.7, 0.9]), (C_2, [0.8, 0.9]), (C_3, [0.7, 0.9])\}, (C_4, [0.7, 0.9])\}$,
$\boldsymbol{R_2} = \{(C_1, [0.5, 0.6]), (C_2, [0.5, 0.7]), (C_3, [0.5, 0.6])\}, (C_4, [0.5, 0.6])\}$,
$\boldsymbol{R_3}= \{(C_1, [0.6, 0.7]), (C_2, [0.6, 0.7]), (C_3, [0.7, 0.7])\}, (C_4, [0.6, 0.8])\}$,
$\boldsymbol{R_4} = \{(C_1, [0.4, 0.4]), (C_2, [0.4, 0.4]), (C_3, [0.4, 0.4])\}, (C_4, [0.5, 0.6])\}$.

The weighting vector of all the criteria $\boldsymbol{C_j}$ (j=1…n) is solicited and aggregated as $\boldsymbol{W}=\{(\omega_1, \omega_2, \omega_3, \omega_4\}=\{[0.7,0.8], [0.6, 0.8], [0.6, 0.9], [1, 1]\}$.

By using the proposed weighted aggregation function (Eq.3), the weightings and ratings for alternatives with respect to all criteria can be aggregated into the weighted aggregated vague values.

$W(E(\boldsymbol{R_1}))= ([0.7, 0.9]\wedge[0.7,0.8])\vee([0.8, 0.9]\wedge[0.6, 0.8])\vee([0.7, 0.9]\wedge[0.6, 0.9])\vee([0.7, 0.9]\wedge[1,1]) = [0.7, 0.9]$,
$W(E(\boldsymbol{R_2}))= ([0.5, 0.6]\wedge[0.7,0.8])\vee([0.5, 0.6]\wedge[0.6, 0.8])\vee([0.5, 0.6]\wedge[0.6, 0.9])\vee([0.5, 0.9]\wedge[1,1])= [0.5, 0.9]$,
$W(E(\boldsymbol{R_3}))= ([0.6, 0.7]\wedge[0.7,0.8])\vee([0.6, 0.7]\wedge[0.6, 0.8])\vee([0.7, 0.7])\wedge[0.6, 0.9])\vee ([0.6, 0.8]\wedge[1,1]) = [0.6, 0.8]$,
$W(E(\boldsymbol{R_4}))= ([0.4, 0.4]\wedge[0.7,0.8])\vee([0.4, 0.4])\wedge[0.6, 0.8])\vee([0.4, 0.4]\wedge[0.6, 0.9])\vee ([0.5, 0.6]\wedge[1,1])= [0.5, 0.6]$,
where $\wedge$ and $\vee$ denote the minimum operator and the maximum operator of the vague values, respectively.

By using the author's score function in Eq.(1), the scores for the weighted aggregated vague values can be transformed as follows:

$S_L((W(E(\boldsymbol{R_1})) = 2\times0.7/3+ 1\times0.9/3=0.7667$
$S_L((W(E(\boldsymbol{R}_2)) = 2\times0.5/3 + 1\times0.9/3=0.6333$
$S_L((W(E(\boldsymbol{R_3})) = 2\times0.6/3+ 1\times0.8/3=0.6667$
$S_L((W(E(\boldsymbol{R}_4)) = 2\times0.5/3+ 1\times0.6/3=0.5333$

Then, by utilizing the transformed scores, ranking of all the alternatives can be obtained in accordance with the transformed crisp scores of the weighted aggregated vague values. Consequently, the ranking order of the seven alternatives is given as follows: $A_1\succ A_3\succ A_2\succ A_4$.

VI. CONCLUSIONSAND FURTHER RESEARCH

The main purpose of this study was to propose a weighted aggregation function and to use Lin's score function for solving vague set based multi-criteria group decision-making problems. By using the proposed weighted aggregation function, employing Lin's score function and following the proposed computational algorithm, a vague set based multi-criteria decision-making method was proposed to rank alternatives for addressing group decision-making problems under vague environment. A numerical example was also illustrated to show that the proposed multi-criteria groupdecision-making method is effective and reasonable to rank the alternatives in handling multi-criteria groupdecision-making problems. For the future researches, the proposed decision-making method could extend for conducting case studies to solve multi-criteria group decision-making problems in business or industry applications.

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