

## Numerical study of the three-dimensional free convection around an inclined ellipsoid of revolution

Dr. Ulrich Canissius

Laboratoire de Mécanique et de Métrologie (LMM), Ecole Normale Supérieure pour l'Enseignement Technique (ENSET), Antsirananana University B.P.0, Antsirananana 201, Madagascar

**ABSTRACT:** This work is devoted to a numerical study of the thermal free convection. In this paper, author solves numerically, using the method of finite differences, the transfer equations, laminar, three-dimensional, between inclined isothermal ellipsoid of revolution, and a Newtonian fluid in vertical upward flow generated by the natural convection. In the boundary layer, the results concerning the dimensionless velocity fields and temperatures as well as the Nusselt number and the friction coefficients, are represented graphically. With respect to the angle of inclination of the ellipsoid, the author put in evidence of the privileged points on the partition of the body.

**Keywords:** three-dimensional free convection, three-dimensional boundary layer, inclined ellipsoid of revolution, heat transfer, numerical study.

### Nomenclature

#### Roman letter symbols

a	thermal diffusivity of the fluid ( $\text{m}^2 \cdot \text{s}^{-1}$ )
a'	length of the semi-axis of the axis of revolution, (m)
b	half-axis length perpendicular to the axis of revolution of the ellipsoid, (m)
C <sub>fu</sub>	meridian friction coefficient
C <sub>fw</sub>	azimuthal friction coefficient
C <sub>p</sub>	specific heat capacity at constant pressure of the fluid, ( $\text{J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$ )
g	acceleration due to gravity, ( $\text{m} \cdot \text{s}^{-2}$ )
L	length reference body, (m)
Nu	local Nusselt number
Pr	Prandtl number
r	normal distance from the projected M of a point P of the fluid to the axis of revolution of the ellipsoid, (m)
S <sub>x</sub> , S <sub>φ</sub>	factors of geometric configuration
T <sub>∞</sub>	temperature of the fluid away from the wall, (K)
T <sub>p</sub>	temperature of the wall, (K)
V <sub>x</sub>	velocity component in x direction, ( $\text{m} \cdot \text{s}^{-1}$ )
V <sub>y</sub>	velocity component in y direction, ( $\text{m} \cdot \text{s}^{-1}$ )
V <sub>φ</sub>	velocity component in φ direction, ( $\text{m} \cdot \text{s}^{-1}$ )
x, y	meridian and normal coordinates, (m)

#### Greek letter symbols

α	angle of inclination, (°)
α <sub>e</sub>	eccentric angle, in the literature, (rad)
φ	azimuthal coordinate, (°)
ρ	density of the fluid, ( $\text{kg} \cdot \text{m}^{-3}$ )
ν	kinematic viscosity, ( $\text{m}^2 \cdot \text{s}^{-1}$ )
λ	thermal conductivity, ( $\text{W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$ )

- $\mu$  dynamic viscosity, ( $\text{kg}\cdot\text{m}^{-1}\cdot\text{s}^{-1}$ )  
 $\beta$  volumetric coefficient and thermal expansion, ( $\text{K}^{-1}$ )  
 $\beta_i$  angle formed by the major axis and a point on the wall, ( $^\circ$ )

### Indices/Exponents

- + dimensionless variables

## I. INTRODUCTION

Heat transfers around the rotationally symmetrical body have been widely studied [1-23] given their practical interest, especially in machinery. Take for example some hydraulic structures, aircraft, turbine engines, propulsion systems for ships, rockets, projectiles, metal deposition techniques in a vapor phase. Most of the literature relating to vertical ellipsoid. Among them, as Mochimaru [1] studied the numerical simulation of the natural convection in a cavity of an ellipsoid of revolution, using a method of spectral finite differences. He found that the strength of the circulating movement of the liquid metal layers due to natural convection can be well controlled by variation of heat transfer through the wall of the ellipsoid. Souad et al [2] studied the heat transfer and unsteady pulse by natural convection within an air-filled ellipsoid of revolution and whose wall is to be brought to a constant temperature, is crossed by a flow of constant density and warmth they have shown the existence of multicellular structures for certain values of the Grashof number and the variation of the form factor. Furthermore, authors observe the transition of the system between two equilibrium states and claim that the Nusselt number is monotonically decreasing function of time when the flow is unicellular and sharply decreases with each appearance or disappearance of cells. Olumuyiwa [3] has contributed to the heat transfer by convection mixed rotationally elliptical vertical and it confirmed that the disturbance parameter is responsible for the lateral displacement of the temperature profile. The author compares the results with the published work by Morris [4-5] to check the calculation code. Alidina [6] contributed to a study of laminar and permanent three-dimensional flows around the ellipsoid of revolution. He showed that on the wall, there is a place where the components do not depend on the position of the body in space. Cherif et al [7] contributed in the hydrodynamic control by mixed convection of the thickness of the vapor deposition of the semiconductor on the symmetrical body. Authors show that the flow and transfer are significantly dependent on this variability and which is possible to control the growth of heat and mass boundary layers by acting on the operating conditions, in particular on the profiles of body. A. Watson et al [8] studied the steady laminar free convection due to an ellipsoid of revolution heated. Xia et al [9] have landed on the natural convection of the low pressure gas in the ellipsoidal chamber induced by combined thermal conditions. After the works, the survey shows that the various thermal conditions non-uniform of stratospheric environment exert a significant influence on both thermal and dynamic characteristics of natural convection of a gas at low pressure in a chamber. Shapiro et al [10] discussed the vortex formation in a thermal elliptical bubble. Lin et al [11] studied the two-dimensional natural convection around the body in the axisymmetric case of variable shape. Authors proposed a fast calculation procedure based on the coordinate transformation which can express the solutions of the conservation equations that govern based on a sequence of universal functions that depend on the Prandtl number and the configuration, determined by the contour of the body and its orientation relative to the strength of the body which generates the movement. Medvinsky et al [12] discussed a study of the conditions to local limits absorption elliptical limits based on the Helmholtz equation and also introducing a new boundary condition of an ellipse based on modal expansion.

Given all the research work published on an ellipsoid, the three-dimensional natural convection between a Newtonian fluid and an inclined elliptical body, also of great interest, given the technological developments in terms of research in the field of heat.

This work, which aims to analyze the influence of the inclination angle on heat transfer. We consider a three-dimensional flow, laminar, continuous, isothermal between a ellipsoid of revolution and a newtonian fluid in vertical upward flow created by the natural convection whose the axis of symmetry is inclined relative to the vertical direction. The conservation equations are discretized using an implicit finite difference scheme.

## II. THEORETICAL FOUNDATIONS

The physical model considered is constituted by an ellipsoid of revolution of length  $L$  and inclined by an angle  $\alpha$  relative to the vertical. The body wall is maintained at a constant temperature  $T_p$ , different from the temperature  $T_\infty$  of fluid away from the wall which is also constant.

The Figure 1 shows the spatial configuration of the physical model studied.

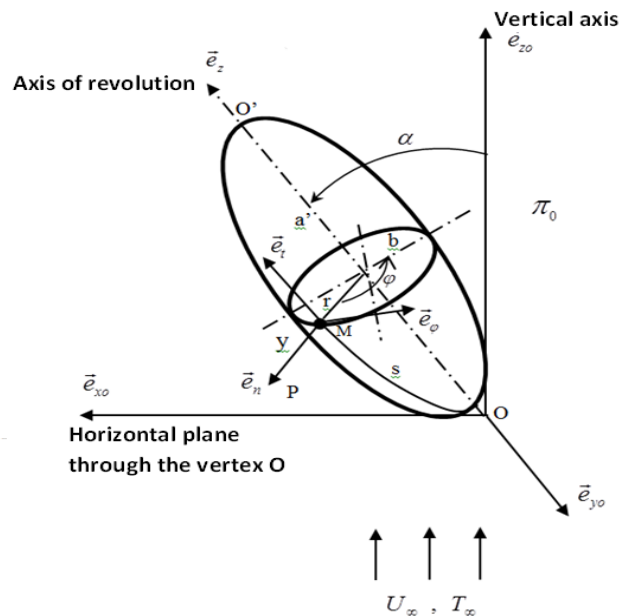


Figure 1: physical model and co-ordinates system

### 2-1. Simplifying assumptions

Besides the classic assumptions of the boundary layer and those of Boussinesq, we pose the following assumptions:

- The ellipsoid is stationary,
- Transfers are laminar and permanent,
- Radiative transfer and viscous energy dissipation are negligible,
- The fluid is air whose physical properties are constant, except for the variations of density are at the origin of the free convection.

### 2-2. Conservation equations in the boundary layer

Let  $\Delta T = T_p - T_\infty$  and the appropriate variables are reduced:

$$x_+ = \frac{x}{L} \quad y_+ = \frac{y}{L} \quad Gr^{\frac{1}{4}} \varphi_+ = \frac{\varphi}{2\pi} \quad r^+ = \frac{r}{L}$$

$$V_x^+ = \frac{V_x}{\sqrt{Lg\beta\Delta T}} \quad V_y^+ = \frac{V_y}{\sqrt{Lg\beta\Delta T}} \quad V_\varphi^+ = \frac{V_\varphi}{\sqrt{Lg\beta\Delta T}} \quad T^+ = \frac{T - T_\infty}{T_p - T_\infty}$$

$$Gr = \frac{g\beta(T_p - T_\infty)L^3}{\nu^2} : \text{Grashofnumber}$$

Then, the dimensionless equations in the boundary layer are written:

- **Equation of continuity**

$$\frac{\partial V_x^+}{\partial x_+} + \frac{\partial V_y^+}{\partial y_+} + \frac{1}{r^+} \frac{\partial V_\varphi^+}{\partial \varphi_+} + \frac{V_x^+}{r^+} \frac{dr^+}{dx_+} = 0 \tag{1}$$

- **Momentumequation**

$$V_x^+ \frac{\partial V_x^+}{\partial x_+} + V_y^+ \frac{\partial V_y^+}{\partial y_+} + \frac{V_\varphi^+}{r^+} \frac{\partial V_x^+}{\partial \varphi_+} - \frac{V_\varphi^{+2}}{r_+} \frac{dr^+}{dx_+} = S_x T^+ + \frac{\partial^2 V_x^+}{\partial y_+^2} \tag{2}$$

$$V_x^+ \frac{\partial V_\phi^+}{\partial x_+} + V_y^+ \frac{\partial V_\phi^+}{\partial y_+} + \frac{V_\phi^+}{r^+} \frac{\partial V_\phi^+}{\partial \phi_+} + \frac{V_x^+ V_\phi^+}{r^+} \frac{dr^+}{dx_+} = S_\phi T^+ + \frac{\partial^2 V_\phi^+}{\partial y_+^2} \tag{3}$$

$S_x$  et  $S_\phi$  are the factors of geometric configuration defined by:

$$S_x = \sin \alpha \cdot \cos \varphi \cdot \cos \beta_i + \cos \alpha \cdot \sin \varphi \tag{4}$$

$$S_\phi = -\sin \alpha \cdot \sin \varphi$$

• **Heat equation**

$$V_x^+ \frac{\partial T^+}{\partial x_+} + V_y^+ \frac{\partial T^+}{\partial y_+} + \frac{V_\phi^+}{r^+} \frac{\partial T^+}{\partial \phi_+} = \frac{1}{Pr} \frac{\partial^2 T^+}{\partial y_+^2} \tag{5}$$

$$Pr = \frac{\mu C_p}{\lambda} = \frac{v}{a}, \text{ Prandtl number}$$

The dimensionless boundary conditions associated with these equations are:

on the wall  $y_+ = 0$

$$T^+ = 1, V_x^+ = V_y^+ = V_\phi^+ = 0 \tag{6}$$

Away from the wall  $y_+ \rightarrow \infty$

$$T^+ = 0, V_x^+ = V_y^+ = V_\phi^+ = 0 \tag{7}$$

**2-3. Nusselt number and friction coefficients**

- *Nusselt number*

$$Nu Gr^{-\frac{1}{4}} = - \left( \frac{\partial T^+}{\partial y_+} \right)_{y_+=0} \tag{8}$$

- *Friction coefficients*

$$Cf_u = Lc_f \left( \frac{\partial V_x^+}{\partial y_+} \right)_{y_+=0}, \quad Cf_\phi = Lc_f \left( \frac{\partial V_\phi^+}{\partial y_+} \right)_{y_+=0} \tag{9}$$

**III. NUMERICAL SOLUTION**

The study area is divided into  $N_x M_x L$  curvilinear parallelepiped attached to the body and defined by the steps dimensionless  $\Delta x_+, \Delta y_+$  and  $\Delta \phi_+$ ,  $N$  and  $L$  being the number of meridians and parallels. For clarity, we note respectively  $U, V, W$  and  $T$  the meridional, normal, azimuthal and dimensionless temperature. The dimensionless conservation equations (1), (2), (3) and (5) are discretized using an implicit finite differences scheme. The calculations are performed at the nodes  $(i,j,k)$  with  $1 \leq i \leq N, 1 \leq j \leq M$  and  $1 \leq k \leq L$ . After arrangement, the discretized equations can each be written in the following form:

$$A X_{j+1} + B X_j + C X_{j-1} = D_j, \tag{10} \quad 2 \leq j \leq JMAX-1$$

Wherein  $X$  is chosen from one of the variables  $U, W$  and  $T$ ,  $JMAX$  index characterizing the thickness of the boundary layer. The algebraic systems (10) associated with the discretized boundary conditions are solved by the Thomas algorithm. As for the dimensionless normal component is calculated from the continuity equation:

$$V_{i+1,j}^k = \frac{1}{4} \left[ 3V_{i+1,j+1}^k + V_{i+1,j-1}^k + 2\Delta y_+ \left( \frac{U_{i+1,j}^k - U_{i,j}^k}{\Delta x_+} + \frac{3W_{i+1,j}^{k+1} - 4W_{i+1,j}^k + W_{i+1,j}^{k-1}}{2\Delta \phi_+ r_{i+1}^+} + \frac{U_{i+1,j}^k}{\Delta x_+} \left( 1 - \frac{r_i^+}{r_{i+1}^+} \right) \right) \right] \tag{11}$$

For  $1 \leq i \leq N-1, 1 \leq k \leq L-1$  and  $2 \leq j \leq JMAX-1$

The convergence within the boundary layer is achieved when the following criteria:

$$\left| \frac{|X^{(p+1)}| - |X^{(p)}|}{\text{Sup}(|X^{(p+1)}|, |X^{(p)}|)} \right| \leq \epsilon \tag{12}$$

is simultaneously checked for  $T, U$  and  $W$ .

$x^{(p)}$  and  $x^{(p+1)}$  are respectively the values of the quantity X of the iterations p and p + 1.

IV. RESULTS AND DISCUSSION

To prove the accuracy of our results, we validated the numerical code by comparing the results of our calculations with those deduced from the literature [14] in the case of an axisymmetric system of an elongated ellipsoid. The table IV-1, illustrating the evolution of the heat exchange ratio based on the eccentric angle in a range of 0 to  $\pi$ , Pr = 1.0, shows that our results are in good agreement with those in the literature [14] and the relative deviation of not more than 1%. In the same table, we also compare the results with those obtained by Merkin [13] and Kumar et al [15], it seems reasonable to conclude that the agreement is good.

Table IV-1: Numerical values of heat transfer coefficient,  $\alpha_e \in [0, \pi]$ , Pr = 1.0, b/a' = 0.25

$\alpha_e$	Present results	M. K. Jaman et al [14]	J. H Merkin [13]	Kumar et al [15]
0.0	0.8412	0.8426	0.8359	0.8428
0.2	0.7714	0.7706	0.7682	0.7722
0.4	0.6622	0.6619	0.6617	0.6632
0.6	0.5790	0.5781	0.5788	0.5794
0.8	0.5184	0.5175	0.5187	0.5191
1.0	0.4736	0.4729	0.4745	0.4747
1.2	0.4397	0.4392	0.4409	0.4410
1.4	0.4146	0.4132	0.4149	0.4150
1.6	0.3936	0.3929	0.3943	0.3944
1.8	0.3772	0.3768	0.3779	0.3779
2.0	0.3644	0.3641	0.3646	0.3646
2.2	0.3539	0.3538	0.3538	0.3537
2.4	0.3450	0.3451	0.3447	0.3446
2.6	0.3368	0.3370	0.3363	0.3362
2.8	0.3264	0.3270	0.3266	0.3262
3.0	0.3072	0.3062	0.3084	0.3070
$\pi$	0.2782	0.2780	0.2785	-

In our results, we set Pr = 0.72 and b/a' = 0.7. The representation of U+, against x+ shows the existence, in the plane of symmetry characterized by  $\phi = 0^\circ$  and  $\phi = 180^\circ$ , of a curvilinear abscissa  $x_+ = 0.43$  privileged wherein x+ do not depend on the inclination angle  $\alpha$  (Figure 2.a). This independence extends within an area from  $x_+ = 0.2$  to  $x_+ = 0.6$  on the meridian defined by  $\phi = 90^\circ$  (Figure 2.b).

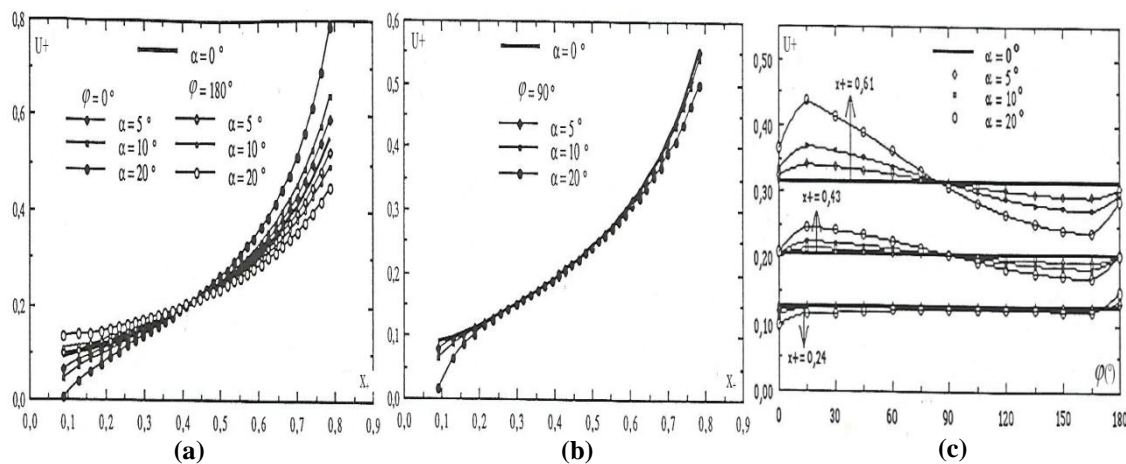
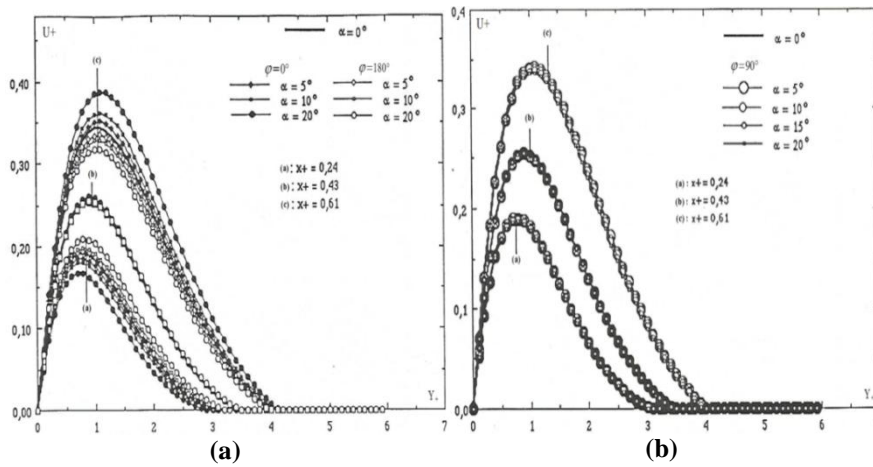


Figure 2: meridian component of the velocity, for different values of  $\alpha$

(a) : U+ against x+,  $\phi = 0^\circ$  and  $\phi = 180^\circ$  ; (b) : U+ against x+,  $\phi = 90^\circ$  ; (c) : U+ against  $\phi$ ,  $x_+ = 0.24, 0.43, 0.61$

In these figures, the curve corresponding to the vertical ellipsoid ( $\alpha = 0^\circ$ ) is a dividing line between the values relative for  $x_+ < 0.43$  and that on  $x_+ > 0.43$ . In the latter, the manners they show increases with the inclination, for  $\phi = 0^\circ$  and decreases on the meridian of equation  $\phi = 180^\circ$ . In these developments, it is thus observed that the variations of the tangential component are reversed for  $x_+ > 0.43$ . For this privileged abscissa, there is a value at which U+ does not depend on the inclination alpha  $\alpha$  and that this region is in the vicinity of  $90^\circ$  (Figure 2.c).

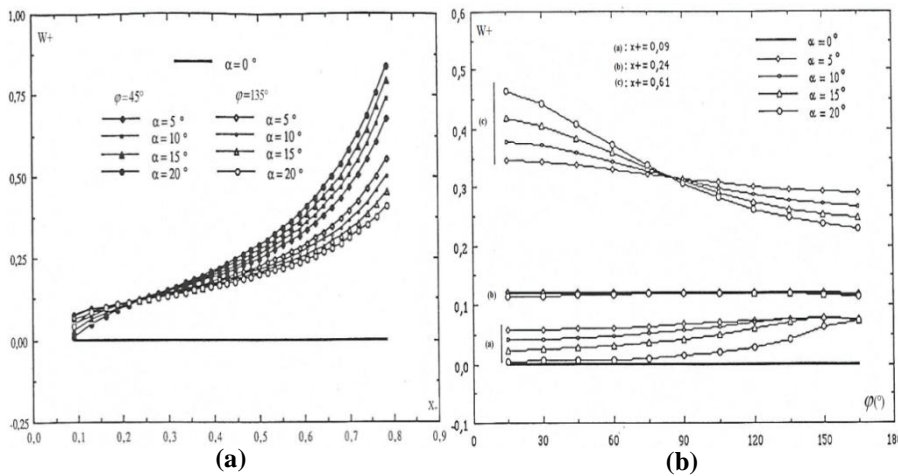
For the privileged abscissa  $x_+ = 0.43$ , the variation curves of  $U_+$  against  $y_+$  do not depend either to  $\alpha$ , for  $\varphi = 0^\circ, 90^\circ, 180^\circ$  and the figures 3.a and 3.b illustrate this phenomenon.



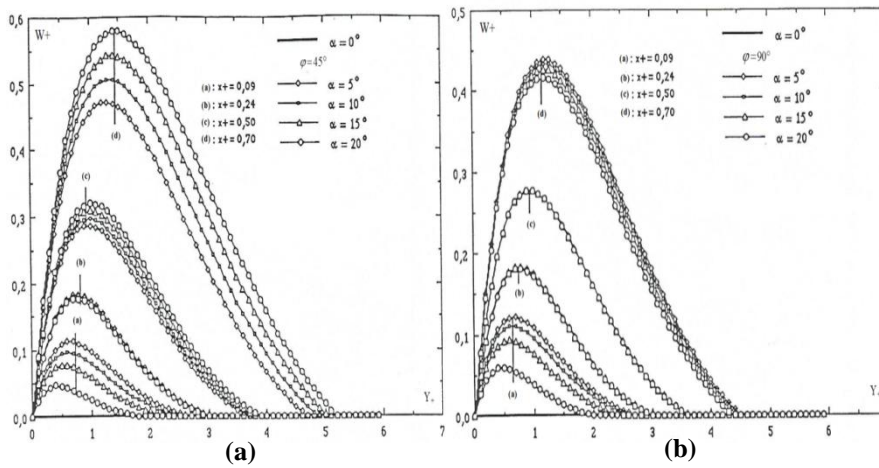
**Figure 3:** meridian component of the velocity against  $y_+$ , for different values of  $\alpha$  and  $x_+ = 0.24, 0.43, 0.61$   
 (a) :  $\varphi = 0^\circ$  et  $180^\circ$  ; (b) :  $\varphi = 90^\circ$

These figures confirm that the thickness of the boundary layer increases with the curvilinear abscissa and the meridional velocity varies from zero at the wall to a zero value outside of the boundary layer through positive values within thereof.

In the case of a non-axisymmetric system, it looks as much on the azimuthal component dimensionless  $W_+$  and, the figures 4.a and 4.b confirm constantly for  $x_+ = 0.24$ , the angle of inclination has no influence on this component and is thus dependent on the latter regardless of the value of  $y_+$  (Figures 5.a and 5.b).

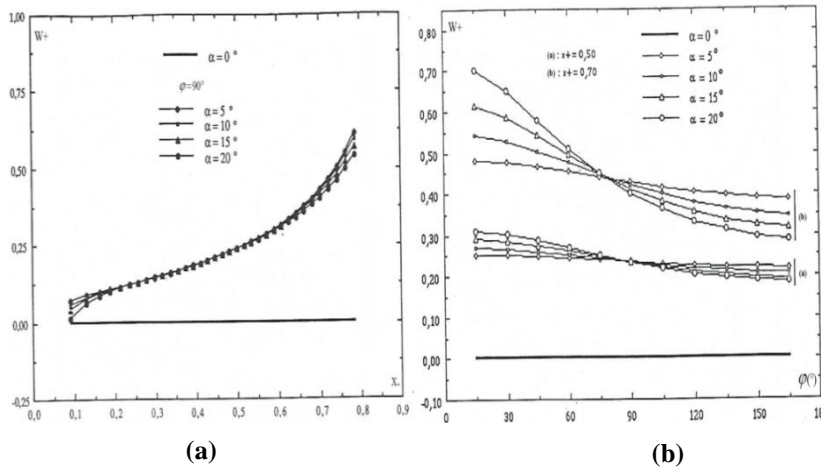


**Figure 4.a:** azimuthal component of the velocity, for different values of  $\alpha$   
 (a) :  $W_+$  against  $x_+$ ,  $\varphi = 45^\circ$  and  $135^\circ$  ; (b) :  $W_+$  against  $\varphi$ ,  $x_+ = 0.09, 0.24, 0.61$

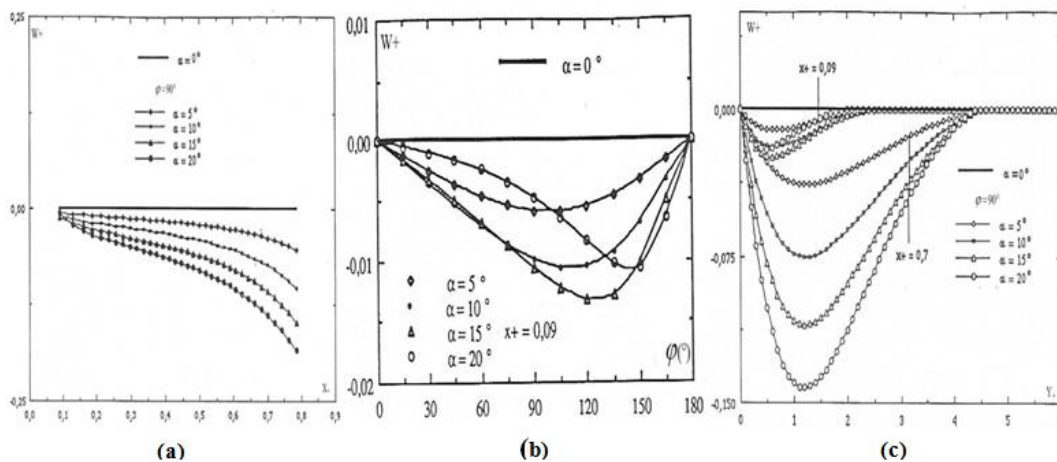


**Figure 5.a:** azimuthal component of the velocity against  $y_+$ , for different values of  $\alpha$  and  $x_+ = 0.09, 0.24, 0.50, 0.70$ .  
**(a):**  $\varphi = 45^\circ$  ; **(b)** :  $\varphi = 90^\circ$

However, we can add notes as, there is one and only one point on the wall of the ellipsoid for which  $W_+$  is not dependent on the inclination, for  $x_+ \geq 0,24$  and even for  $\varphi = 90^\circ$  and  $0, 20 \leq x_+ \leq 0, 60$  (Figures 5.c and 5.d).

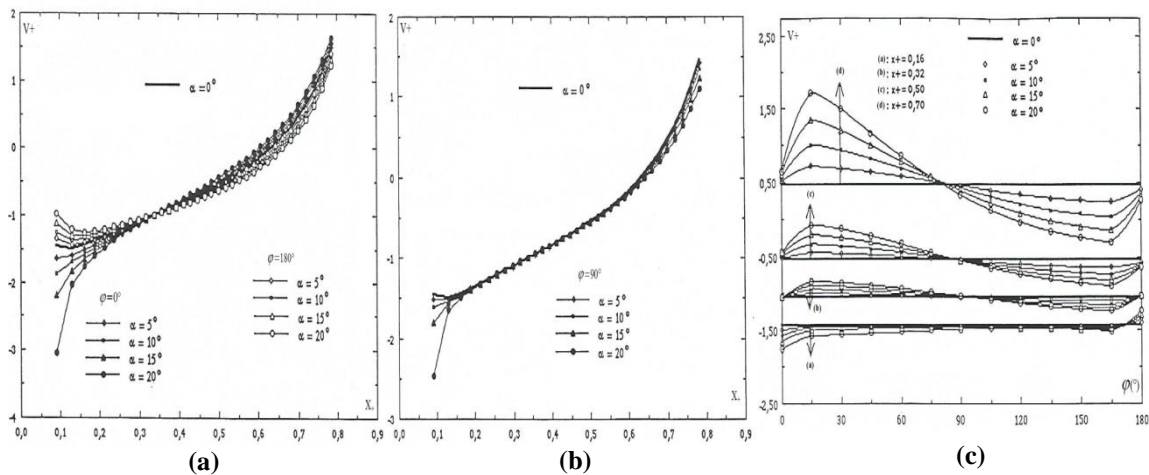


**Figure 5.c:** azimuthal component of the velocity, for different values of  $\alpha$  ,  
**(a)** :  $W_+$  against  $x_+$ ,  $\varphi = 90^\circ$  ; **(b)** :  $W_+$  against  $\varphi$  ,  $x_+ = 0.50, 0.70$



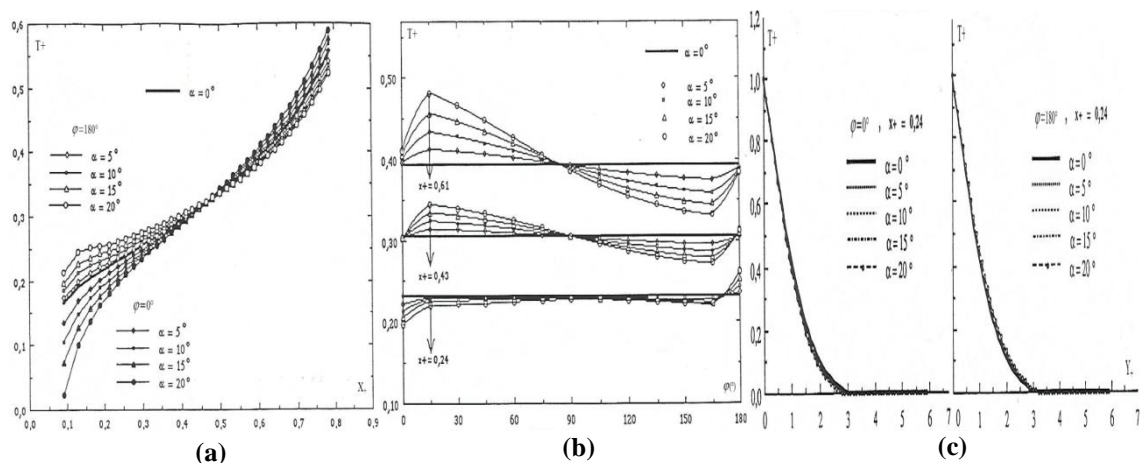
**Figure 6:** azimuthal component of the velocity, for different values of  $\alpha$   
**(a)** :  $W_+$  against  $x_+$ ,  $\varphi = 90^\circ$  ; **(b)** :  $W_+$  against  $\varphi$  ,  $x_+ = 0.09$  ;  
**(c)** :  $W_+$  against  $y_+$ ,  $\varphi = 90^\circ$ ,  $x_+ = 0.09$  and  $0.7$

The curves in the figures 6 show in general that the component  $W_+$  increases with  $\alpha$ . Moreover, these paces further confirm that the thickness of the boundary layer increases with increasing  $x_+$ . The Figures 7.a, 7.b illustrate the evolution of the normal component dimensionless  $V_+$ , for several values of  $\alpha$ ,  $\varphi = 0^\circ$  and  $180^\circ$ . The curves corresponding to  $\varphi = 0^\circ$  and  $\varphi = 180^\circ$ , evolves from either side and relating to the axisymmetric flow ( $\alpha = 0^\circ$ ). On the meridian defined by  $\varphi = 90^\circ$ ,  $V_+$  no longer depends on the inclination for  $0,2 \leq x_+ \leq 0,6$ . However,  $V_+$  admits one to three privileged points for a fixed value, for example  $x_+ = 0.32$ , then, the coordinates points are defined by  $(x_+ = 0.32, \varphi = 0^\circ)$ ,  $(x_+ = 0.32, \varphi = 90^\circ)$  and  $(x_+ = 0.32, \varphi = 180^\circ)$  (Figure 7.c).



**Figure 7:** normal component of the velocity, for several values of  $\alpha$ .  
 (a) :  $V_+$  against  $x_+$ , for  $\varphi = 0^\circ$  and  $180^\circ$ ; (b) :  $V_+$  against  $x_+$ ,  $\varphi = 90^\circ$ ; (c) :  $V_+$  against  $\varphi$ , for  $x_+ = 0.16, 0.32, 0.50, 0.70$

The dimensionless temperature field has the same features as that of  $U_+$  and the figures 8.a, 8.b show the existence of privileged points on the wall of the ellipsoid for which  $T_+$  is independent of the inclination  $\alpha$ . We also note that the temperature varies weakly with  $\alpha$  in the plane of symmetry  $\pi_0$  (Figure 8.c).

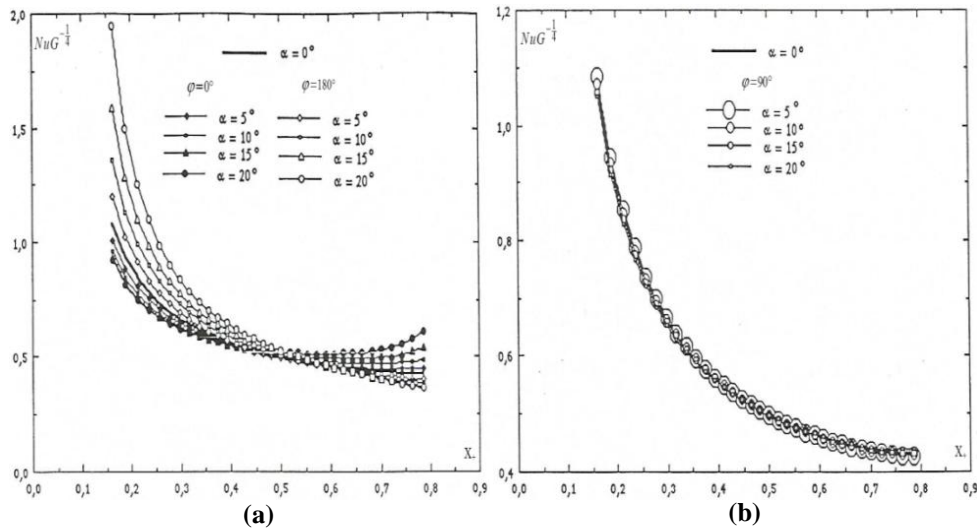


**Figure 8:** temperature profile, for several values of  $\alpha$ .  
 (a) :  $T_+$  against  $x_+$ ,  $\varphi = 0^\circ$  and  $\varphi = 180^\circ$ ; (b) :  $T_+$  against  $\varphi$ ,  $x_+ = 0.24, 0.43, 0.61$ ;  
 (c) :  $T_+$  against  $y_+$ ,  $x_+ = 0.24$ ,  $\varphi = 0^\circ$  and  $\varphi = 180^\circ$

In the case of dimensionless quantity  $NuG^{-\frac{1}{4}}$ , we constantly observe the peculiarities concerning the temperature. In case of presence of convection due to movement of a fluid in laminar flow, the heat transfer will be made primarily by fluid displacement and certainly, the number is none other than the dimensionless temperature gradient at the wall, while its variations depend to exchanges between the wall and the fluid. In our case, given the hypothesis relating thereto, we note that the dimensionless quantity  $NuG^{-\frac{1}{4}}$  present in the

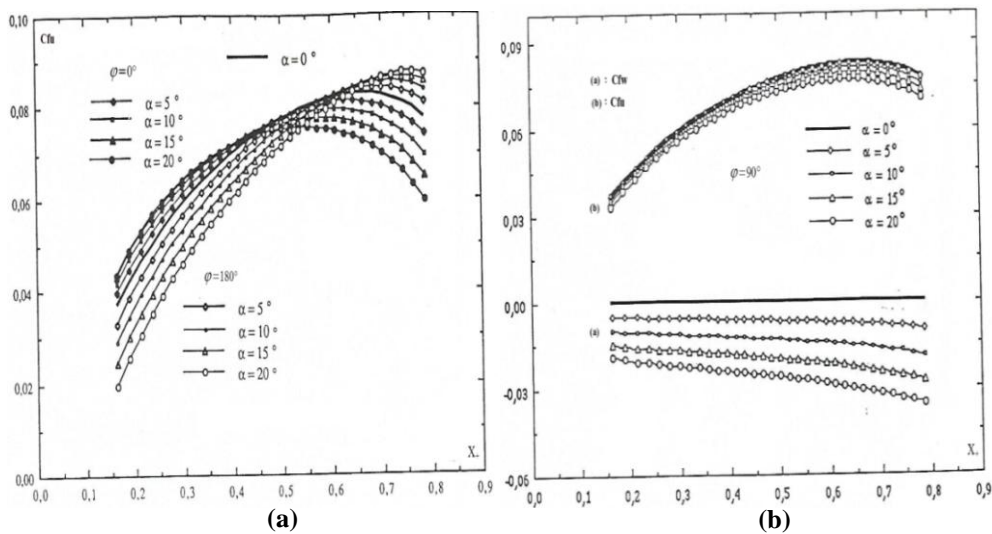


mennerregressive. This indicates that, the heat exchange decreases gradually as one moves the wall according to the movements of the particles following dimensionless directions  $x_+$  and  $y_+$ . Furthermore, it is independent of angle  $\alpha$  with the dimensionless direction  $x_+$  on the meridian equation  $\varphi = 90^\circ$  (Figure 9).



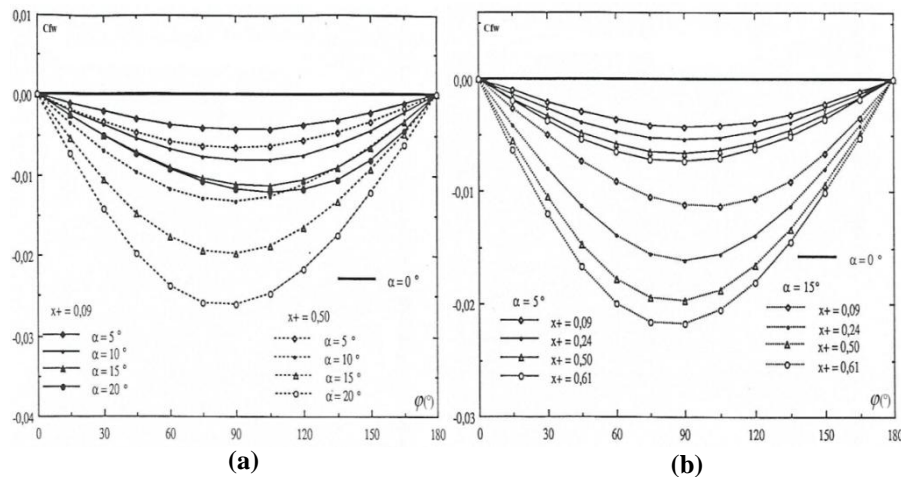
**Figure 9:** nusselt number against  $x_+$ , for several values of  $\alpha$  .  
 (a):  $\varphi = 0^\circ$  and  $\varphi = 180^\circ$  ; (b) :  $\varphi = 90^\circ$

Figures 10.a and 10.b show some curves of changes in the coefficient of friction  $Cfu$  to show the existence of maximum warning of separation of the boundary layer. These points are naturally closer to the pole of ellipsoid for  $\varphi = 0^\circ$  that for  $\varphi = 180^\circ$ .



**Figure 10:** tangential and azimuthal friction coefficients against  $x_+$ , for several values of  $\alpha$  .  
 (a) :  $Cfu$  for  $\varphi = 0^\circ$  and  $\varphi = 180^\circ$  ; (b) :  $Cfu$  and  $Cfw$  for  $\varphi = 90^\circ$

Figure 11, showing the changes in  $Cfw$  against  $\varphi$ , shows that it is zero in the plane of symmetry and confirms that this size increases with  $\alpha$  .



**Figure 11:** azimuthal friction coefficient against  $\phi$ , for different values of  $\alpha$ .

(a) :  $C_{fw}$  for  $x_+ = 0.09$  and  $0.50$ ; (b) :  $C_{fw}$  for  $x_+ = 0.09, 0.24, 0.50, 0.70$ ,  $\alpha = 5^\circ$  and  $15^\circ$

Increasing the amplitude of this dimensionless magnitude in the negative pole confirms, that a strong adherence of fluid particles to the wall when the body is strongly inclined.

## V. CONCLUSION

The orthogonal coordinate systems curvilinear related to body are well suited to the study of three-dimensional hydrodynamic and thermal boundary layers around an ellipsoid of revolution inclined. In this article, we presented the distributions of speed and temperature as well as the local values of Nusselt number and friction coefficients. In the case of a pure natural convection, it appears in the calculations that on the wall of the ellipsoid, there seems exist the privileged coordinate values for which the tilt angle has little effect on the dynamic and thermal quantities. Their position depends of course on the curvilinear abscissa and in the vicinity of the meridian equation  $\phi = 90^\circ$ .

In these results, a presence of suction of the particles on the lower meridian, when the body is strongly inclined and this phenomenon causes a slight disruption. After the analyzes, we find that the thickness of the boundary layer varies and depends on the curvilinear abscissa and of the tilt. The results for the normal component according to the normal coordinate illustrate its evolution in terms of thickness.

In this work, we reported the effects of the inclination angle of the body, by considering the form factor is constant. Soon, it would be desirable to consider the transfer of heat and pulse based on variations of the form factor and inclination or even coupled to a material transport and take into account the unstable boundary conditions.

## ACKNOWLEDGEMENTS

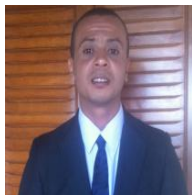
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#### Author Biography



Dr. U. Canissius was born in Diego Suarez, Madagascar. He did his studies from the prestigious Antsirana University, in the area of factory mechanics and fluid mechanics, in the energy option. He has published a number of papers in various national and international journals & conferences. Since 2012, he is teacher researcher, attached to the department of Mechanical and Metrology at the Antsirana University, Madagascar. Currently, he is responsible of Master parcourse in Civil Engineering and of the Metallic Structure to the ENSET School.