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Non Newtonian Behavior of Blood in Presence of Arterial Occlusion

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ABSTRACT: *The objective of the present numerical model is to investigate the effect of shape of stenosis on blood flow through an artery using Bingham plastic fluid model. Blood is modeled as Bingham plastic fluid in a uniform circular tube with an axially symmetric but radially non symmetric stenosis. The expressions for flux, dimensionless resistance to flow with stenosis shape parameter, stenosis length and stenosis size have been shown graphically.*

Keywords: *Stenosis, flux, resistance to flow, Bingham-plastic fluid*

I. INTRODUCTION

Human system, its growth functions are very complicated in nature, so the knowledge of blood flow through arteries is very much important for better understanding the anatomy and physiology of an organic system. For over centuries, in most of the cases cardiovascular diseases have been noticed as one of the major cause of death in the industrialized world. Among the cardiovascular diseases the familiar one such as stroke and atherosclerosis are closely related to abnormality, disorder and malfunction of blood flow characteristics in human body, owing to this, blood flow related problems have shown significant interest for biomedical researchers.

Blood flow characteristics can be altered significantly by the arterial diseases, such as stenosis and aneurysm. Stenosis is a serious cardiovascular disease. The medical term stenosis means narrowing of body passage. Though actual formation of stenosis is unclear to us, it is believed that stenosis is formed by the deposition of fatty substances like cholesterol, fats/ lipids in the inner wall of the artery and unnatural growth of the connective tissue in the lumen of the artery (Young and Tsai [1]).

If stenosis is formed resistance to flow is increased and hence normal blood flow is disturbed in a significant manner, so blood flow is insufficient to reach each cell and this resist the nutrient supplement whose consequences cause several diseases like hypertension, stroke, brain haemorrhage etc. This may be caused by unhealthy living conditions such as heavy alcohol use, exposure to Tabaco smoke, lack of physical activity and improper dietary habits.

In view of these, several bio-medical researchers (Young [2], Lee and Fung [3], Shukla et.al [4], Chaturani and PonnalagarSamy[5]) have considered various mathematical models for blood flow through single stenosed artery. In all those studies they have considered the blood as a Newtonian fluid. But since it has been observed that whole blood being a suspension of erythrocytes in an aqueous solution, blood behaves as a non-Newtonian fluid at low shear rates in micro vessels (Charm and Kurland [6], Blair [7], Aroesty and Gross [8], Majhi and Nair [9], Cokelet [10], Lih [11]). Halder [12] presented a mathematical model of blood flow through stenosed artery by considering blood as power–law fluid and observed that maximum resistance to flow is attained at the throat of the stenosis, in case of a symmetrical stenosis. Biswaset. al [13], Siddiqui et. al [14] have presented a non-Newtonian fluid model by considering blood as Herschel-Bulkley type fluid. Blair and Spanner [15] have presented a mathematical model by considering blood to be Casson type non-Newtonian fluid.

Dechant [16] has presented a perturbation model for the oscillatory flow of a Bingham-plastic in rigid and periodically displaced tube. Biswaset. al [17] have studied two layered pulsatile flow of blood through arterial tube by considering the core layer as Bingham-plastic fluid and the peripheral as Newtonian fluid. Many researchers (Parmar et. al [18], Richard et. al [19]) have presented mathematical models to study the effect of stenosis on non-Newtonian flow of blood. However, all those investigations considered the effect of single stenosis, but the constrictions may develop in series or may be irregular in shape or overlapping. Chakravarthy et. al [20], Srivastava et. al [21] have studied effect of overlapping stenosis on arterial flow of blood.

In the present analysis I propose to discuss the effect of overlapping stenosis on blood flow through an arterial tube by considering the blood as Bingham-plastic type non-Newtonian fluid.

II. MATHEMATICAL FORMULATION

Let us consider the steady flow of blood through an axially symmetric but radially non-symmetric overlapping stenosed artery.
The geometry of stenosis can be taken as [21]:

$$h = \frac{R(z)}{R_0}$$
$$= 1 - \frac{3\delta}{2R_0 L_0^4}[11(z-d)L_0^3 - 47(z-d)^2 L_0^2 + 72(z-d)^3 L_0 - 36(z-d)^4], \quad d \le z \le d + L_0$$
$$= 1, \text{ otherwise}, \quad (1)$$

where $R(z)$ is the radius of the tube in the stenotic region, R_0 is the radius of the tube outside the stenotic region, R_p is the radius in the plug flow region , L_0 is the length of the stenosis and d indicates its location, δ is the maximum height of the stenosis. Projection of stenosis at the two positions is denoted by z as $z = d + \frac{L_0}{6}$, $z = d + \frac{5L_0}{6}$. The critical height is taken as $\frac{3\delta}{4}$at $z = d + \frac{L_0}{2}$ from the origin.

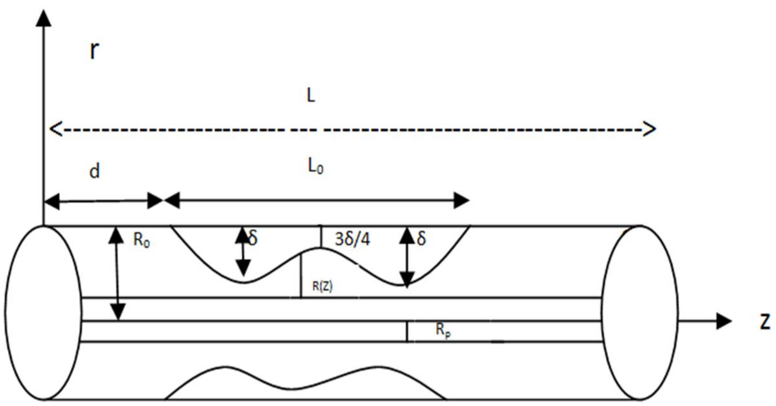

Fig.1: Geometry of a uniform tube of circular- cross section with overlapping stenosis.

The equation governing the flow is given by

$$-\frac{\partial p}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}), \quad (2)$$

in which τ_{rz} represents the shear stress of blood for Bingham-plastic fluid and p is the pressure gradient.
The relationship between shear stress and shear rate is given by

$$\tau_{rz} = \mu\left(-\frac{\partial u}{\partial r}\right) + \tau_0 , \quad \tau_{rz} \ge \tau_0$$
$$\frac{\partial u}{\partial r} = 0, \quad \tau_{rz} < \tau_0 \quad (3)$$

where u stands for the axial velocity of blood; τ_0, the yield stress and μ , the coefficient of viscosity of blood.
The boundary conditions are:

(i) τ_{rz} is finite at $r = 0$
(ii) $u = 0$ at $r = h(z)$
(iii) $\frac{\partial u}{\partial r} = 0$ if $\tau_{rz} < \tau_0$

(4)

III. SOLUTIONS

Integrating (2) and using the boundary condition (i) of (4) we get

$$\tau_{rz} = \frac{Pr}{2}$$

From (3), we get by using the boundary condition (ii) of (4)

$$u = \frac{Ph^2}{4\mu}\left[(1 - \frac{r^2}{h^2}) - \frac{4\tau_0}{Ph}(1 - \frac{r}{h})\right] \quad (5)$$

Since $\frac{\partial u}{\partial r} = 0$ at $r = r_0$, the upper limit of the plug flow region is obtained as

$$r_0 = \frac{2\tau_0}{P}$$

Thus we get

$$u = \frac{Ph^2}{4\mu}[(1-\frac{r^2}{h^2}) - \frac{2r_0}{h}(1-\frac{r}{h})] \quad (6)$$

The plug velocity u_p is given by

$$u_p = \frac{Ph^2}{4\mu}(1-\frac{r_0}{h})^2 \quad (7)$$

The volumetric flow rate i.e, the flux is given by

$$Q = 2[\int_0^{r_0} u_p r dr + \int_{r_0}^{h} u r dr]$$

$$= \frac{Ph^4}{24\mu}[3(1-\frac{r_0}{h}) - \frac{\tau_0}{h}(1-\frac{r_0^3}{h^3})]$$

$$= \frac{Ph^4}{24\mu}[3-4\frac{r_0}{h} + \frac{r_0^4}{h^4}] \quad (8)$$

Thus

$$\frac{\partial p}{\partial z} = -P = \frac{-24\mu Q}{h^4(3-4\frac{r_0}{h}+\frac{r_0^4}{h^4})} \quad (9)$$

The pressure drop Δp across the stenosis between z = 0 to z = L is obtained as

$$\Delta p = \int_0^L \frac{\partial p}{\partial z} dz = -24\mu \int_0^L \frac{Q}{h^4(3-4\frac{r_0}{h}+\frac{r_0^4}{h^4})} dz \quad (10)$$

Introducing the following non-dimensional quantities we get

$$\bar{z} = \frac{z}{L}, \quad \bar{\delta} = \frac{\delta}{R_0}, \quad \bar{R}(z) = \frac{R(z)}{R_0}, \quad \bar{Q} = \frac{Q}{\pi U R_0^2},$$

$$\bar{\tau_0} = \frac{\tau_0}{\mu U/R_0}, \quad \bar{\tau_{rz}} = \frac{\tau_{rz}}{\mu U/R_0}, \quad \bar{P} = \frac{P}{\mu U L/R_0^2}, \quad (11)$$

In equation (10) we finally get (after dropping the bars)

$$\Delta p = \int_0^1 \frac{-24\mu Q}{h^4(3-4\frac{r_0}{h}+\frac{r_0^4}{h^4})} dz \quad (12)$$

The resistance to flow λ is defined as

$$\lambda = \frac{\Delta p}{Q} = -\int_0^1 \frac{24\mu}{h^4(3-4\frac{r_0}{h}+\frac{r_0^4}{h^4})} dz \quad (13)$$

The pressure drop in the absence of stenosis (h =1) is denoted by Δp_N and is obtained from (12) as

$$\Delta p_N = \int_0^1 \frac{-24\mu Q}{(3-4r_0+r_0^4)} dz \quad (14)$$

The resistance to flow in the absence of stenosis as

$$\lambda_N = \frac{\Delta p_N}{Q} = \int_0^1 \frac{-24\mu}{(3-4r_0+r_0^4)} dz = \frac{-24\mu}{(3-4r_0+r_0^4)}$$

Hence the normalized resistance to flow $\bar{\lambda}$ is given by

$$\bar{\lambda} = \frac{\lambda}{\lambda_N}$$

$$= (3-4r_0+r_0^4)\int_0^1 \frac{1}{(3h^4-4r_0h^3+r_0^4)} dz \quad (15)$$

IV. RESULTS AND DISCUSSIONS

To illustrate the flow behavior the results are shown graphically with the help of MATLAB-7.6.The numerical results are shown graphically and discussed for various values of the shape parameters.

Fig. 2 - Fig. 4 depict the variations of flux Q for different values of yield stress τ_0 , stenosis length L_0 and z with the variation of stenosis height δ. It is observed that Q decreases with the increase of δ and τ_0 for fixed values of the other parameters, but it increases when stenosis length L_0 and z increase.

Fig. 5 - Fig. 7 show the variations of resistance to flow with the variations of stenosis height for different values of τ_0, L_0 and d. It is found that as stenosis increases, resistance to flow $\bar{\lambda}$ increases with the increase of τ_0 andL_0 , but the reverse effect occurs when d increases.

V. CONCLUSIONS

Blood flow characteristics through human artery are greatly influenced by the flux and resistance to flow, whose consequences cause several diseases, like hypertension, heart attack, brain haemorrhage etc. In the present analysis we observe that resistance to flow increases within the stenotic region as stenosis developed. So the present investigation may be helpful for further study of the various types of cardiovascular diseases.

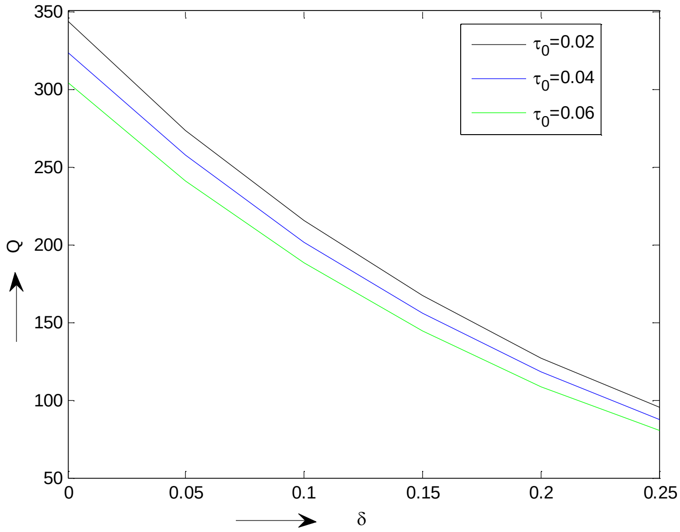

Fig. 2: Variation of flux for different values of yield stress.

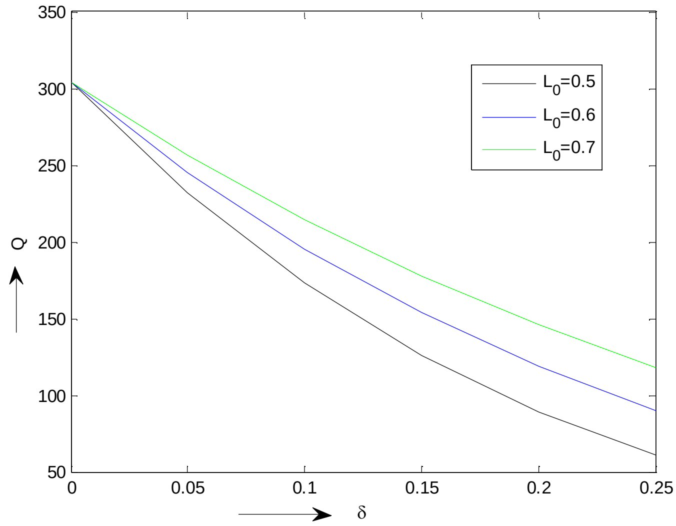

Fig. 3: Variation of flux for different values of stenosis length.

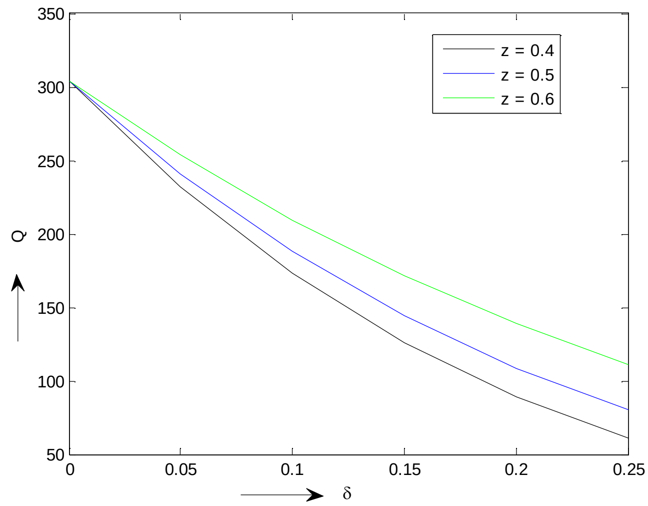

Fig. 4: Variation of flux for different values of z.

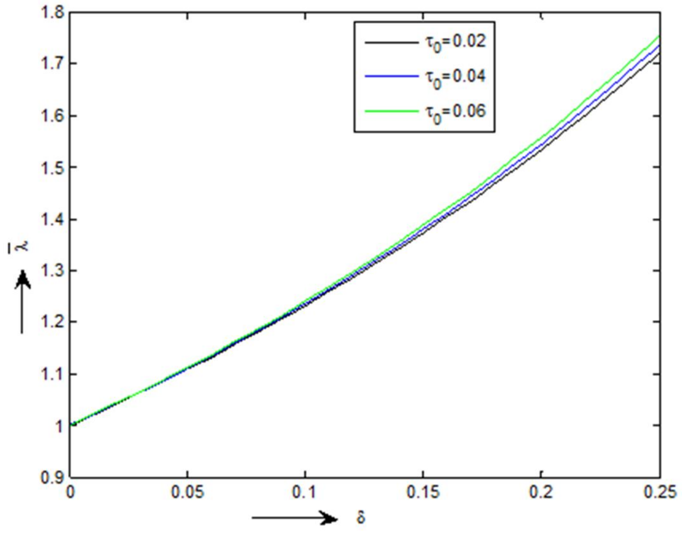

Fig. 5: Variations of resistance to flow with the variations of stenosis height for different values of τ_0.

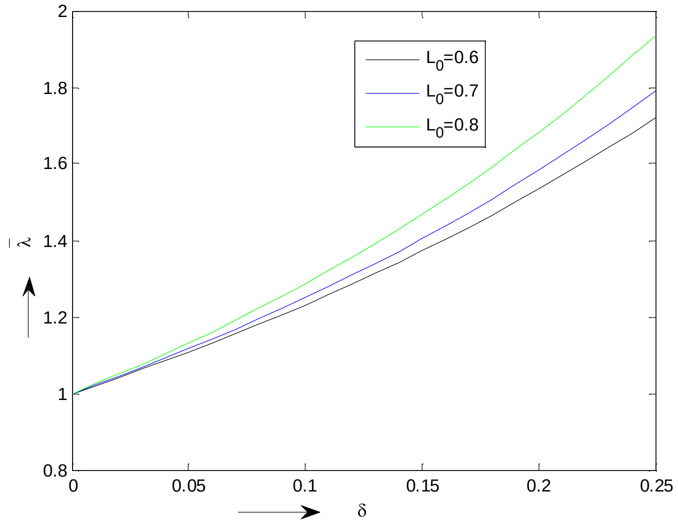

Fig. 6: Variations of resistance to flow with the variations of stenosis height for different values of L_0.

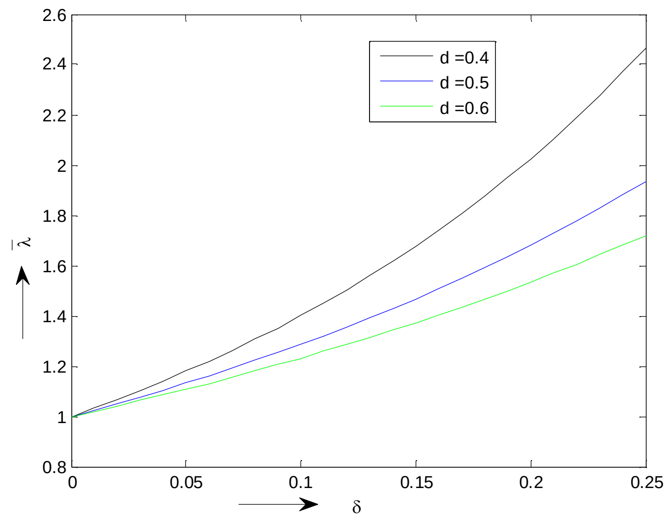

Fig. 7: Variations of resistance to flow with the variations of stenosis height for different values of d.

REFERENCES

[1]. D. F.Young and F. Y. Tsai, Flow characteristics in models of arterial stenosis- steady flow, *J. of Biomechanics, 6,* 1973 395-410.

[2]. D. F. Young, Effects of a time-dependency stenosis on flow through a tube, *J. Engg.Ind., Trans ASME, 90,* 1968, 248-254.

[3]. J. S. Lee and Y. C. Fung, Flow in locally constricted tubes and low Reynolds number, *J. Appl. Mech., Trans ASME, 37,* 1970, 9-16.

[4]. J. B. Shukla,,R. S. Parihar and B. R. P.Rao, Effects of stenosis on non-Newtonian flow through an artery with mild stenosis, *Bull. Math. Biol., 42,* 1980, 283-294.

[5]. P. Chaturaniand R.PonnalagarSamy, Pulsatile flow ofCasson's fluid through stenosed arteries with applications to blood flow, *Biorheol., 23,* 1986, 491-511.

[6]. S. Charmand G. Kurland,Viscometry of human blood for shear rates of 0-100000 sec-1, *Nature, 206(4984),*1965, 617-618.

[7]. G. W. S. Blair, An equation for the flow of blood, plasma and serum through glass capillaries, *Nature, 183(4661),*1959, 613-614.

[8]. J. Aroestyand J. F.Gross, Pulsatile flow in small blood vessels.I.Casson Theory, *Biorheology, 9(1),* 1972, 33-43.

[9]. S. N. Majhiand V. R. Nair, Pullsatile flow of third grade fluids under body acceleration-modelling blood flow, *Int. J. Engg. Sci. 32(5),* 1996, 839-846.

[10]. G. R. Cokelet, The rheology of human blood, *Biomechanics, Ed. by Y. C. Fung et. al , 63, 1972,* Englewood Cliffs: Prentice Hall.

[11]. M. M. Lih, *Transport phenomena in Medicine and Biology* (Wiley, New York, 1975).

[12]. K. Halder, Effects of the shape of stenosis on the resistance to blood flow through an artery, *Bull. Mathe.Bio., 47,* 1985, 545-550.

[13]. D. Biswas and R. B.Laskar, Steady flow of blood through a stenosed artery: A non-Newtonian fluid model, *Journal of Science and Technology, 7(11),* 2011, 144-153.

[14]. S. U. Siddqui, N. K. Verma and R. S. Gupta, A Mathematical model for pulsatile flow of Herschel-Bulkley fluid through stenosed artery, *Journal of Science and Technology, 4(5),* 2010, 49-66.

[15]. G. W. Scott Blair and D. C. Spanner, *An introduction to Bio rheology* (Elsevier Scientific pub, Company, Amsterdam, Oxford and New York, 1974).

[16]. L. J. Dechant, A Perturbation model for the oscillatory flow of a Bingham plastic in rigid and periodically displaced tubes, *J. Biomech. Eng. 121(5),* 1999, 502-504.

[17]. D. Biswas and U. S.Chakraborty, Two layered pulsatile blood flow in a stenosed artery with body acceleration and slip at wall, *International Journal of Applications and Applied Mathematics (AAM),5(2),* 2010, 303-320.

[18]. L. Parmar,S. B. Kulshreshthaand D. P.Sing, Effects of stenosis on Casson flow of blood through arteries, *International Journal of Advanced Computer and Mathematical Sciences, 4(4),* 2013, 257-268.

[19]. L. K. Richard, D. F Youngand N. R.Chalvin, Wall vibrations induced by flow through simulated stenosis in Models and arteries, *Biomech., Vol.10(431),* 1977.

[20]. S. Chakravarthy,T. K. Mandal, Mathematical modelling of blood flow through an overlapping stenosis, *Math. Compute. Model., 19,* 1994, 59-73.

[21]. V. P. Srivastava and R. Rastogi, Blood flow through stenosed catheterized artery : effects of hematocrit and stenosis shape, *Comput. Math. Applc.59, 2010, 1377-1385*