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Some Fixed Point Theorems in Fuzzy Metric Spaces

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ABSTRACT: *In this paper we present some common fixed point theorems for occasionally weakly compatible mapping in fuzzy metric spaces.*
Keywords*: Occasionally weakly compatible mappings, fuzzy metric space.*

I. INTRODUCTION

The concept of fuzzy set was introduced by Zadeh[27] in 1965 and the concept of fuzzy metric space introduced by Kramosiland Michalek [15]in 1975.George and Veeramani [7] modifiedthe notion of fuzzy metric with the help f continuoust-norms. Continuing Pant[19,20,21]introduced the new concept of reciprocally continuous mapping and establised common fixed point theorems .Balasubramaniam et al.[5]gives more results on fuzzy metric space.C.T Aage and J.N.Salunke [3]establised some results on fixed point theorem in fuzzy metric space.The purpose of this paper is to find some commonfixed point theorems for occasionally weakly compatible mapping in fuzzy metric spaces.

II. PRELIMINARIES

Definition 2.1 A fuzzy set A in X is a function with domain X and values in [0,1]. **Definition 2.2**A binary operation $*$: $[0, 1] \times [0, 1] \longrightarrow [0, 1]$ is a continuous t-norms if $*$ is satisfying conditions:

i. * is an commutative and associative;
ii. * is continuous;
iii. $a * 1 = a$ for all $a \in [0, 1]$;
iv. $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, and $a, b, c, d \in [0, 1]$.

Definition 2.3 A 3-tuple $(X,M, *)$ is said to be a fuzzy metric space if X is an arbitrary set, $*$is a continuous t-norm and M is a fuzzy set on $X^2 \times (0,\infty)$ satisfying the following conditionor all $x, y, z\in X$, $s, t > 0$,
(f1) $M(x, y, t) > 0$;
(f2) $M(x, y, t) = 1$ if and only if $x = y$
(f3) $M(x, y, t) = M(y, x, t)$;
(f4) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$;
(f5) $M(x, y, \cdot) : (0, \infty) \rightarrow (0, 1]$ is continuous.

Then M is called a fuzzy metric on X. Then M(x, y, t) denotes the degree of nearness between x and y with respect to t.

Example 2.4(Induced fuzzy metric) Let (X, d) be a metric space. Denote $a*b = ab$ for all $a, b\in[0, 1]$ and let $M_d$be fuzzy sets on $X^2 \times (0,\infty)$ defined as follows:

$$M_d(x, y, t) = \frac{t}{t+d(x,y)}$$

Then $(X,M_{d,}*)$ is a fuzzy metric space. We call this fuzzy metric induced by a metric d as the standard intuitionistic fuzzy metric.

Definition 2.5 :Let $(X,M,*)$ be a fuzzy metric space. Then
(a) a sequence $\{x_n\}$ in X is said to converges to x in X if for each $\in > 0$ and each $t > 0$, there exists $n_0\in N$ such that $M(x_n, x, t) > 1 -\epsilon$ for all $n \in n_0$.
(b) a sequence {xn} in X is said to be Cauchy if for each $\in > 0$ and each $t > 0$, there exists $n_0\in N$ such that $M(x_n, x_m, t) > 1 - \epsilon$ for all $n,m \in n_0$.
(c) A fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.

Definition 2.6 A pair of self-mappings (f, g) of a fuzzy metric space (X,M,∗) is said to be
(i) weakly commuting if M(fgx, gfx, t) ≥M(fx, gx, t) for all x ∈X and t > 0.
(ii) R-weakly commuting if there exists some R > 0 such that
M(fgx, gfx, t) ≥M(fx, gx, t/R) for all x ∈ X and t > 0.

Definition 2.7 Two self-mappings f and g of a fuzzy metric space (X,M,∗) are called compatible if $\lim_{n\to\infty} M(fgxn, gfxn, t) = 1$whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = x$ for some x in X.

Definition 2.8: Two self maps f and g of a fuzzy metric space (X,M,∗) are called reciprocally continuous on X if $\lim_{n\to\infty} fgxn = fx$ and$\lim_{n\to\infty} gfxn = gx$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fxn = \lim_{n\to\infty} gx_n = x$ for some x in X.

Lemma 2.9 Let (X,M,∗) be a fuzzy metric space. If there exists q ∈(0, 1) such that
M(x, y, qt) ≥ M(x, y, t) for all x, y∈ X and t > 0, then x = y.

Definition 2.10 Let X be a set, f, g selfmaps of X. A point x in X is called a coincidence point of f and g ifffx = gx. We shall call w = fx = gx a point of coincidence of f and g.

Definition 2.11- A pair of maps S and T is called weakly compatible pair if they commute at coincidence points. The concept occasionally weakly compatible is introduced by M. A-Thagafi and NaseerShahzad [4].

Definition 2.12- Two self-maps f and g of a set X are occasionally weakly compatible (owc) iff there is a point x in X which is a coincidence point of f and g at which f and g commute. Al-Thagafi and NaseerShahzad [4] shown that occasionally weakly is weakly compatible but converse is not true.

Example 2.13 Let R be the usual metric space. Define S, T : R →R by Sx = 2x and Tx = x^2 for all x ∈R. Then Sx = Tx for x = 0, 2 but ST0 = TS0, and
ST2 ≠ TS2. S and T are occasionally weakly compatible self-maps but not weakly compatible**.**

Lemma 2.14 Let X be a set, f, g owcself-maps of X. If f and g have a unique point of coincidence, w = fx = gx, then w is the unique common fixed point of f and g.

MAIN RESULTS

Theorem-3.1– Let $(X, M,*)$ be a fuzzy metric space and let P,Q,R,S be self-mapping of X.Let the Pair {P,,R},{Q,S} be owc. If there exists $q \in (0,1)$ such that

$$M(Px,Qy,qt) \geq \min\{\frac{M(Rx,Sy,t).M(Qy,Rx,t)}{M(Px,Sy,t)}, \frac{M(Rx,Sy,t).M(Qy,Sy,t)}{M(Px,Sy,t)}, \frac{M(Rx,Sy,t).M(Qy,Sy,t)}{M(Qy,Rx,t)}, \frac{M(Qy,Sy,t).M(Px,Sy,t)}{M(Qy,Rx,t)}, \frac{M(Px,Sy,t).(Qy,Rx,t)}{M(Rx,Sy,t)}\} \quad (1)$$

For all $x, y \in X$ and for all $t > 0$,then there exist $w \in X$ such that $Pw = Rw = w$ and a unique point $z \in X$ such that $= Sz = z$. Moreover $z = w$,so that there is a unique common fixed point of P,Q,R, and S.

Proof- Let the pairs {P,R} and {Q,S} be two owc , so there are points $x, y \in X$ such that $Px = Rx$ *and* $Qy = Sy$. We claim that $Px = Qy$. if not ,then by inequality (1),

$$M(Px,Qy,qt) \geq \min\{\frac{M(Rx,Sy,t).M(Qy,Rx,t)}{M(Px,Sy,t)}, \frac{M(Rx,Sy,t).M(Qy,Sy,t)}{M(Px,Sy,t)}, \frac{M(Rx,Sy,t).M(Qy,Sy,t)}{M(Qy,Rx,t)}, \frac{M(Qy,Sy,t).M(Px,Sy,t)}{M(Qy,Rx,t)}, \frac{M(Px,Sy,t).(Qy,Rx,t)}{M(Rx,Sy,t)}\}$$

$$= \min\{\frac{M(Px,Qy,t).M(Qy,Px,t)}{M(Px,Qy,t)}, \frac{M(Px,Qy,t).M(Qy,Qy,t)}{M(Px,Qy,t)}, \frac{M(Px,Qy,t).M(Qy,Qy,t)}{M(Qy,Px,t)}, \frac{M(Qy,Qy,t).M(Px,Qy,t)}{M(Qy,Px,t)}, \frac{M(Px,Qy,t).(Qy,Px,t)}{M(Px,Qy,t)}\}$$

$$= \min\{ M(Px,Qy,t), M(Qy,Qy,t), M(Qy,Qy,t), M(Qy,Qy,t), M(Px,Qy,t)\}$$

$$= \min\{ M(Px,Qy,t)$$

Therefore $Px = Qy$, $i.e$ $Px = Rx = Qy = Sy$. Suppose that there is a another point z such that $Pz = Rz = Qy = Sy, so\ Px = Pz\ and\ w = Px = Rx$ is the unique point of coincident of$P and\ R$. By the lemma 2.14 w is the only fixed point of $P and\ R$. Similarly there is a unique point $z \in X$ such that $z = Qz = Sz$.Assume that $w \neq z$. We have

$$(w,z,qt) = M(Pw,Qz,qt)$$
$$\geq \min\{\frac{M(Rw,Sz,t).M(Qz,Rw,t)}{M(Pw,Sz,t)}, \frac{M(Rw,Sz,t).M(Qz,Sz,t)}{M(Pw,Sz,t)}, \frac{M(Rw,Sz,t).M(Qz,Sz,t)}{M(Qz,Rw,t)}, \frac{M(Qz,Sz,t).M(Pw,Sz,t)}{M(Qz,Rw,t)}, \frac{M(Pw,Sz,t).(Qz,Rw,t)}{M(Rw,Sz,t)}$$

$$\geq min\{\frac{M(w,z,t).M(z,w,t)}{M(w,z,t)}, \frac{M(w,z,t).M(z,z,t)}{M(w,z,t)}, \frac{M(w,z,t).M(z,z,t)}{M(z,w,t)},$$
$$\frac{M(z,z,t).M(w,z,t)}{M(z,w,t)}, \frac{M(w,z,t).(z,w,t)}{M(w,z,t)}\}$$

$$= \min\{M(w,z,t), M(z,z,t), M(z,z,t), M(z,z,t), M(w,w,t)\}$$
$$= M(w,z,t)$$

Therefore we have z = w by Lemma 2.14 and z is a common fixed point of P,Q,$R$and S. The uniqueness of the fixed point holds from (1).

Theorem-3.2-Let $(X,M,*)$be a fuzzy metric space and let P,Q,R,S be the self mapping of X.Let the pair {P,R}and {Q,S} be owc.If there exists $q \in (0,1)$, such that

$$M(Px,Qy,qt) \geq \emptyset[\min\{\frac{M(Rx,Sy,t).M(Qy,Rx,t)}{M(Px,Sy,t)}, \frac{M(Rx,Sy,t).M(Qy,Sy,t)}{M(Px,Sy,t)}, \frac{M(Rx,Sy,t).M(Qy,Sy,t)}{M(Qy,Rx,t)}, \frac{M(Qy,Sy,t).M(Px,Sy,t)}{M(Qy,Rx,t)}, \frac{M(Px,Sy,t).(Qy,Rx,t)}{M(Rx,Sy,t)}\}] \quad (2)$$

For all $x,y \in X$ and $\emptyset:[0,1] \to [0,1]$ such that $\emptyset(t) > t for\ all\ 0 < t < 1$, then there exist a unique common fixed point of P,Q,R and S.

Proof-The proof follows from Theorem 3.1.

Theorem-3.3-Let $(X,M,*)$be a fuzzy metric space and let P,Q,R and S be self mapping of X. Let the pair {P,R} and {Q,S} be owc.If there exists $q \in (0,1)$, such that

$$M(Px,Qy,qt) \geq \emptyset\{\frac{M(Rx,Sy,t).M(Qy,Rx,t)}{M(Px,Sy,t)}, \frac{M(Rx,Sy,t).M(Qy,Sy,t)}{M(Px,Sy,t)}, \frac{M(Rx,Sy,t).M(Qy,Sy,t)}{M(Qy,Rx,t)}, \frac{M(Qy,Sy,t).M(Px,Sy,t)}{M(Qy,Rx,t)}, \frac{M(Px,Sy,t).(Qy,Rx,t)}{M(Rx,Sy,t)}\} \quad (3)$$

For all $x,y \in X$ and $\emptyset:[0,1]^5 \to [0,1]$ such that $\emptyset(t,1,1,1,t) > t for\ all\ 0 < t < 1$, then there exist a unique common fixed point of P,Q,R and S.

Proof-Let the pairs {P,R} and {Q,S} be two owc , so there are points $x,y \in X$ such that $Px = Rx$ and $Qy = Sy$. We claim that $Px = Qy$ .By inequality (3),

$$M(Px,Qy,qt) \geq \emptyset\{\frac{M(Rx,Sy,t).M(Qy,Rx,t)}{M(Px,Sy,t)}, \frac{M(Rx,Sy,t).M(Qy,Sy,t)}{M(Px,Sy,t)}, \frac{M(Rx,Sy,t).M(Qy,Sy,t)}{M(Qy,Rx,t)}, \frac{M(Qy,Sy,t).M(Px,Sy,t)}{M(Qy,Rx,t)}, \frac{M(Px,Sy,t).(Qy,Rx,t)}{M(Rx,Sy,t)}\}$$
$$= \emptyset\{\frac{M(Px,Qy,t).M(Qy,Px,t)}{M(Px,Qy,t)}, \frac{M(Px,Qy,t).M(Qy,Qy,t)}{M(Px,Qy,t)}, \frac{M(Px,Qy,t).M(Qy,Qy,t)}{M(Qy,Px,t)}, \frac{M(Qy,Qy,t).M(Px,Qy,t)}{M(Qy,Px,t)}, \frac{M(Px,Qy,t).(Qy,Px,t)}{M(Rx,Qy,t)}\}$$
$$= \emptyset\{M(Px,Qy,t), M(Qy,Qy,t), M(Qy,Qy,t), M(Qy,Qy,t), M(Px,Qy,t)\}$$
$$> M(Px,Qy,t)$$

This is contradiction. Therefore$Px = Qy$,$i.e$ $Px = Rx = Qy = Sy$. Suppose that there is a another point z such that$Pz = Rz$ $then$ by $(3)Pz = Rz = Qy = Sy, so$ $Px = Pz$ and $w = Px = Rx$ is the unique point of coincident of$P$ and S. By the lemma 2.14 w is the only fixed point of $P and$ R. Similarly there is a unique point $z \in X$ such that $z = Qz = Sz$.Thus z is a common fixed point of P,Q,R and S.Uniqueness of the fixed point holds from (3).

Theorem3.4-Let $(X, M,*)$be a fuzzy metric space and let P, Q, R, S be the self mapping of X.Let the pair *{P,R}* and {Q,S} be owc.If there exists $q \in (0,1)$, such that

$$M(Px, Qy, qt) \geq \frac{M(Rx, Sy, t). M(Qy, Rx, t)}{M(Px, Sy, t)} * \frac{M(Rx, Sy, t). M(Qy, Sy, t)}{M(Px, Sy, t)} * \frac{M(Rx, Sy, t). M(Qy, Sy, t)}{M(Qy, Rx, t)} * \frac{M(Qy, Sy, t). M(Px, Sy, t)}{M(Qy, Rx, t)} * \frac{M(Px, Sy, t). (Qy, Rx, t)}{M(Rx, Sy, t)} \qquad (4)$$

For all $x, y \in X$, then there exist a unique common fixed point of P, Q, R *and* S.

Proof-Let the pairs {P,R} and {Q,S} be two owc , so there are points $x, y \in X$ such that $Px = Rx$ *and* $Qy = Sy$. We claim that $Px = Qy$ .By inequality (4),

$$M(Px, Qy, qt) \geq \frac{M(Rx, Sy, t). M(Qy, Rx, t)}{M(Px, Sy, t)} * \frac{M(Rx, Sy, t). M(Qy, Sy, t)}{M(Px, Sy, t)} * \frac{M(Rx, Sy, t). M(Qy, Sy, t)}{M(Qy, Rx, t)} * \frac{M(Qy, Sy, t). M(Px, Sy, t)}{M(Qy, Rx, t)} * \frac{M(Px, Sy, t). (Qy, Rx, t)}{M(Rx, Sy, t)}\}$$

$$= \frac{M(Px, Qy, t). M(Qy, Px, t)}{M(Px, Qy, t)} * \frac{M(Px, Qy, t). M(Qy, Qy, t)}{M(Px, Qy, t)} * \frac{M(Px, Qy, t). M(Qy, Qy, t)}{M(Qy, Px, t)} * \frac{M(Qy, Qy, t). M(Px, Qy, t)}{M(Qy, Px, t)} * \frac{M(Px, Qy, t). (Qy, Px, t)}{M(Rx, Qy, t)}$$

$$\geq M(Px, Qy, t) * M(Qy, Qy, t) * M(Qy, Qy, t) * M(Qy, Qy, t) * M(Px, Qy, t)$$

$$\geq M(Px, Qy, t)$$

Therefore $Px = Qy$, $i.e$ $Px = Rx = Qy = Sy$. Suppose that there is a another point z such that$Pz = Rz = Qy = Sy$, *so* $Px = Pz$ *and* $w = Px = Rx$ is the unique point of coincident of*Pand R*. By the lemma 2.14 w is the only fixed point of *Pand R*. Similarly there is a unique point $z \in X$ such that $z = Qz = Sz$. Thus w is a common fixed point of *P*,*Q*,*R*and *S*.

Corollary 3.5- Let $(X, M,*)$be a fuzzy metric space and let P, Q, R, S be the self mapping of X.Let the pair *{P,R}* and {Q,S} be owc.If there exists $q \in (0,1)$, *For all* $x, y \in X$
$t > 0$, such that

$$M(Px, Qy, qt) \geq M(Px, Qy, t)$$

then there exist a unique common fixed point of P, Q, R *and* S.

Proof: The Proof follows from Theorem 3.5

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