American Journal of Engineering Research (AJER)
e-ISSN: 2320-0847 p-ISSN : 2320-0936
Volume-5, Issue-6, pp-62-73
www.ajer.org
Research Paper Open Access

"An Inventory Model for Two Warehouses with Constant Deterioration and Quadratic Demand Rate under Inflation and Permissible Delay in Payments"

U. B. Gothi[1], Prachi Saxena[2], Kirtan Parmar[3]

[1]*Head & Asso. Prof., Dept. Of Statistics, St. Xavier's College (Autonomous), Ahmadabad, Gujarat, India.*
[2]*Research Scholar, Dept. Of Statistics, St. Xavier's College (Autonomous), Ahmadabad, Gujarat, India.*
[3]*Adhyapak Sahayak, Dept. Of Statistics, St. Xavier's College (Autonomous), Ahmadabad, Gujarat, India.*

ABSTRACT: *In this paper, we have analysed a two-warehouse inventory model for deteriorating items with quadratic demand with time varying holding cost. The effect of permissible delay in payments is also considered, which is usual practice in most of the businesses i.e. purchasers are allowed a period to pay back for the goods brought without paying any interest. To make it more suitable to the present environment the effect of inflation is also considered. Our objective is to minimize the average total cost per time unit under the influence of inflation. Numerical examples are provided to illustrate the model and sensitivity analysis is also carried out for the parameters.*

Keywords: *Inventory model, Two-warehouse, Deterioration, Quadratic demand.*

I. INTRODUCTION

The main problem in an inventory management is to decide where to stock the goods. Generally, when the products are seasonal or the suppliers provide discounts on bulk purchase, the retailers purchase more goods than the capacity of their owned warehouse (OW). Therefore, the excess units over the fixed capacity w of the owned warehouse are stored in rented warehouse (RW). Usually, the unit holding charge is higher in rented warehouse than the owned warehouse, as the rented warehouse provides a better preserving facility resulting in a lower rate of deterioration in the goods than the owned warehouse. And thus, the firm stores goods in owned warehouse before rented warehouse, but clears the stocks in rented warehouse before owned warehouse.

Inventory models for deteriorating items were widely studied in the past but the two-warehouse inventory issue has received considerable attention in recent years. Hartley [10] was the first person to develop the basic two-warehouse inventory model. Chung and Huang [5] proposed a two-warehouse inventory model for deteriorating items under permissible delay in payments, but they assumed that the deteriorating rate of two warehouses were the same. An inventory model with infinite rate of replenishment with two-warehouse was considered by Sarma [12]. An optimization inventory policy for a deteriorating items with imprecise lead-time, partially/fully backlogged shortages and price dependent demand under two-warehouse system was developed by Rong *et al.* [18]. Lee and Hsu [13] investigated a two-warehouse production model for deteriorating items with time dependent demand rate over a finite planning horizon.

Earlier, in Economic Order Quantity (EOQ), it was usually assumed that the retailer must pay to the supplier for the items purchased as soon as the items were received. In the last two decades, the influence of permissible delay in payments on optimal inventory management has attracted attention of many researchers. Goyal [9] first considered a single item EOQ model under permissible delay in payments. Aggarwal and Jaggi [1] extended Goyal's [9] model to the case with deteriorating items. Aggarwal and Jaggi's [1] model was further extended by Jamal et al. [2] to consider shortages. Chung and Huang [7] further extended Goyal's [9] model to the case that the units are replenished at a finite rate under delay in payments and developed an easy solution procedure to determine the retailer's optimal ordering policy. A literature review on inventory model under trade credit is given by Chang et al. [8]. Teng *et al.* [19] developed the optimal pricing and lot sizing under permissible delay in payments by considering the difference between the selling price and the purchase cost and also the demand is a function of price. For the relevant papers related to permissible delay in payments see Chung and Liao [6], Liao ([14], [15]), Huang and Liao [11].

Recently, Kirtan Parmar and U. B. Gothi [16] have developed order level inventory model for deteriorating items under time varying demand condition. Devyani Chatterji and U. B. Gothi [4] have developed an integrated inventory model with exponential amelioration and two parameter Weibull deterioration. Ankit Bhojak and U. B. Gothi [3] have developed inventory models for ameliorating and deteriorating items with time dependent demand and inventory holding cost.

Parekh R.U. and Patel R.D. [17] have developed a two-warehouse inventory model in which they assumed that the demand is linear function of time t. They took different deterioration rates and different inventory holding costs in both OW and RW under inflation and permissible delay in payments.

In this paper, we have tried to develop a two-warehouse inventory model under time varying holding cost and quadratic demand under inflation and permissible delay in payments. In the present work we have considered same deterioration rate and same linear holding cost throughout the period [0, T]. In this model t_r and T are taken as decision variables. Numerical examples are provided to illustrate the model and sensitivity analysis of the optimal solutions for major parameters is also carried out. The purpose of this study is to make the model more relevant and applicable in practice.

II. NOTATIONS

1. $I_r(t)$: Inventory level for the rented warehouse (RW) at time t.
2. $I_o(t)$: Inventory level for the owned warehouse (OW) at time t.
3. w : The capacity of the owned warehouse.
4. D(t) : Demand rate.
5. $\theta(t)$: Rate of deterioration per unit time.
6. R : Inflation rate.
7. A : Ordering cost per order during the cycle period.
8. C_d : Deterioration cost per unit per unit time.
9. C_h : Inventory holding cost per unit per unit time.
10. Q : Order quantity in one cycle.
11. k : Purchase cost per unit.
12. p : Selling price per unit.
13. Ie : Interest earned per year
14. Ip : Interest charge per year.
15. M : Permissible period of delay in settling the accounts with the supplier
16. t_r : time at which the inventory level reaches zero in RW in two warehouse system.
17. T : The length of cycle time.
18. TCi : Total cost per unit time in the i^{th} case. (i = 1, 2, 3)

III. ASSUMPTIONS

1. The demand rate of the product is $D(t) = a + bt + ct^2$ (where a, b, c > 0).
2. Holding cost is a linear function of time and it is $C_h = h+rt$ (h, r > 0) for both OW and RW
3. Shortages are not allowed.
4. Replenishment rate is infinite and instantaneous.
5. Repair or replacement of the deteriorated items does not take place during a given cycle.
6. OW has a fixed capacity W units and the RW has unlimited capacity.
7. First the units kept in RW are used and then of OW.
8. The inventory costs per unit in the RW are higher than those in the OW.

IV. MATHEMATICAL MODEL AND ANALYSIS

At time t = 0 the inventory level is S units. From these 'w' units are kept in owned warehouse (OW) and rest in the rented warehouse (RW). The units kept in rented warehouse (RW) are consumed first and then of owned warehouse (OW). Due to the market demand and deterioration of the items, the inventory level decreases during the period [0, t_r] and the inventory in RW reaches to zero. Again with the same effects, the inventory level decreases during the period [t_r, T] and the inventory in OW will also become zero at t = T.
The pictorial presentation is shown in the Figure – 1.

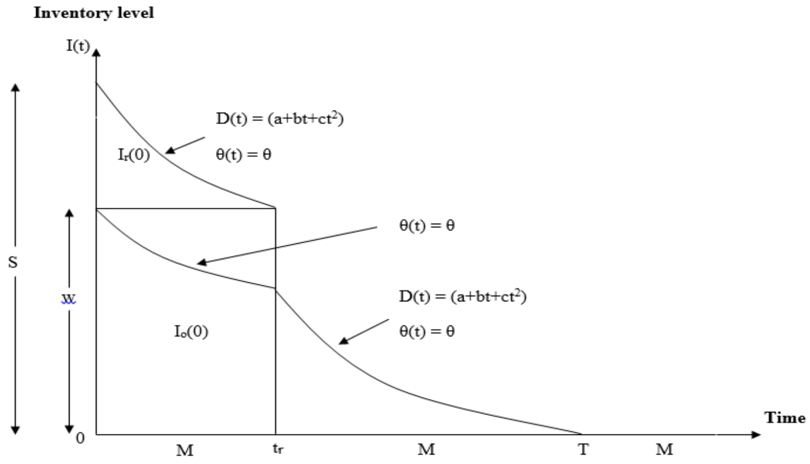

Figure – 1: Graphical presentation of the inventory system

The differential equations which describe the instantaneous state of inventory at time t over the period [0, T] are given by

$$\frac{dI_r(t)}{dt} + \theta I_r(t) = -\left(a + bt + ct^2\right) \qquad (0 \le t \le t_r) \qquad (1)$$

$$\frac{dI_0(t)}{dt} + \theta I_0(t) = 0 \qquad (0 \le t \le t_r) \qquad (2)$$

$$\frac{dI_0(t)}{dt} + \theta I_0(t) = -\left(a + bt + ct^2\right) \qquad (t_r \le t \le T) \qquad (3)$$

Under the boundary conditions $I_r(t_r) = 0$, $I_0(0) = w$, and $I_0(T) = 0$, solutions of equations (1) to (3) are given by

$$I_r(t) = a(t_r - t) + (b + a\theta)\frac{\left(t_r^2 - t^2\right)}{2} + (c + b\theta)\frac{\left(t_r^3 - t^3\right)}{3} + c\theta\frac{\left(t_r^4 - t^4\right)}{4} - a\theta(t_r - t)t - b\theta\frac{\left(t_r^2 - t^2\right)t}{2} - c\theta\frac{\left(t_r^3 - t^3\right)t}{3} \qquad (4)$$

$$I_0(t) = we^{-\theta t} \qquad (5)$$

$$I_0(t) = a(T - t) + (b + a\theta)\frac{\left(T^2 - t^2\right)}{2} + (c + b\theta)\frac{\left(T^3 - t^3\right)}{3} + c\theta\frac{\left(T^4 - t^4\right)}{4} - a\theta(T - t)t - b\theta\frac{\left(T^2 - t^2\right)t}{2} - c\theta\frac{\left(T^3 - t^3\right)t}{3} \qquad (6)$$

V. COSTS COMPONENTS

The total cost per replenishment cycle consists of the following cost components.

1) Ordering Cost

The operating cost (OC) over the period [0, T] is

$$OC = A \qquad (7)$$

2) Deterioration Cost

The deterioration cost (DC) over the period [0, T] is

$$DC = C_d\left\{\int_0^{t_r} \theta \cdot I_r(t) \cdot e^{-Rt}dt + \int_0^{t_r} \theta \cdot I_0(t) \cdot e^{-Rt}dt + \int_0^{T} \theta \cdot I_0(t) \cdot e^{-Rt}dt\right\}$$

$$\Rightarrow DC = C_d \left\{ \left[\frac{\theta}{12R^5} \left\{ \begin{array}{l} \left[\begin{array}{l} 4R^4\left(6at_r + 6a\theta t_r^2 + 6bt_r^2 + 4b\theta t_r^3 + 4ct_r^3 + 3c\theta t_r^4\right) - 6R^3\left(6a + 6a\theta t_r + 3b\theta t_r^2 + 2c\theta t_r^3\right) \\ +24R^2(a\theta - b) + 12R(b\theta - 2c) + 24c\theta \end{array} \right] \\ +12e^{-Rt_r}\left[3R^3\left(a + bt_r + ct_r^2\right) - 2R^2\left(a\theta - b + b\theta t_r - 2ct_r + c\theta t_r^2\right) - R\left(b\theta - 2c + 2c\theta t_r\right) - 2c\theta\right] \\ + \left[\begin{array}{l} 4R^4\left(6a\theta T^2 + 6bT^2 + 4b\theta T^3 + 4cT^3 + 3c\theta T^4 + 12T\right) - 6R^3\left(6a + 6a\theta T + 3b\theta T^2 + 2c\theta T^3\right) \\ +24R^2(a\theta - b) + 12R(b\theta - 2c) + 24c\theta \end{array} \right] \\ +12e^{-RT}\left[3R^3\left(a + bT + cT^2\right) - 2R^2\left(a\theta - b + b\theta T - 2cT + c\theta T^2\right) - R\left(b\theta - 2c + 2c\theta T\right) - 2c\theta\right] \end{array} \right\} \right] - \left[\frac{w\theta\left(e^{-t_r(R+\theta)} - 1\right)}{(R + \theta)} \right] \right\}$$

(8)

3) Inventory Holding Cost

The inventory holding cost (IHC) over the period [0, t_r] is

IHC = Holding cost during the cycle period T in RW [HC(RW)]
+ Holding cost during the cycle period T in RW [HC(OW)] (9)

where, $HC(RW) = \int_0^{t_r} (h + rt) \cdot I_r(t) \cdot e^{-Rt} dt$

$$\Rightarrow HC(RW) = \frac{1}{12R^6} \left\{ \begin{array}{l} e^{-Rt_r} \left[\begin{array}{l} h\left\{48R^4\left(a + bt_r + ct_r^2\right) - 36R^3\left(a\theta - b + bt_r\theta + 2ct_r + ct_r^2\theta\right) + 24R^2\left(-b\theta + 2c - 2ct_r\theta\right) - 24c\theta R\right\} \\ +r\left\{ \begin{array}{l} 48R^4\left(at_r + bt_r^2 + ct_r^3\right) + 36R^3\left(2a - a\theta t_r + 3bt_r - b\theta t_r^2 + 4ct_r^2 - c\theta t_r^3\right) \\ +24R^2\left(-3a\theta + 3b - 4bt_r\theta + 8ct_r - 5ct_r^2\theta\right) + 24R\left(-2b\theta + 4c - 5ct_r\theta\right) - 120c\theta \end{array} \right\} \end{array} \right] \\ +h\left\{ \begin{array}{l} \left[5R^5\left(12at_r + 6a\theta t_r^2 + 6bt_r^2 + 4b\theta t_r^3 + 4ct_r^3 + 3c\theta t_r^4\right)\right] \\ -8R^4\left\{6a + 6a\theta t_r + 2c\theta t_r^3 + 3b\theta t_r^2\right\} + 36R^3(a\theta - b) + 24R^2(b\theta - 2c) + 24c\theta R \end{array} \right\} \\ +r\left\{ \begin{array}{l} 4R^4\left(12at_r + 6a\theta t_r^2 + 6bt_r^2 + 4b\theta t_r^3 + 4ct_r^3 + 3c\theta t_r^4\right) \\ +R^3\left(-72R^3a - 24R^3c\theta t_r^3 - 36R^3b\theta t_r^2 - 72R^3a\theta t_r\right) \\ +72R^2(a\theta - b) + 48R(b\theta - 2c) + 120c\theta \end{array} \right\} \end{array} \right\}$$

(10)

$$HC(OW) = \int_0^{t_r} (h + rt) \cdot I_0(t) \cdot e^{-Rt} dt + \int_{t_r}^{T} (h + rt) \cdot I_0(t) \cdot e^{-Rt} dt$$

$$
= \left\{
\begin{array}{l}
-\dfrac{1}{(R+\theta)^2}\left[\left\{e^{-t_r(R+\theta)}\left[r(Rt_r+\theta t_r+1)+h(R+\theta)\right]-r-h(R+\theta)\right\}w\right] \\[2ex]
+\dfrac{e^{-Rt_r}}{12R^6}\left\{
\begin{array}{l}
h\left[
\begin{array}{l}
24c\theta R+R^2\left[24b\theta+48c(-1+t_r\theta)\right]+R^3\left[36a\theta+36b(-1+t_r\theta)+36c(-2t_r+t_r{}^2\theta)\right] \\
+R^4\left[48a(-1+\theta t_r-T\theta)+24b(-2t_r+\theta t_r{}^2-T^2\theta)+16c(-3t_r{}^2+\theta t_r{}^3-T^3\theta)\right] \\
+R^5\left[
\begin{array}{l}
30a(-2t_r+\theta t_r{}^2+2T-2Tt_r\theta+T^2\theta)+10b(-3t_r{}^2+\theta t_r{}^3+3T^2-3T^2t_r\theta \\
+2T^3\theta)+5c(-4t_r{}^3+\theta t_r{}^4+4T^3-4T^3t_r\theta+3T^4\theta)
\end{array}
\right]
\end{array}
\right] \\[2ex]
+r\left[
\begin{array}{l}
R\left[48b\theta+24c(-4+5t_r\theta-5T\theta)\right]+R^2\left[72a\theta+24b(-3+4t_r\theta)+24c(-8t_r+5t_r{}^2\theta)\right] \\
+R^3\left[36a(-2+3\theta t_r-2T\theta)+36b(-3t_r+2\theta t_r{}^2-T^2\theta)+12c(-12t_r{}^2+5\theta t_r{}^3-2T^3\theta)\right] \\
+R^4\left[
\begin{array}{l}
24a(-4t_r+3\theta t_r{}^2+2T-4Tt_r\theta+T^2\theta)+8b(-9t_r{}^2+4\theta t_r{}^3+3T^2-6T^2t_r\theta+2T^3\theta) \\
+4c(-16t_r{}^3+5\theta t_r{}^4+4T^3-8T^3t_r\theta+3T^4\theta)
\end{array}
\right] \\
+R^5\left[
\begin{array}{l}
30a(-2t_r{}^2+t_r{}^3\theta+2Tt_r-2Tt_r{}^2\theta+T^2t_r\theta) \\
+10b(-3t_r{}^3+t_r{}^4\theta+3T^2t_r-3T^2t_r{}^2\theta+2T^3t_r\theta)+5c(-4t_r{}^4+t_r{}^5\theta+4T^3t_r-4T^3t_r{}^2\theta \\
+3T^4t_r\theta)
\end{array}
\right] \\
+120c\theta
\end{array}
\right]
\end{array}
\right\} \\[2ex]
+\dfrac{e^{-RT}}{12R^6}\left\{
\begin{array}{l}
h\left[
\begin{array}{l}
-24c\theta R+24R^2\left[-b\theta+2c(1-T\theta)\right] \\
+36R^3\left[-a\theta+b(1-T\theta)+c(2T-T^2\theta)\right]+48R^4(a+bT+cT^2)
\end{array}
\right] \\
+r\left[
\begin{array}{l}
24R(-2b\theta+4c)+24R^2\left[-3a\theta+b(3-4T\theta)+c(8T-5T^2\theta)\right] \\
+36R^3\left[a(2-T\theta)+b(3T-T^2\theta)+c(-T^3\theta+4cT^2)\right] \\
+48R^4\left[(aT+T^2b+T^3c)-120c\theta\right]
\end{array}
\right]
\end{array}
\right\}
\end{array}
\right\}
\tag{11}
$$

4) Interest Earned: There are two cases

<u>Case 1</u>: (M □ T)

In this case, interest earned is:

$$IE_1 = p\cdot I_e\cdot\int_0^M (a+bt+ct^2)t\cdot e^{-Rt}dt$$

$$\Rightarrow IE_1 = -\frac{p\cdot I_e}{R^4}\left\{e^{-RM}\left[\begin{array}{l}2R\left(b+3Mc\right)+2R^2\left(a+2Mb+3M^2c\right)\\+3R^3\left(Ma+M^2b+M^3c\right)+6c\end{array}\right]-2Rb-2R^2a-6c\right\} \tag{12}$$

<u>Case 2</u>: (M > T)

In this case, interest earned is:

$$IE_2 = p\cdot I_e[\int_0^T (a+bt+ct^2)t\cdot e^{-Rt}dt+(a+bT+cT^2)T(M-T)]$$

$$\Rightarrow IE_2 = p \cdot I_e \begin{Bmatrix} \frac{-1}{R^4}\left\{ e^{-RT}\left[2R\left(b+3cT\right)+2R^2\left(a+2bT+3cT^2\right)+3R^3\left(aT+bT^2+cT^3\right)+6c\right]-2aR^2-2bR-6c\right\} \\ +(a+bT+cT^2)T(M-T) \end{Bmatrix} \tag{13}$$

5) Interest Payable: There are three cases described in figure-1

<u>Case 1</u>: ($M \le t_r \le T$)

In this case, annual interest payable is:

$$IP_1 = k \cdot I_p \cdot \left[\int_M^{t_r} I_r(t) \cdot e^{-Rt} dt + \int_M^{t_r} I_0(t) \cdot e^{-Rt} dt + \int_{t_r}^{T} I_0(t) \cdot e^{-Rt} dt \right]$$

$$\Rightarrow IP_1 = kI_p \begin{Bmatrix} \frac{1}{12R^5} \begin{Bmatrix} e^{-RM} \begin{bmatrix} 12R\left[b\theta+2c(-1+M\theta)\right]+24R^2\left[a\theta+b(-1+M\theta)+c(M^2\theta-2M)\right] \\ +6R^3\left[6a(-1-\theta t_r+M\theta)+3b(-\theta t_r^2-2M)+2c(-\theta t_r^3-3M^2+M^3\theta)\right] \\ +R^4 \begin{Bmatrix} 24a\left[2t_r+\theta t_r^2-2M-2M\theta t_r+M^2\theta\right] \\ +2b\left[12t_r^2+8\theta t_r^3-12M\theta t_r^2-12M^2+9M^2\theta+4M^3\theta\right] \\ +4c\left[4t_r^3+3\theta t_r^4-4M\theta t_r^3-4M^3+M^4\theta\right]+24c\theta \end{Bmatrix} \end{bmatrix} \\ +e^{-Rt_r} \begin{Bmatrix} 12R\left[-b\theta+2c(1-t_r\theta)\right]+24R^2\left[-a\theta+b(1-t_r\theta)+c(2t_r-t_r^2\theta)\right] \\ +36R^3(a+bt_r+ct_r^2)-24c\theta \end{Bmatrix} \end{Bmatrix} \\ +\frac{w\left(e^{-M(R+\theta)}-e^{-t_r(R+\theta)}\right)}{(R+\theta)} \\ +\frac{1}{12R^5} \begin{Bmatrix} e^{-RT} \begin{bmatrix} 12R\left[-b\theta+2c(-\theta T+1)\right]+24R^2\left[-a\theta+b(1-T\theta)+c(2T-T^2\theta)\right] \\ +36R^3(a+bT+cT^2) \end{bmatrix} \\ +e^{Rt_r} \begin{Bmatrix} 12R\left[b\theta+2c(-1+t_r\theta)\right]+24R^2\left[a\theta+b(-1+t_r\theta)+c(-2t_r+t_r^2\theta)\right] \\ +6R^3\left[6a(-1+\theta t_r-T\theta)+3b(-2t_r+\theta t_r^2-T^2\theta)+2c(-3t_r^2+\theta t_r^3-T^3\theta)\right] \\ +4R^4 \begin{bmatrix} 6a(-2t_r+\theta t_r^2+2T-2T\theta t_r+T^2\theta)+2b(-3t_r^2+\theta t_r^3+3T^2-3T^2\theta t_r+2T^3\theta) \\ +c(-4t_r^3+\theta t_r^4+4T^3-4T^3\theta t_r+3T^4\theta) \end{bmatrix} \end{Bmatrix} \end{Bmatrix} \end{Bmatrix} \tag{14}$$

<u>Case 2</u>: ($t_r \le M \le T$)

In this case, interest payable is:

$$IP_2 = k \cdot I_p \cdot \int_M^T I_0(t) \cdot e^{-Rt} dt$$

$$\Rightarrow IP_2 = \frac{k \cdot I_p}{12R^5}\left[\begin{array}{l} e^{-RT}\left\{12R\left[-b\theta + 2c\left(1-\theta T\right)\right] + 24R^2\left[-a\theta + b\left(1-T\theta\right) + c\left(2T - T^2\theta\right)\right] + 36R^3\left(bT + cT^2 + 1\right) - 24c\theta\right\} \\ +e^{-RM}\left\{\begin{array}{l} 12R\left[b\theta + 2c\left(-1+M\theta\right)\right] + 24R^2\left[a\theta + b\left(-1+M\theta\right) + c\left(-2M + M^2\theta\right)\right] \\ +6R^3\left[6a\left(-1-T\theta+M\theta\right) + 3b\left(-T^2\theta - 2M + M^2\theta\right) + 2c\left(-T^3\theta - 3M^2 + M^3\theta\right)\right] \\ +4R^4\left[\begin{array}{l} 6a\left(2T + T^2\theta - 2M - 2MT\theta + M^2\theta\right) + 2b\left(3T^2 - 3MT^2\theta + 2T^3\theta - 3M^2 + M^3\theta\right) \\ +c\left(4T^3 + 3T^4\theta - 4M^3 + M^4\theta - 4MT^3\theta\right) \end{array}\right] + 24c\theta \end{array}\right\} \end{array}\right] \quad (15)$$

Case 3: (M □ T)

In this case, no interest charges are paid for the item and so

$$IP_3 = 0 \quad (16)$$

Substituting values from equations (7) to (11) and equations (12) to (16) in equations (17) to (19), the retailer's total cost during a cycle in three cases will be as under:

$$TC_1 = \frac{1}{T}[A + HC(OW) + HC(RW) + DC + IP_1 - IE_1] \quad (17)$$

$$TC_2 = \frac{1}{T}[A + HC(OW) + HC(RW) + DC + IP_2 - IE_1] \quad (18)$$

$$TC_3 = \frac{1}{T}[A + HC(OW) + HC(RW) + DC + IP_3 - IE_2] \quad (19)$$

Our objective is to determine the optimum values t_r* and T* of t_r and T respectively so that TC_i is minimum. Note that t_r* and T* can be obtained by solving the equations

$$\frac{\partial TC_i}{\partial t_r} = 0 \quad \text{and} \quad \frac{\partial TC_i}{\partial T} = 0 \quad (i = 1, 2, 3) \quad (20)$$

$$\left.\begin{array}{l} \left[\left(\frac{\partial^2 TC_i}{\partial t_r^2}\right)\left(\frac{\partial^2 TC_i}{\partial T}\right) - \left(\frac{\partial TC_i}{\partial t_r \partial T}\right)^2\right]_{t_r = t_r^*, T = T^*} > 0 \\ \left[\frac{\partial^2 TC_i}{\partial t_r^2}\right]_{t_r = t_r^*, T = T^*} > 0 \end{array}\right\} \quad (21)$$

The optimum solution of the equations (20) can be obtained by using appropriate software. The above developed model is illustrated by the means of the following numerical example.

Numerical Example – 1

To illustrate the proposed model, an inventory system with the following hypothetical values is considered. By taking A = 150, w = 100, a = 8, b = 0.5, c = 0.2, k = 10, p = 15, θ = 0.2, h = 1, r = 0.5, R = 0.06, M = 10, C_d = 4, Ip = 0.15 and Ie = 0.12 (with appropriate units). The optimal values of t_r and T are t_r* = 13.71456792, T* = 22.25988127 units and the optimal total cost per unit time TC = 3.346889173 units.

Numerical Example – 2

By taking A = 150, w = 100, a = 8, b = 0.5, c = 0.2, k = 10, p = 15, θ = 0.2, h = 1, r = 0.5, R = 0.06, M = 16, C_d = 4, Ip = 0.15 and Ie = 0.12 (with appropriate units). The optimal values of t_r and T are t_r* = 13.51613807, T* = 22.37726544 units and the optimal total cost per unit time TC = 3.345597534 units.

Numerical Example – 3

By taking A = 150, w = 100, a = 8, b = 0.5, c = 0.2, k = 10, p = 15, θ = 0.2, h = 1, r = 0.5, R = 0.06, M = 25, C_d = 4, Ip = 0.15 and Ie = 0.02 (with appropriate units). The optimal values of t_r and T are t_r* = 13.18456160, T* = 21.82708029 units and the optimal total cost per unit time TC = 3.354324259 units.

VI. SENSITIVITY ANALYSIS

Sensitivity analysis depicts the extent to which the optimal solution of the model is affected by the changes in its input parameter values. Here, we study the sensitivity for total cost per time unit TC with respect to the changes in the values of the parameters A, w, a, b, k, p, θ, h, r, R, M, C_d, Ip and Ie.

The sensitivity analysis is performed by considering variation in each one of the above parameters keeping all other remaining parameters as fixed.

Table – 1: Partial Sensitivity Analysis Based Numerical Example – 1

Parameter	%	t_r	T	TC
A	– 20	13.71447314	22.25972159	3.346912472
	– 10	13.71452053	22.25980143	3.346900821
	+ 10	13.71461530	22.25996110	3.346877526
	+ 20	13.71466269	22.26004094	3.346865879
w	– 20	13.71431025	22.25885504	3.347103806
	– 10	13.71443908	22.25936817	3.346996485
	+ 10	13.71467456	22.25995064	3.346673450
	+ 20	13.71482346	22.26063452	3.346262374
a	– 20	13.84152478	22.41111398	3.343914723
	– 10	13.76345536	22.22684594	3.344374543
	+ 10	13.65129742	22.18409989	3.348403083
	+ 20	13.58816752	22.10821091	3.349934358
b	– 20	13.8042342	22.42534248	3.336424245
	– 10	13.7378448	22.25457424	3.346352560
	+ 10	13.65365824	22.16879359	3.356457972
	+ 20	13.59333966	22.07839783	3.366016024
c	– 20	13.40809897	21.84679176	3.374305795
	– 10	13.57726226	22.07546066	3.359071177
	+ 10	13.74245234	22.24245492	3.327533453
	+ 20	13.93224244	22.46575234	3.289724248
k	– 20	13.67366618	22.27401292	3.346956312
	– 10	13.69435261	22.26692751	3.346918734
	+ 10	13.72634634	22.25354735	3.346874345
	+ 20	13.75234245	22.23045532	3.346844245
p	– 20	13.69744424	22.23103278	3.347317011
	– 10	13.70601765	22.24547596	3.347104047
	+ 10	13.72309523	22.27424896	3.346672374
	+ 20	13.74735242	22.30465234	3.344693510
h	– 20	13.83833426	22.38734515	3.342301127
	– 10	13.77588804	22.32329591	3.344583519
	+ 10	13.70534424	22.25552357	3.346834535
	+ 20	13.62068895	22.17354374	3.349345345
r	– 20	13.20357242	22.59345423	3.327351645
	– 10	13.53183825	21.99245885	3.338248686
	+ 10	13.87425830	22.49184150	3.354473847
	+ 20	14.01504787	22.69505830	3.361185927
M	– 20	13.58566195	22.14684808	3.347400392
	– 10	13.65213269	22.20358478	3.347388352
	+ 10	13.77312758	22.31551645	3.345980907
	+ 20	13.89424234	22.38093463	3.343414543
θ	– 20	13.89752709	22.42210880	3.151146989
	– 10	13.80659136	22.34812012	3.264790826
	+ 10	13.62292868	22.16236931	3.407779167
	+ 20	13.51435235	22.02543452	3.553623423
C_d	– 20	14.31247578	23.09785921	3.390785280
	– 10	14.00173743	22.66127789	3.367959896
	+ 10	13.44791651	21.88887087	3.327316573
	+ 20	13.19928734	21.54435448	3.309038613
Ip	– 20	13.67366619	22.27401293	3.346956309
	– 10	13.69435261	22.26692751	3.346918732
	+ 10	13.71559399	22.25534246	3.346876232
	+ 20	13.74230052	22.23734623	3.346834321
Ie	– 20	13.69744424	22.23103278	3.347317012
	– 10	13.70601764	22.24547596	3.347104047
	+ 10	13.72309523	22.27424897	3.346672377
	+ 20	13.74455743	22.29057395	3.346042343

Table – 2: Partial Sensitivity Analysis Based Numerical Example – 2

Parameter	%	t_r	T	TC
A	– 20	13.51604909	22.37711758	3.345621306
	– 10	13.51609358	22.37719151	3.345609420
	+ 10	13.51618256	22.37733936	3.345585645
	+ 20	13.51622705	22.37741330	3.345573756
w	– 20	13.51591199	22.37631039	3.345813273
	– 10	13.51602503	22.37678794	3.345705399
	+ 10	13.51625110	22.37774291	3.345489679
	+ 20	13.51636414	22.37822034	3.345381840
a	– 20	13.64333127	22.52035345	3.342216825
	– 10	13.57965806	22.44885384	3.343898291
	+ 10	13.45277320	22.30559047	3.347314233
	+ 20	13.38956532	22.23383113	3.349048078
b	– 20	13.63481162	22.54865775	3.326087512
	– 10	13.57516715	22.46261813	3.335847649
	+ 10	13.45771684	22.29259521	3.355336689
	+ 20	13.39989606	22.20860292	3.365064684
c	– 20	13.21581454	21.99050041	3.373941719
	– 10	13.38150471	22.20451234	3.358192800
	+ 10	13.62768352	22.51962059	3.335297864
	+ 20	13.72160264	22.63894021	3.326720367
k	– 20	13.52523031	22.39237401	3.345042694
	– 10	13.52064253	22.38475036	3.345320005
	+ 10	13.51171465	22.36991547	3.345875258
	+ 20	13.50737009	22.36269679	3.346153170
p	– 20	13.48614532	22.32743475	3.347000083
	– 10	13.50117694	22.35240706	3.346302662
	+ 10	13.53102955	22.40201124	3.344884651
	+ 20	13.54585219	22.42664574	3.344163978
h	– 20	13.63360874	22.49599813	3.340773135
	– 10	13.57429871	22.43635440	3.343175195
	+ 10	13.45908327	22.31871718	3.348039373
	+ 20	13.40309302	22.26069621	3.350500007
r	– 20	13.09635691	21.86323698	3.326199607
	– 10	13.32131908	22.13846509	3.336626683
	+ 10	13.68655241	22.58653786	3.353410265
	+ 20	13.83692044	22.77152068	3.360283803
M	– 20	13.28075404	21.98653331	3.342745114
	– 10	13.40434365	22.19159195	3.344844285
	+ 10	13.61670836	22.54444818	3.345332544
	+ 20	13.70665010	22.69408388	3.344337937
θ	– 20	13.7357502	22.58242423	3.123457239
	– 10	13.60291751	22.45823933	3.262047752
	+ 10	13.42959475	22.28682626	3.407275777
	+ 20	13.34412799	22.19014080	3.454355134
C_d	– 20	13.79534277	22.75459060	3.367050204
	– 10	13.65323617	22.56236200	3.356125211
	+ 10	13.38374436	22.19884321	3.335438851
	+ 20	13.30585885	22.05758387	3.315277583
Ip	– 20	13.52523031	22.39237401	3.345042693
	– 10	13.52064253	22.38475036	3.345320005
	+ 10	13.51171465	22.36991547	3.345875258
	+ 20	13.50737009	22.36269679	3.346153170
Ie	– 20	13.48614532	22.32743476	3.347000084
	– 10	13.50117694	22.35240706	3.346302664
	+ 10	13.53102956	22.40201124	3.344884652
	+ 20	13.54585219	22.42664575	3.344163979

Table – 3: Partial Sensitivity Analysis Based Numerical Example – 3

Parameter	%	t_r	T	TC
A	– 20	13.18446186	21.82691502	3.354349092
	– 10	13.18451173	21.82699765	3.354336674
	+ 10	13.18461148	21.82716293	3.354311843
	+ 20	13.18466134	21.82724556	3.354299426
w	– 20	13.18432625	21.82603960	3.354549367
	– 10	13.18467927	21.82760057	3.354211723
	+ 10	13.18479693	21.82812078	3.354099199
	+ 20	13.18479693	21.82812078	3.354099198
a	– 20	13.31931477	21.98110765	3.350336381
	– 10	13.24755683	21.90558324	3.352374535
	+ 10	13.11733530	21.74978254	3.356341913
	+ 20	13.05021083	21.67229643	3.358374908
b	– 20	13.31054574	22.01073005	3.333785661
	– 10	13.24726480	21.91859734	3.344061525
	+ 10	13.12242898	21.73617509	3.364573350
	+ 20	13.06085992	21.64587791	3.374808297
c	– 20	12.86485735	21.40993104	3.384654039
	– 10	13.04146032	21.64109189	3.367810636
	+ 10	13.20457248	21.90587584	3.334523534
	+ 20	13.40229561	22.10781077	3.334085768
p	– 20	13.22525283	21.89451611	3.353281835
	– 10	13.20479079	21.86060235	3.353796635
	+ 10	13.16455998	21.79394096	3.354864818
	+ 20	13.14478079	21.76117570	3.355418422
h	– 20	13.30282615	21.94904273	3.349173527
	– 10	13.24312262	21.88778028	3.351739476
	+ 10	13.12709971	21.76692854	3.356927110
	+ 20	13.07069582	21.70731147	3.359547320
r	– 20	12.68846444	21.18106899	3.338032570
	– 10	12.95550184	21.52931617	3.346696600
	+ 10	13.38337921	22.08494842	3.361083736
	+ 20	13.55768506	22.31066367	3.367111872
M	– 20	13.05396416	21.61080343	3.336519252
	– 10	13.11901453	21.71850074	3.345348582
	+ 10	13.25061016	21.93655146	3.363450676
	+ 20	13.31716504	22.04692377	3.372732410
θ	– 20	13.34646353	22.79342342	3.195525833
	– 10	13.25268542	21.88025799	3.270702764
	+ 10	13.11295366	21.75863725	3.415650030
	+ 20	13.03955684	21.67944485	3.462226585
C_d	– 20	13.46399819	22.20643506	3.375664255
	– 10	13.32175500	22.01316892	3.364796485
	+ 10	13.05211161	21.64771033	3.344219928
	+ 20	12.92412517	21.47464179	3.334458659
Ie	– 20	13.22525283	21.89451611	3.353281835
	– 10	13.20479079	21.86060235	3.353796633
	+ 10	13.16455998	21.79394096	3.354864818
	+ 20	13.14478079	21.76117570	3.355418421

VII. GRAPHICAL PRESENTATION

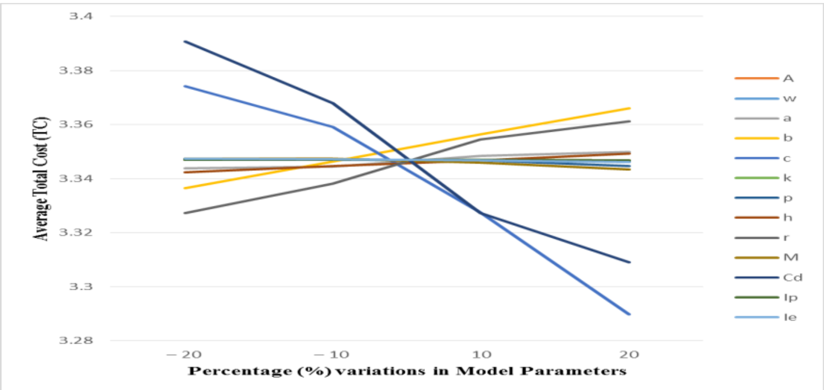

Figure – 2

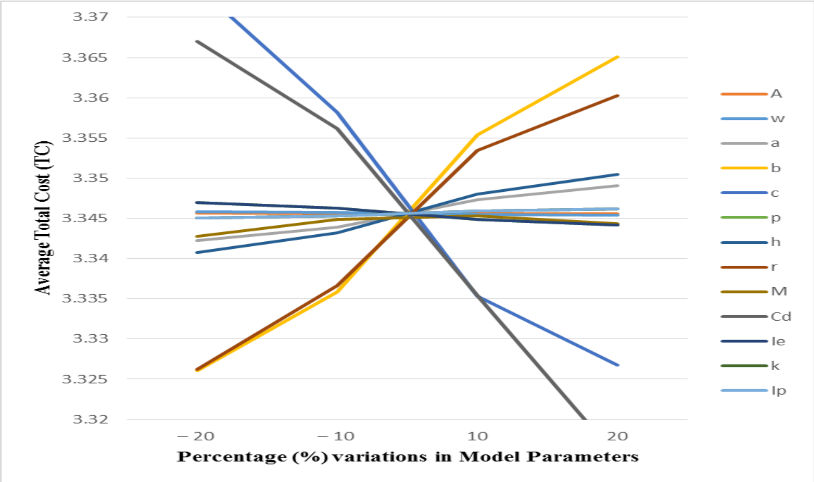

Figure – 3

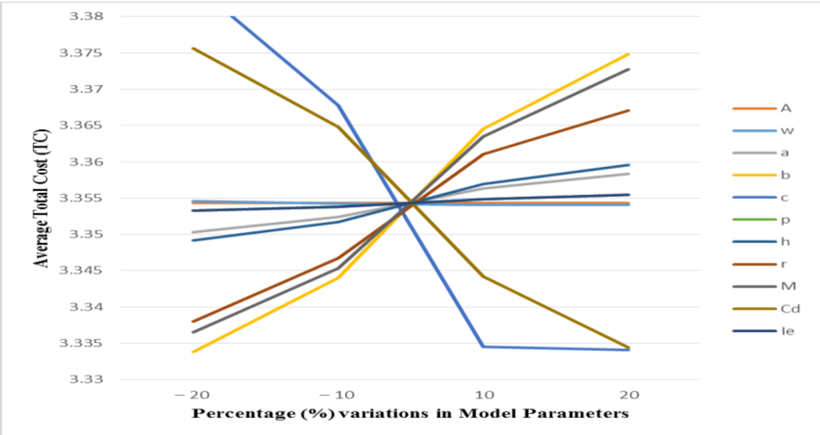

Figure – 4

VIII. CONCLUSIONS

- From the **Table – 1**, we observe that as the values of the parameters a, b, h, r and θ increase the average total cost also increases and for the values of the parameters A, w, c, k, p, M, C_d, Ip and I_e the average total cost decreases.

- **Table – 2** shows that as the values of the parameters a, b, k, h, r, θ, Ip and M increase the average total cost also increases and for the values of the parameters A, w, c, p, C_d and I_e the average total cost decreases.

- From the **Table – 3**, we note that as the values of the parameters a, b, p, h, r, M, θ and Ie increase the average total cost also increases and for the values of the parameters A, w, c and C_d the average total cost decreases.

- From the **Figure – 2**, we observe that the total cost per time unit is highly sensitive to changes in the values of c, C_d, moderately sensitive to changes in the values of b, r and less sensitive to changes in the values of A, w, a, k, p, h, M, C_d, Ip, I_e.

- From the **Figure – 3** we note that the total cost per time unit is highly sensitive to changes in the values of c, b, r, C_d, moderately sensitive to changes in the values of a, h and less sensitive to changes in the values of A, w, p, M, k, Ip, I_e.

- **Figure – 4** shows that the total cost per time unit is highly sensitive to changes in the values of b, c, M, r, C_d, moderately sensitive to changes in the values of a, h and less sensitive to changes in the values of A, w, p, I_e.

REFERENCES

[1]. Aggarwal S. P., and Jaggi C. K., Ordering Policies of Deteriorating Items under Permissible Delay in Payments, Journal of Operational Research Society, 1995, 46, 658–662.

[2]. A.M.M. Jamal, B.R. Sarkar. and S. Wang, An ordering policy for deteriorating items with allowable shortages and permissible delay in payment, Journal of Operational Research Society, 1997, 48, 826–833.

[3]. Bhojak A. and Gothi U. B., Inventory Models for Ameliorating and Deteriorating Items with Time Dependent Demand and IHC. International Journal of Management and Social Science Research Review, 2015, Vol. 1, Issue 2, 85–98.

[4]. Chatterji, D. A., Gothi U. B. and Parmar K. C., An Integrated Inventory Model with Exponential Amelioration and Two Parameter Weibull Deterioration, Journal of Statistics and Mathematical Engineering, MAT Journals, 2016, Vol. 2, No. 2.

[5]. Chung K. J., and Huang Y. F., The Optimal Cycle Time for EPQ Inventory Model Under Permissible Delay in Payments, International Journal of Production Economics, 2003, 84, 307–318.

[6]. Chung K. J., and Liao J. J., Lot-Sizing Decisions Under Trade Credit Depending on the Ordering Quantity, Computers and Operations Research, 2004, 31, 909–928.

[7]. Chung K. J., and Huang T. S., The Optimal Retailer's Ordering Policies for Deteriorating Items with Limited Storage Capacity Under Trade Credit Financing, International Journal of Production Economics, 2007, 106, 127–145.

[8]. C.T. Chang, J.T. Teng and S.K. Goyal, Inventory lot sizing models under trade credits, Asia Pacific Journal Operational Research, 2008, 25, 89–112.

[9]. Goyal S. K., Economic Order Quantity under Conditions of Permissible Delay in Payments, Journal of the Operational Research Society, 1985, 36, 335–338.

[10]. Hartley V. R., Operations Research – A Managerial Emphasis, 1976, Goodyear, Santa Monica.

[11]. Huang K. N., and Liao J. J., A Simple Method to Locate the Optimal Solution for Exponentially Deteriorating Items Under Trade Credit Financing, Computers and Mathematics with Applications, 2008, 56, 965–977.

[12]. K.V.S. Sarma, A deterministic inventory model for deteriorating items with two storage facilities; European Journal of Operational Research, 1987, 29, 70–72.

[13]. Lee C. C., and Hsu S. L., A Two-Warehouse Production Model for Deteriorating Inventory Items with Time-Dependent Demands, European Journal of Operational Research, 2009, 194, 700–710.

[14]. Liao J. J., On an EPQ Model for Deteriorating Items Under Permissible Delay in Payments, Applied Mathematical Modelling, 2007, 31, 393–403.

[15]. Liao J. J., An Inventory Control System Under Deferrable Delivery Conditions, Mathematical and Computer Modelling, 2008, 47, 247–258.

[16]. Parmar, K., Gothi U. B., Order level inventory model for deteriorating items under time varying demand condition, Sankhaya Vignan, 2015, NSV 11, No. 1, 20–29.

[17]. Parekh R. U. and Patel R. D., Deteriorating items inventory models for two warehouses with linear demand, time varying holding cost under inflation and permissible delay in payments, International Journal of Mathematics and Statistics Invention, 2014, Vol. 2, Issue 5, pp. 39-48.

[18]. Rong M., Mahapatra N. K., and Maiti M., A Two Warehouse Inventory Model for a Deteriorating Item with Partially/Fully Backlogged Shortage and Fuzzy Lead Time, European Journal of Operational Research, 2008, 189, 59–75.

[19]. Teng J. T., Change C. T., and Goyal S. K., Optimal Pricing and Ordering Policy Under Permissible Delay in Payments, International Journal of Production Economic, 2005, 97, 121–129.