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Unsteady Similarity Solution of Free convective boundary layer flow over porous plate with variable properties considering viscous dissipation and Slip Effect

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Abstract: *The combined effects of viscous dissipation and slip effect on the momentum and thermal transport for the unsteady boundary layer flow over porous plate have been carried out. We have applied free parameter method to solve governing partial differential equations. The governing non-linear partial differential equations are transformed into a system of coupled non-linear ordinary differential equations using similarity transformations and then solved numerically using the Runge–Kutta method with shooting technique for better accuracy. The flow and temperature fields as well as the free convective parameter and heat transfer coefficient are determined and displayed graphically involved in the similarity transformation. Effects of the slip parameter, free convection parameter, Prandtl number and unsteadiness parameter on the flow and heat transfer are examined and analyzed.*

Keywords: *Free convection, Partial Slip, Similarity solution, Unsteady* **,** *Viscous dissipation*

I. INTRODUCTION

In the recent years free convective boundary layer flows have a great interest from both theoretical and practical point of views because of its vast and significant applications in cosmic fluid dynamics ,solar physics, geophysics, electronics, paper production, wire and fiver coating, composite processing and storage system of agricultural product etc.

The study of boundary layer flow over porous surface moving with constant velocity in an ambient fluid was initiated by Sakiadis [1]. Erickson et al. [2] extended Sakiadis [1] problem to include blowing or suction at the moving porous surface. Subsequently Tsou et al. [3] presented a combined analytical and experimental study of the flow and temperature fields in the boundary layer on a continuous moving surface. R. Ellahi et al. [4] investigated numerical analysis of unsteady flows with viscous dissipation and nonlinear slip effects. Excellent reviews on this topic are provided in the literature by Nield and Bejan [5], Vafai [6], Ingham and Pop [7] and Vadasz [8]. Recently, Cheng and Lin [9] examined the melting effect on mixed convective heat transfer from a permeable over a continuous Surface embedded in a liquid saturated porous medium with aiding and opposing external flows. The unsteady boundary layer flow over a stretching sheet has been studied by Devi et al. [10], Elbashbeshy and Bazid [11], Tsai et al. [12] and Ishak [13].

Andersson et al. [14] investigated using a similarity transformation the flow of a thin liquid film of a power-law fluid by unsteady slip surface. Ellahi et al. [15] discussed Analysis of steady flows in viscous fluid with heat/mass transfer and slip effects. Zeeshan and Ellahi [16] studied Series solutions for nonlinear partial differential equations with slip boundary conditions for non-Newtonian MHD fluid in porous space. Williams *et al.*, [17] studied the unsteady free convection flow over a vertical flat plate under the assumption of variations of the wall temperature with time and distance. They found possible semi-similar solutions for a verity of classes of wall temperature distributions. Kumari *et al.*, [18] observed that the unsteadiness in the flow field was caused by the time dependent velocity of the moving sheet. Hong et al. [19], Chen and Lin [20] and Jaisawal and Soundalgekar [21] studied the free convection in a porous medium with high porosity. After a pioneering work of Sakiadis [22,23] the study of flow and heat transfer characteristics past continuous stretching surfaces has drawn considerable attention, and a good amount of the literature has been generated on this problem for instance [24]. The effects of slip boundary condition on the flow of Newtonian fluid due to a stretching sheet were explained by Andersson [25] and Wang [26]. Although various aspects of this class of boundary layer problems have been tackled, the effect of buoyancy force was ignored. It will be demonstrated that the system of time-dependent governing equations can be reduced to a

Some parameter problem by introducing a suitable transformation variables. Accurate numerical solutions are generated by aplying shooting method. A comprehensive parametric study is conducted and a representative set of graphical results for the velocity slip and thermal slip parameter are reported and discussed. The analysis showed that the unsteadiness parameter, buoyancy force, free convection parameter and viscous dissipation have significant influence on the flow and thermal fields.

II. MATHEMATICAL ANALYSIS

Consider the unsteady-two-dimensional free convection boundary layer flow of viscous dissipating incompressible fluid embedded in a porous medium and moving with variable velocity. Under the usual boundary-layer approximation, the governing equations for the flow and heat and mass transfer have the form:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=v\frac{\partial^2 u}{\partial y^2}+g\beta^*(T-T_\infty) \tag{2}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} \tag{3}$$

where x and y are axes along and perpendicular to the plate respectively, u and v are velocity components in x- and y-directions respectively, ρ is the fluid density, μ is the coefficient of fluid viscosity,$v$is the kinematic fluid viscosity, β is the volumetric coefficient of thermal expansion, g is the acceleration due to gravity, T is the temperature, T_∞ is the free stream temperature, k is the thermal conductivity of the fluid and $c_p$is the specific heat and the other symbols have their usual meanings.

The corresponding boundary conditions are:

$$u=L_1\left(\frac{\partial u}{\partial y}\right), v=0\ at\ y=0; u\to U_\infty\ as\ y\to\infty \tag{4}$$

$$\text{and } T=T_w+D_1\left(\frac{\partial t}{\partial y}\right) at\ y=0; T\to T_\infty\ as\ y\to\infty \tag{5}$$

Here L_1 and D_1 is the velocity slip factor and thermal slip factor respectively.

The continuity equation (1) is satisfied by introducing the stream function ψ(x, y),

such that $u=\frac{\partial\psi}{\partial y}$, $v=-\frac{\partial\psi}{\partial x}$

The momentum and energy equations (2)and (3) can be transformed to the corresponding ordinary differential equations by introducing the following similarity transformations:

$$\psi=\sqrt{U_\infty xv\ \ t}\ f(\eta) \qquad \eta=y\sqrt{\frac{U_\infty}{v\ \ xt}}, \theta(\eta)=\frac{(T-T_\infty)}{(T_w-T_\infty)} \tag{6}$$

where υ_∞ is a reference kinematic viscosity.

If $U=U_\infty=ax, x=\delta, t=\mu$ then the momentum and energy equations (2) – (3) after some simplifications, reduce to the following forms:

$$f'''+\frac{1}{2}\ Aff''+A_1\eta f''+\lambda\theta=0 \tag{7}$$

$$where\ \lambda=\frac{g\beta^*(T-T_\infty)\delta\mu}{U_\infty{}^2}(Free\ convection\ parameter), A=\mu\ (Unsteadiness\ parameter)\ \ and\ A_1=\frac{1}{a}$$

$$\theta''+A_1P_r\eta\theta'+AP_rf\theta'=0$$

$$where\ P_r=\frac{\rho C_p v}{2k}\ (Prandtl\ number)\ . \tag{8}$$

The corresponding boundary conditions are:

$$f=0;\ f'=\xi f''; at\ \eta=0; f'\to 1\ \ as\ \eta=\infty \tag{9}$$

$$and\ \theta=1+\tau\theta'\ at\ \eta=0; \theta\to 0\ \ as\ \eta=\infty \tag{10}$$

Where the prime ($'$) denotes differentiation with respect to η and ξ is the velocity slip parameter , τ is the thermal slip parameter.

It is important to note that for liquids (Pr > 1.0) and for gases (Pr < 1.0).

The equations (8) and (9) constitute a non-linear coupled boundary value problem prescribed at two boundaries, the analytical solution of which is not feasible. Therefore, these equations have been solved numerically on computer using Newton's shooting techniques with the Runge-kutta Gill method with a step size of 0.01. The corresponding velocity and temperature profiles are shown in figure.

III. RESULT AND DISCUSSION

In order to get clear insight into the physics of the problem, a parametric study is performed and the obtained numerical results are displayed with the help of graphical illustrations. The profiles for velocity and temperature are shown in fig.1 to fig.6.

3.1 Effect of initial boundary condition:

It is seen from fig.1 that the velocity and shear stress profiles for free convection flow for no-slip condition at the boundary $\lambda = 0, \tau = 0, \xi = 0$

3.2 Effect of free convection parameter:

In the fig.2(a), the buoyancy aiding flow (λ > 0), increase in free convection parameter will increase the velocity inside the boundary layer due to favourable buoyancy effects in both slip and no-slip cases and consequently heat transfer rate from the plate will increase. In the fig.2(b), the shear stress profile $f''(\eta)$ though initially increases with λ but it decreases for large η.From the fig.2(c)it is found that for the increase of λ the temperature distribution is suppressed in case of slip as well as no-slip condition and consequently the thermal boundary layer thickness becomes thinner. Physically λ > 0 means heating of the fluid or cooling of the surface of the plate (assisting flow).

3.3 Effect of the velocity slip parameter:

In Fig.3(a)-3(b) the velocity $f'(\eta)$and shear stress $f''(\eta)$ profiles exhibit opposite character before and after some points. With increasing values of ξ, the velocity increases up to $\eta \approx 2.72$ and then decreases. Also, the dimensionless shear stress decreases up to $\eta \approx 3.52$ and after that it increases.In fig.3(c) it is observed that the temperature decreases significantly with the increase in slip parameter ξ and also the thickness of the thermal boundary layer reduces.

3.4 Effect of the Prandtl number:

From the fig.4(a)-4(c) it is observed that The velocity $f'(\eta)$ along the plate decreases with increase in Pr for both slip and no-slip cases and the profile $f''(\eta)$decreases up to a point, then increases.

In both cases, as Prandtl number increases, the temperature at every location in the thermal boundary layer decreases. The thickness of the boundary layer decreases as Prandtl number increases which is usual case of an isothermal flat plate with no-slip.

3.5 Effect of the thermal slip parameter:

Variations of velocity, shear stress and temperature due to thermal slip parameter are presented in fig.5(a)-5(c). As the thermal slip parameter increases, the velocity increases and the shear stress at first increases and then after a point ($\eta \approx 1.26$) it decreases. This happens due to the combined effects of free convection and velocity slip. From fig.5(c) we observed that with the increasing thermal slip, the temperature rises above the plate temperature T_w before it decays to the ambient temperature T_∞.

3.6 Effect of the Skin-friction coefficient and Temperature gradient:

It is seen from the fig.6(a)-6(b) that it indicates the effects of velocity slip and thermal slip on skin friction coefficient and temperature gradient at the plate. Although the skin friction coefficient increases with increasing thermal slip parameter but it decreases with increasing velocity slip. Plate temperature gradient is found to increase with the increasing thermal slip parameter as well as with the increasing velocity slip parameter.

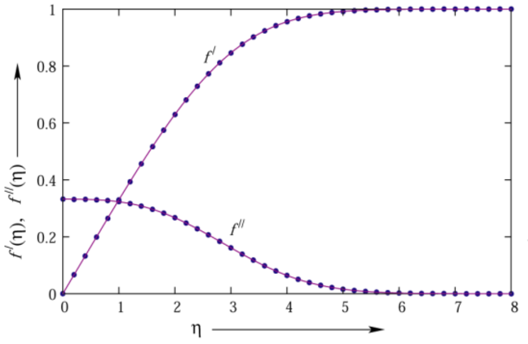

Fig. 1 Velocity profile $f'(\eta)$, and shear stress profiles $f''(\eta)$ for $\lambda = 0,\ \xi = 0,\ \tau = 0$.

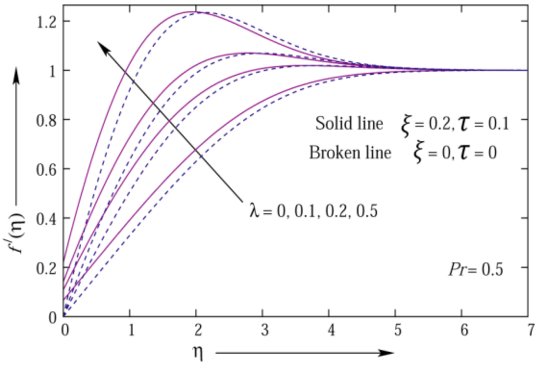

Fig. 2a Velocity profiles $f'(\eta)$ for several values of λ

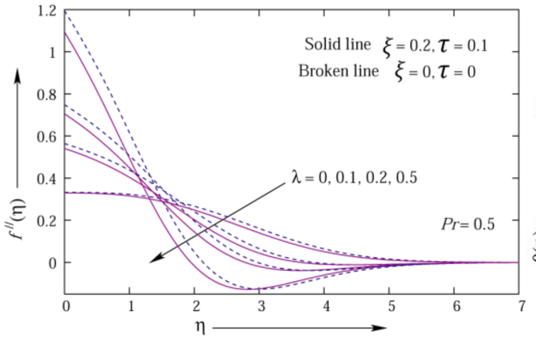

Fig. 2b Shear stress profiles $f''(\eta)$ for several values of λ

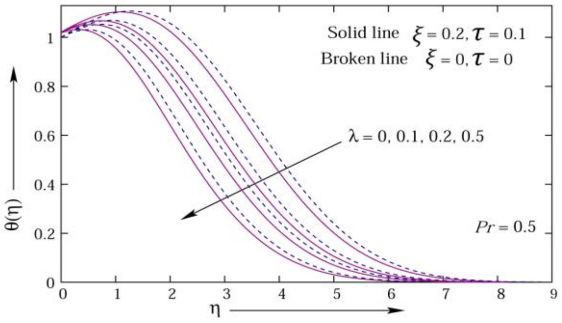

Fig. 2c Temperature profiles $\theta(\eta)$ for several values of λ

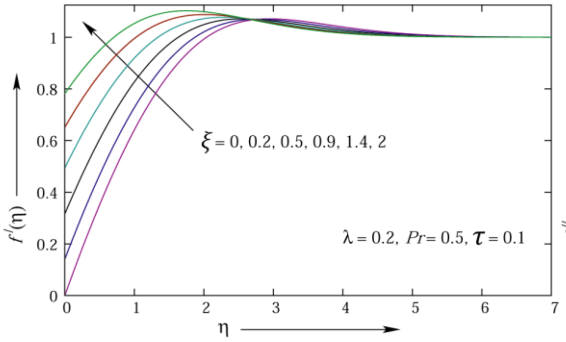

Fig. 3a Velocity profiles $f'(\eta)$ for several values of ξ

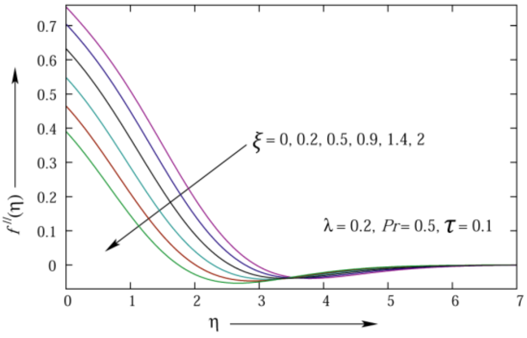

Fig. 3b Shear stress profiles $f''(\eta)$ for several values of ξ

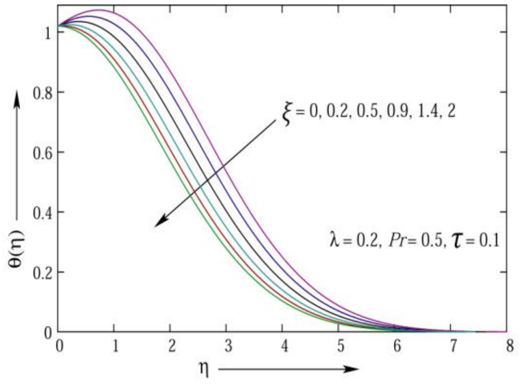

Fig. 3c Temperature profiles $\theta(\eta)$ for several values of ξ

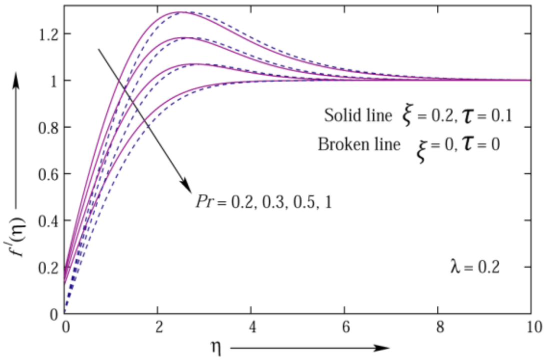

Fig. 4a Velocity profiles $f'(\eta)$ for several values of Pr

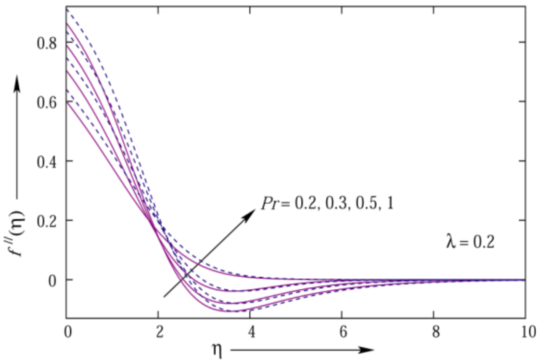

Fig. 4b Shear stress profiles $f''(\eta)$ for several values of Pr

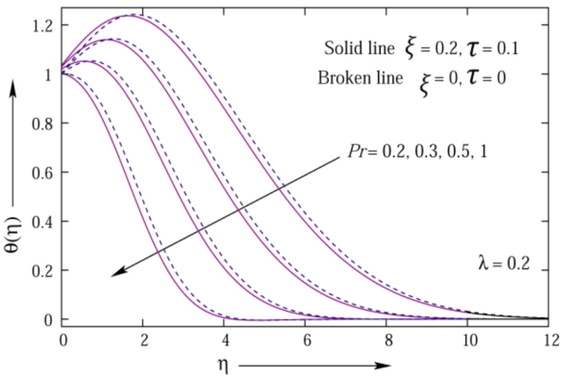

Fig. 4c Temperature profiles θ(η) for several values of Pr

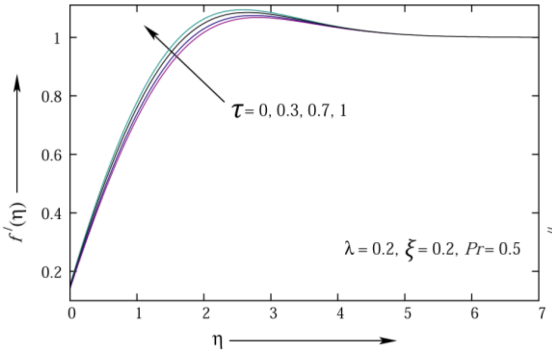

Fig. 5a Velocity profiles $f'(\eta)$ for several values of τ

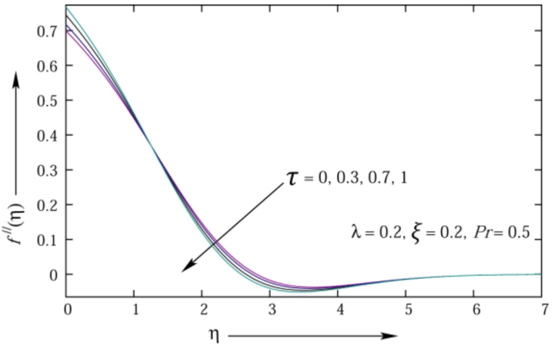

Fig. 5b Shear stress profiles $f''(\eta)$ for several values of τ

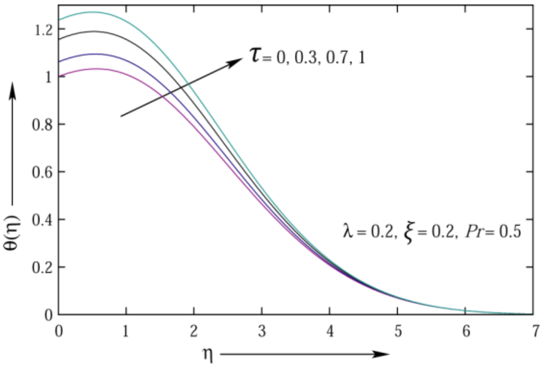

Fig. 5c Temperature profiles θ(η) for several values of τ

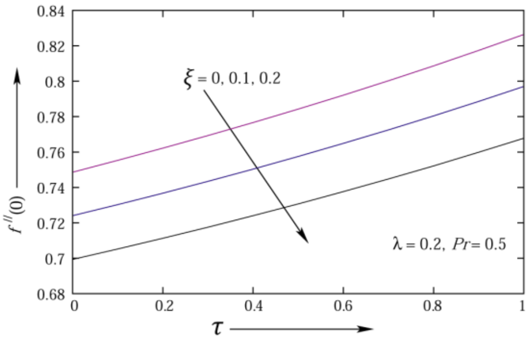

Fig. 6a Skin-friction coefficient $f''(0)$ against τ for various values of ξ

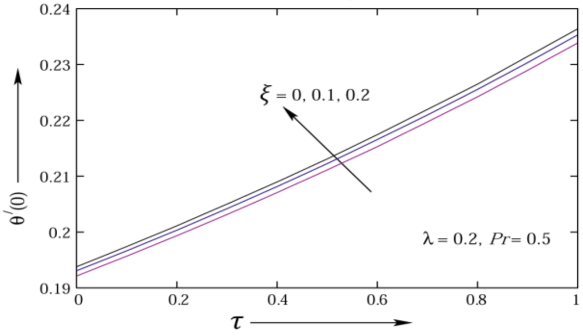

Fig. 6b Temperature gradient at the plate θ′(0) against τ for various values of ξ

IV. CONCLUSION

From the present study, we have derived a new set of non-linear ordinary differential equations (7) to (8) with boundary conditions (9) and (10) of the unsteady boundary layer free convection flow over porous plate and slip effect.In this paper we have discussed the effects of velocity and thermal slip parameter on an unsteady two dimensional free convective boundary layer flow. The numerical results have been presented in the form of graphs. From the present numerical investigations the following major conclusions may be drawn:
(i) Velocity and temperature in the unsteady case is observed to be lesser than those of the steady case.
(ii) It is observed that an increase the free convection parameter, the velocity increases and the temperature decreases.
(iii) With increasing the values of velocity slip, the velocity increases at a fixed point and then it decreases.
(iv) The increase of Prandtl number reduces the velocity along the plate as well as the temperature.
(v)Increasing the thermal slip parameter both velocity and temperature be increased.

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