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Adaptation of compromise programming approach for multi-criteria material selection

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ABSTRACT: *Selection of proper materials for new products and continuous improvement of existing ones to meet the ever changing service requirements in order to gain and or maintain competitive edge is a challenging task. The material selected for a component determines its performance in terms of functionality, manufacturability, maintainability, environmental imparts and life cycle costs. Material selection requires multi-criteria decision analysis approach that is able to take the relative importance of each criterion and the deviations of the achievement levels of each criterion from their respective ideal values into account simultaneously. In this paper, the Minkowski distance metric as used in compromise programming is adapted to solve material selection problem. Two examples are presented to illustrate the potential of the proposed approach. The results show that the proposed method is effective for material selection and sensitive to the level of intensity of the designer's concern over the deviations of achievement levels from their respective ideal values and provides useful insights on optimal trade-offs among the alternative materials.*

KEYWORDS: *Compromise programming, Distance metric, Material selection, Multi-criteria decision making, Trade-off, Utopia*

I. INTRODUCTION

Material selection is an important element in product design. It is the task of identifying the material(s) that after being manufactured have the properties, dimensions and shape needed for the product to serve its purpose in the most effective and efficient manner and also at minimum costs to the manufacturer, user and the environment/society [1-4]. The material selected for a component or product determines its performance in terms of functionality, manufacturability, maintainability, environmental impact and life cycle costs [3,4-6]. Hence, it is crucial to select appropriate material for a particular design.

Material selection is not limited to the design of new products. Existing products are often redesigned in order to gain and/or maintain a competitive edge in the market and most of such redesigns necessitate the use of new materials. Service requirements of products are not static. They are ever changing; for instance, turbine discs for aero-engines developed in the 1950's were made of forged steel components when turbine gas temperatures were relatively low, typically 450^0C. The steel disc met all requirements at this temperature, but strength and oxidation resistance fell rapidly at higher temperatures. Higher temperatures are needed in order to increase the thermodynamic cycle efficiency, induce fuel savings and reduce the emission of pollutants. To meet this requirement of higher temperature, Ni-Fe alloys replaced forged steel discs in the mid 1960's. With continuous improvement, the 1970's saw a further increase in the disc temperature to over 600^0C where the stability of Ni-Fe alloys became inadequate. In order to extend disc capability above 600^0C, Ni-based superalloys with increased precipitation hardening and higher thermal stability were introduced. Efficiency of aero-engine turbine is proportional to temperature; hence the research community in turbine machinery and power plants are seeking means to further drive temperature higher [7-9]. The operating temperatures of the rim sections of present day high-pressure turbine discs now approach 760^0C and even as high as 815^0C for some specialized military applications [10,11]. Hence, material selection is very crucial for new product design and the continuous improvement drive for existing products in order to gain technical and commercial benefits in the present day market [5].

Material selection presents a big challenge in product design and development for many reasons: (1) Material selection focuses on the entire product/component life cycle including manufacturing, operation and maintenance and product retirement. Manufacturing costs, total cost of ownership over the life of the product, including retirement and environmental impact are becoming increasingly important to manufacturers/business owners, customers/product users and regulatory agencies. The selection of materials that best meet the technical, economic and environmental criteria over the life of the product is not a trivial problem [3,12-17]. (2) Over 40,000 metal alloys and almost the same number of non-metals, ceramics, polymers and composites are at the designers' disposal. A plethora of new materials with varying degrees of properties improvements has been discovered by the research community in the last decade. Today, materials are developing faster than any other time in history and as a consequence the design space is ever expanding. It is difficult for designers, although educated in the fundamentals of materials and engineering, to still be able to make optimum decisions on materials to satisfy design problems given the vast range of materials available and new materials being developed [5, 17,18]. (3) The requirements the product/component is expected to meet are numerous and conflicting. For instance, the material must meet the service requirements and for a mechanical design, these may depend on many properties such as creep, wear resistance, ultimate tensile strength, toughness, etc… Since the material has to be processed to achieve the dimensions and shape needed for the component to serve its purpose in the most cost effective manner, other criteria such as manufacturability (weldability, castability, machinabilty, etc…) and economic factors (unit cost, cost-to-mass ratio, recoverability, etc…) must be considered. These requirements are of different degrees of importance and often incompatible because it is not possible to improved one requirement without reducing the satisfaction of one or more of the other requirements. (4) Apart from the conflict among the numerous requirements, there is also conflict among stakeholders. An instance is a case where the designer's interest is in composite light weight material with extreme strength-to-mass ratio while the interest of the recycler is in pure and easy-to-recycle material [19-23].

The challenge confronting the designer is how to choose from the vast number of materials, the one that best fulfill the numerous conflicting requirements. This requires systematic approach/mathematical tool to guide the designer in the material selection decision. Material selection is ultimately a multi-criteria decision making process involving assessment of trade-offs among various conflicting and divergent performance criteria [16,24]. Since it is not possible to achieve the ideal values of all the criteria simultaneously, the designer needs an approach that will give the best compromise solution [25,26].

An appreciable number of research works has appeared on material selection using different multi-criteria decision making (MCDM) methods. Athawale and Chakraborty [27] presented a review and comparative study of various MCDM methods such as VIKOR, ELECTRE, TOPSIS, PROMETHEE, simple additive weighting (SAW), Weighted product method (WPM), grey relational analysis (GRA), range of value method and graph theory and matrix approach. Various extensions of these methods either applied singly or in combination with other methods have also appeared in the literature [16,17,28]. Complex proportional assessment of alternatives (COPRAS) and its extensions, genetic algorithm with neural networks, desirability function, and multi-objective optimization on the basis of ratio analysis (MOORA) have also been used for material selection [6, 29-32].

These approaches mostly consider three characteristics of the material selection problem: (a) performance criteria (b) relative importance of each criterion and (c) alternatives. The alternatives are ranked and the one that gives the best compromise among the criteria is then selected for the given application. However, there is an aspect of performance criteria which has not been fully addressed in the literature. Criteria are assigned weights to reflect their relative importance, but the preferences of the designer concerning the deviations are often not taken into consideration. For instance, in some material selection situations it may be only the largest deviation that counts. In other words, the intensity of his concern over large deviations is high. In some other situations the designer may weigh all the deviations equally which implies the intensity of his concern over large deviations is low. He may also weigh the deviations in proportion to their magnitudes depending on his needs. Therefore, a fourth characteristic of material selection problem which is the intensity of designer's concern over deviations should be included in the material selection model. Although a plethora of multi-criteria methods has been proposed for material selection, there is still a need for simple as well as a systematic approach that incorporates the intensity of designer's concern over the large deviations and also provides opportunities for trade-off explorations based on these concerns. In this paper, an approach developed from the $L_P - norm$ as used in the compromise programming (CP) method is proposed for the selection of most suitable material for a given engineering application.

II. MATERIAL SELECTION PROBLEM

Material selection problem has the following characteristics; (i) there exist a finite set of performance criteria, usually conflicting, with non-commensurable units and different order of magnitudes (ii) the criteria are of varying degree of importance and weights are assigned to each to reflect their relative importance (iii) there exist a finite set of alternative materials from which the most appropriate/best is to be selected (iv) the intensity of the designer's concern over the large deviations. The problem is that of selecting the best material from the set of alternative materials while taking the existing situations into account such that there is a maximum realization of designer's objectives. The problem will be trivial if there exists a material that achieves the ideal performance levels of all criteria simultaneously. Unfortunately, it is often not feasible to get such material, so a compromise is needed to resolve the problem. A procedure for identifying the material that gives the best compromise is proposed in the following section.

III. THE PROPOSED APPROACH

Compromise programming (CP) was first proposed by Zeleny [33,34] and has become one of the widely used multi-criteria decision making (MCDM) methods [25,26,36-38]. The basic idea in CP is to identify the utopian solution, which in our case is the material that achieves the ideal values of all criteria simultaneously. Achievement of utopia is not practically feasible because of the inherent conflict in the criteria but may be used as a base point. The designer therefore seeks a compromise solution. His decision is based on Zeleny's axiom of choice where the solutions that are closer to the utopian are preferred to those that are farther [33,34]. To achieve this closeness, Minkowski distance metric ($L_P - norm$) is introduced into the analysis. The $L_P - norm$ is used to calculate the distances between the achievement levels belonging to the solution set and the utopian point to identify the one that is closest to the utopia. We shall adapt the Minkowski distance metric as used in CP to material selection problem. The procedural steps on its adaptation are presented below:

Step 1: Identification of performance criteria.

Identify the set **I** of all performance requirements/criteria on which the evaluation of the materials will be based. The set **I** has the following properties: (i) there is a set $\mathbf{B} \subset \mathbf{I}$ of beneficial criteria for which higher values indicates better performance (ii) there also exist a set $\mathbf{C} \subset \mathbf{I}$ of non-beneficial criteria of which lower values imply better performance (iii) observe that $\mathbf{B} \cap \mathbf{C} = \mathbf{\Phi}$ since a criterion is either beneficial or non-beneficial and not both.

Step 2: Assignment of weight to criteria.

Assign weight $(w_b, w_c \in [0,1])$ to each criteria $b \in \mathbf{B}$ and $c \in \mathbf{C}$ to reflect their relative importance using the appropriate method. Analytical hierarchy process (AHP) and entropy methods are commonly used for criteria weighting [28]. The authors have proposed a method for criteria weighting but its description is beyond the scope of this paper. Observe that;

$$\sum_{i \in I} w_i = \sum_{b \in B \subset I} w_b + \sum_{c \in C \subset I} w_c = 1 \tag{1}$$

***Step 3: Assignment of aspiration levels and veto thresholds*.**

Assign aspiration levels and veto thresholds to each of the identified criteria. For each beneficial criterion, determine the largest/best performance value $\{ x_b^{\max} | b \in \mathbf{B}\}$ that is practically attainable and the smallest/worst performance value $\{ x_b^{\min} | b \in \mathbf{B}\}$ that is admissible. Recall that for beneficial criteria, larger values imply better performance. Hence, $\{ x_b^{\max} | b \in \mathbf{B}\}$ and $\{ x_b^{\min} | b \in \mathbf{B}\}$ are the aspiration levels and veto thresholds respectively. Similarly, for non-beneficial criteria, determine the smallest/best performance level $\{ x_c^{\min} | c \in \mathbf{C}\}$ that is practically achievable and the largest/worst performance value $\{ x_c^{\max} | c \in \mathbf{C}\}$ that is acceptable. $\{ x_c^{\min} | c \in \mathbf{C}\}$ and $\{ x_c^{\max} | c \in \mathbf{C}\}$ are the respective aspiration levels and veto thresholds for non-beneficial criteria. In order to adapt the $L_P - norm$ as used in compromise programming to material selection, we shall call $\mathbf{U}_{\mathbf{B},asp.} = \{ x_b^{\max} | b \in \mathbf{B}\}$ and $\mathbf{U}_{\mathbf{C},asp.} = \{ x_c^{\min} | c \in \mathbf{C}\}$ the ideal/anchor values of the beneficial and non-beneficial criteria respectively while $\mathbf{U}_{\mathbf{B},veto} = \{ x_b^{\min} | b \in \mathbf{B}\}$ and $\mathbf{U}_{\mathbf{C},veto} = \{ x_c^{\max} | c \in \mathbf{C}\}$ are denoted as the anti-ideal/nadir values of the beneficial and non-beneficial criteria respectively. Material that satisfies all criteria at their ideal/aspiration levels is the utopian. Observe that the utopian **U** point is given by;

$$\mathbf{U} = \{\mathbf{U}_{\mathbf{B},asp.}, \mathbf{U}_{\mathbf{C},asp.}\} = \{ x_b^{\max}, \ x_c^{\min} \mid b \in \mathbf{B} \text{ and } c \in \mathbf{C}\} \qquad (2)$$

The utopian is practically not feasible, so according to Zeleny's axiom of choice we seek for a material whose performance rating on all the criteria is closest to it.

Step 4: Sorting through material database.

Sort through the material database to identify the set **A** of alternative materials that meet these performance requirements. Any material whose achievement level on beneficial criteria falls below the veto threshold $x_b^{\min}$ for any $b \in \mathbf{B}$ is screened out while for non-beneficial criteria; a material is screened out if its achievement level is above the veto threshold $x_c^{\max}$ for any $c \in \mathbf{C}$. Note that only the materials which fulfill the membership conditions as stated in Eq. (3) below are included in **A**. The set **A** may also be referred to as the solution set.

$$\text{Achievement level (AL)} = \{ x_{ab} \geq x_b^{\min}, x_{ac} \leq x_c^{\max} \mid \forall\ (b \in \mathbf{B} \text{ and } c \in \mathbf{C}) \},\ a=1,2,\ldots,m. \qquad (3)$$

where x_{ab} is the achievement level of material "a" on beneficial criterion "b" and x_{ac} is the achievement level of material "a" on non-beneficial criterion "c". Next, the alternative materials are ranked in order to identify the best.

Step 5: Developing the distance metrics.

Develop the $L_p - norm$/Minkowski distance metrics. Let D_{ab} be the deviation/distance of the achievement of material "a" from the aspiration level, $x_b^{\max}$ of beneficial criterion "b". Then D_{ab} is given by;

$$D_{ab} = x_b^{\max} - x_{ab} \qquad (4)$$

Because of non-commensurable units and different order of magnitudes of the criteria, normalized distances are used rather than the absolute distances [25]. The normalized distance (D_{ab}^N) is given by;

$$D_{ab}^N = \left(\frac{x_b^{\max} - x_{ab}}{x_b^{\max} - x_b^{\min}} \right); \text{ and } 0 \leq D_{ab}^N \leq 1 \qquad (5)$$

Equation (5) above shows how far the performance rating of material "a" on criterion "b" is from the aspiration level. The degree of closeness, DC_{ab}^N to the aspiration level may be expressed as;

$$DC_{ab}^N = 1 - D_{ab}^N = 1 - \left(\frac{x_b^{\max} - x_{ab}}{x_b^{\max} - x_b^{\min}} \right) \qquad (6)$$

Simplification of Eq. (5) gives,

$$DC_{ab}^N = \left(\frac{x_{ab} - x_b^{\min}}{x_b^{\max} - x_b^{\min}} \right) \text{ and } 0 \leq DC_{ab}^N \leq 1 \qquad (7)$$

The degree of closeness DC_{ab}^{N} may also be express as percentage in which case the value of DC_{ab}^{N} lies between 0 and 100% (i.e. $0 \le DC_{ab}^{N} \le 100\%$). If the level of achievement of criterion "b" is at the veto threshold, then $DC_{ab}^{N} = 0$, and $DC_{ab}^{N} = 100$ if achievement is at the aspiration level. Note that while Eq. (5) expresses how far the achievement is from the aspiration level, Eq. (7) expresses how close it is to the aspiration level.

Similarly, the degree of closeness, DC_{ac}^{N} of the achievement of material "a" with respect to the non-beneficial criterion "c" is given by;

$$DC_{ac}^{N} = \left(\frac{x_c^{\max} - x_{ac}}{x_c^{\max} - x_{cb}^{\min}} \right) \qquad (8)$$

The value of DC_{ac}^{N} also lies between 0 and 100%. Because of the divergent nature of the criteria, it is not feasible to get a material which achieves the aspiration levels of all performance criteria simultaneously. Hence, we seek for a material within **A** whose overall achievement with respect to all criteria is closest to **U**. So aggregate the normalized degrees of closeness to obtain the composite degree of closeness to **U**. The degree of closeness of overall achievement of material "a" to **U** is given by;

$$L_{p,a} = \left[\sum_{b \in B \subset I} w_b \left(\frac{x_{ab} - x_b^{\min}}{x_b^{\max} - x_b^{\min}} \right)^p + \sum_{c \in C \subset I} w_c \left(\frac{x_c^{\max} - x_{ac}}{x_c^{\max} - x_{cb}^{\min}} \right)^p \right]^{\frac{1}{p}} \qquad (9)$$

where $p \ge 1$ and $\sum_{b \in B \subset I} w_b + \sum_{c \in C \subset I} w_c = 1$

The properties of $L_{p,a}$ are:

Property i: The weights $(w_b, w_c \in [0,1])$ express the relative importance of each beneficial and non-beneficial criterion respectively.

Property ii: According to Eq. (9), it is obvious that $L_{p,a} \ge 0$. Since $L_{p,a}$ is normalized by the exponent $1/p$ it can be guaranteed that $L_{p,a} \le 1$, hence, $0 \le L_{p,a} \le 1$ for all a $\in$ **A**. If the degree of closeness is expressed as percentage then, $0 \le L_{p,a} \le 100\%$.

Property iii: the parameter p explicitly expresses the intensity of the concern of the designer over the deviations from the aspiration levels. The distance from **U** decreases as p increases. On the other hand, the degree of closeness increases with increasing value of p. All the possible distances are bounded by $L_{1,a}$ (i.e. p = 1; Manhattan distance) and $L_{\infty,a}$ (i.e. $p = \infty$; Tchebycheff distance). Note that, the value of parameter p is chosen to express the designer's preferences regarding the larger deviations. Manhattan distance is used when the deviations of the achievement from their respective aspiration values are of equal concern to the designer. If only the largest deviation counts to the designer, then the Tchebycheff distance is used and the problem becomes a mini-max problem. If the designer weighs each deviation in proportion to its magnitude, then the Euclidean distance (p = 2) is used to rank the materials. The greater the concern of the designer over the maximum deviation the larger the value of parameter p; when p = ∞ the largest deviation completely dominates the distance measure [25, 38, 39].

Step 6: find material "a" from the solution set A, so as to maximize $L_{p,a}$. To achieve this, compute $L_{p,a}$ for each $a \in \mathbf{A}$ and rank the materials in descending order of $L_{p,a}$ values. The material whose overall achievement (AL) corresponds to $\max_a \{L_{P,a} | a = 1,2,...,m\}$ is the closest to the utopia **U**.

IV. NUMERICAL EXAMPLE

In this section, material selections for some given engineering applications are used to demonstrate the feasibility of the proposed approach to evaluate and find the best material. Two examples will be used: (i) Material selection for non-heat-treatable cylinder cover from the literature [41] and (ii) material selection for armature shaft.

Example 1: Non-heat-treatable cylinder cover material

In this example, the problem of selecting the best material for non-heat-treatable cylinder cover is considered using the procedure described in section 3. Firstly, all necessary performance criteria were identified based on service requirements of the non-heat-treatable cylinder cover as well as the manufacturability and cost requirements. A total of twelve criteria were listed out of which eight were beneficial while four are non-beneficial. The criteria with their respective weights, aspiration levels and veto thresholds are presented in Table 1. Eight alternative materials that fulfilled the conditions of Equation (3) were selected (see Table 2) and their respective properties/achievement on each criterion is presented in Table 3.

Table 1: Material Selection Data for Non-heat-treatable Cylinder Cover

Criteria	Criteria Type	Aspiration Level	Veto threshold	Criteria Weight (%)
Density, D (Mg/m^3)	Non-beneficial	2.67	8.95	5.3
Compressive strength, CS (MPa)	Beneficial	690	50	8.9
Ultimate tensile strength, UTS (MPa)	Beneficial	1030	210	7.3
Spring back index, SBI	Non-beneficial	0.08	1.55	10.3
Bending force index, BFI	Non-beneficial	1355	20317	10.3
Static load index, SLI	Beneficial	2916	260	8.7
Hardness, H (Vickers)	Beneficial	380	45	6.7
Yield strength, YS (MPa)	Beneficial	800	50	9.4
Elastic modulus, EM (GPa)	Beneficial	205	73.59	7.4
Thermal diffusivity, TD (cm^2/h)	Beneficial	741	174	8.2
Thermal conductivity, TC (W/m K)	Beneficial	398	17	11.2
Cost of base material, C (CAN$/kg)	Non-beneficial	1.04	18.64	6.5

Source: Shanian and Savadogo, (2006)

Table 2: List of Alternative Materials

Alternative materials	Code
Copper-2-beryllium (cast) UNS C82400	A1
Copper-cobalt-beryllium (cast) UNS C82000	A2
Electrolytic tough-pitch, h.c. copper, soft (wrought) UNS C11000	A3
Electrolytic tough-pitch, h.c. copper, hard (wrought) UNS C11000	A4
Wrought aluminum alloy 5052 H34	A5
Wrought austenitic stainless steel AISI 304, HT grade D	A6
Commercial bronze, cuzn10, soft (wrought) UNS C22000	A7
Carbon steel (annealed) AISI 1020	A8

Source: Shanian and Savadogo, (2006)

Table 3: Material properties (achievement level for each criterion)

Alt. Mat.	D	CS	UTS	SBI	BFI	SLI	H	YS	EM	TD	TC	C
A1	8.25	560	940	0.78	15183	2916	380	560	138	465	105	18.64
A2	8.65	460	600	0.71	12472	2395	220	460	125	465	205	13.99
A3	8.94	50	210	0.08	1355	260	45	50	122	460	398	3.00
A4	8.95	340	380	0.48	9218	1770	115	340	135	460	390	3.46
A5	2.67	190	295	0.25	20317	1966	87	191	73.59	741	152	2.81
A6	8.06	690	1030	1.55	5909	2174	350	800	190	189	17	5.99
A7	8.63	95	270	0.17	2711	520	63	100	116	174	185	3.22
A8	7.08	267	355	0.48	1957	720	110	265	205	329	50	1.04

Source: Shanian and Savadogo, (2006)

Using Equation (9), the degree of closeness to the utopian were computed for different values of parameter, p (i.e. p = 1, p = 2, p = 10 and p = 100). The degree of closeness as percentages and the ranking are presented in Table 4a below. For material selection situation where the deviations are of equal concern to the designer (i.e. $L_1 - norm$) the ranking of alternative materials obtained was $A6 \succ A1 \succ A4 \succ A2 \succ A8 \succ A3 \succ A5 \succ A7$. $A6$, $A1$ and $A4$ are the 1st, 2nd and 3rd choice materials respectively. When the deviations are weighed in proportion to their magnitude (i.e. $L_2 - norm$), the ranking was $A6 \succ A3 \succ A1 \succ A4 \succ A5 \succ A8 \succ A2 \succ A7$. The same $A6$ remains the best choice but $A3$ that ranked 6th when $L_1 - norm$ was used is now 2nd and $A1$ that ranked 2nd is now ranked 3rd. The ranking obtained with $L_{10} - norm$ is $A3 \succ A6 \succ A8 \succ A5 \succ A1 \succ A7 \succ A4 \succ A2$. The $L_{10} - norm$ expressed the increased concern (high intensity) of the designer over larger deviations, and alternative material $A3$ that ranked 6th became the best choice instead of $A6$ which is now 2nd and $A8$ 3rd. The $L_{100} - norm$ shows that the intensity of the designer's concern over the large deviations is higher compared to his concern when the $L_{10} - norm$ was used. The ranking of the alternatives with the $L_{100} - norm$ is $A3 \succ A6 \succ A1 \succ A8 \succ A5 \succ A4 \succ A7 \succ A2$. The alternative materials $A3$, $A6$ and $A1$ are ranked 1st, 2nd and 3rd respectively when compared to the ranking obtained with the $L_{10} - norm$ where $A8$ is ranked 3rd. The suitability of this approach to reflect the preferences of the designer concerning the larger deviations is made clearer by Table 4b. The differences in the rankings at different intensity levels of the designer's concern show the necessity of incorporating the concern of the designer into the material selection process. It is clear that the level of designer's preferences regarding larger deviations determines the ranking of candidate materials. Hence rankings obtained without the incorporation of the intensity of designer's concern regarding the large deviations may be misleading.

Table 4a: Degree of Closeness ($L_{p,a}$) and Ranking of alternatives

Alt. Mat.	L_1 **(p = 1)**		L_2 **(p = 2)**		L_{10} **(p = 10)**		L_{100} **(p = 100)**	
	Closeness (%)	Rank	Closeness (%)	Rank	Closeness (%)	Rank	Closeness (%)	Rank
A1	54.58	2	62.74	3	84.62	5	98.15	3
A2	49.55	4	52.13	7	64.15	8	78.44	8
A3	44.45	6	63.26	2	89.72	1	98.86	1
A4	53.22	3	59.08	4	80.24	7	95.78	6
A5	43.32	7	57.23	5	84.64	4	98.02	5
A6	57.99	1	71.34	1	89.13	2	98.65	2
A7	35.52	8	51.12	8	80.97	6	92.03	7
A8	45.97	5	57.20	6	85.87	3	98.07	4

Table 4b: Sensitivity of $L_{p,a}'s$ to the intensity of designer's concern over deviations

Rank	**Example 1: Non-heat-treatable Cylinder Cover**			
	L_1	L_2	L_{10}	L_{100}
1st	A6	A6	A3	A3
2nd	A1	A3	A6	A6
3rd	A4	A1	A8	A1

Example 2: Armature shaft material

Following the procedure of Section 3, ten criteria were identified out of which seven were beneficial and three were non-beneficial. The list of criteria with their respective aspiration level, veto threshold and weights is presented in Table 5. The list of materials that fulfilled the selection criteria of Eq. (3) are presented in Table 6 while Table 7 shows the properties or achievement levels of the alternative materials on each criterion. Next, Eq. (9) was used to compute the $L_{p,a}$ values for (p = 1, 2, 10 and 100) and the alternatives were ranked in descending order of $L_{p,a}$ (see Table 8a).

Table 5: Material Selection Data for Armature Shaft

Criteria	Criteria Type	Aspiration Level	Veto threshold	Criteria Weight (%)
Ultimate tensile strength, UTS (MPa)	Beneficial	790	330	14.4
Yield strength, YS (MPa)	Beneficial	605	140	14.4
Elastic modulus, EM (GPa)	Beneficial	202	105	11.0
Ductility, DU (%)	Beneficial	55	10	8.4
Hardness, H (Vickers)	Beneficial	93	55	11.6
Density, D (Kg/m^3)	Non-beneficial	7.80	8.44	6.2
Thermal conductivity, TC (W/m K)	Beneficial	120	16.2	9.2
Thermal diffusivity, TD (cm^2/h)	Beneficial	415.4	127.3	6.8
Thermal expansion, TE ($\mu m / mK$)	Non-beneficial	11.5	15.7	8.8
Cost of base material, C ($/Kg)	Non-beneficial	3.94	7.98	9.3

Table 6: List of Alternative Materials (Armature shaft)

Alternative materials	Code
Carbon Steel SAE 1006	B1
Carbon Steel SAE 1010	B2
Carbon Steel SAE 1020	B3
Carbon Steel SAE 1030	B4
Carbon Steel SAE 1070	B5
Carbon Steel SAE 1090	B6
Carbon Steel SAE 1117	B7
Carbon Steel SAE 1547	B8
Stainless Steel AISI 201	B9
Forging Brass, UNS C 37700	B10

Table 7: Material properties (Armature shaft)

Alt. Mat.	UTS	YS	EM	DU	H	D	TC	TD	TE	C
B1	330	285	200	20	55	7.872	64.9	171.4	12.6	5.90
B2	365	305	200	20	60	7.872	51.9	147.2	12.6	7.08
B3	420	205	200	15	73	7.872	51.9	135.7	11.9	5.59
B4	525	440	200	12	80	7.872	48.7	127.3	11.7	3.94
B5	640	495	201	10	91	7.872	51.2	132.7	12.1	4.57
B6	696	540	202	10	92	7.872	49.8	133.7	11.5	7.08
B7	475	400	200	12	86	7.872	51.2	135.2	11.5	7.67
B8	710	605	200	10	93	7.872	51.2	137.5	11.5	7.98
B9	790	380	197	55	90	7.80	16.2	415.4	15.7	4.60
B10	360	140	105	30	74	8.44	120	374.2	12.5	5.06

Source: MatWeb (Material Property Data) http://matweb.com

The ranking obtained for $L_1 - norm$, $L_2 - norm$, $L_{10} - norm$ and $L_{100} - norm$ as displayed in Tables 8a and 8b, reflect the sensitivity of this approach to the intensity of the concern of the designer over the deviations. For clarity, see the extract from Table 8a displayed in Table 8b showing the 1st, 2nd and 3rd best alternative materials for all the $L_{p,a}$'s. The best material for the armature shaft with $L_1 - norm$, $L_2 - norm$ and $L_{10} - norm$ is $B9$ while 2nd and or 3rd vary (Table 8b). With greater concern over the large deviations (i.e. $L_{100} - norm$), $B8$, became the best material followed by $B9$ and $B6$ as 2nd and 3rd respectively, while $B1$, $B2$ and $B3$ are the worst with the same degree of closeness (95.80%) to the utopia Table 8a. Note that for the $L_{100} - norm$, $B8$ was ranked 1st while $B9$ is ranked 2nd. However, it may be expedient for the designer to choose alternative $B9$ instead of $B8$ since the difference between the closeness of $B8$ and $B9$ to the utopia is marginal (98.99% and 98.98%) and $B9$ is ranked higher than $B8$ for all the other distance metrics (see table 8a). This demonstrates another merit of this approach in that it provides opportunity for trade-off exploration with different values of parameter p in order to gain useful insights on optimal trade-offs among alternatives before making the final choice of material. The degree of closeness (Tables 4a and 8a) agrees with previous research that the distance from utopia decreases as the value of parameter p increases or conversely, the degree of closeness increases as value of p increases.

Table 8a: Degree of Closeness ($L_{p,a}$) and Ranking of alternatives for armature shaft

Alt. Mat.	L_1 (p = 1)		L_2 (p = 2)		L_{10} (p = 10)		L_{100} (p = 100)	
	Closeness (%)	Rank	Closeness (%)	Rank	Closeness (%)	Rank	Closeness (%)	Rank
B1	39.27	8	51.70	8	80.37	8	95.80	8
B2	38.07	9	49.34	9	80.36	9	95.80	8
B3	44.36	7	55.26	7	82.21	3	95.80	8
B4	60.24	5	68.73	5	87.35	4	97.79	4
B5	66.70	2	74.26	4	86.48	5	96.82	7
B6	65.64	4	75.43	3	90.09	3	98.43	3
B7	51.51	6	62.47	6	85.62	6	97.74	5
B8	66.31	3	78.19	2	92.87	2	98.99	1
B9	72.13	1	81.17	1	93.11	1	98.98	2
B10	33.10	10	48.77	10	79.56	10	97.64	6

Table 8b: Sensitivity of $L_{p,a}'s$ to the intensity of designer's concern over deviations

Rank	Example 2: Armature Shaft			
	L_1	L_2	L_{10}	L_{100}
1st	B9	B9	B9	B8
2nd	B5	B8	B8	B9
3rd	B8	B6	B3	B6

V. CONCLUSION

The proposed method for material selection has been shown to be a suitable tool for incorporating the intensity of designer's concern over larger deviations in the material selection process. The model ranked candidate materials from best to worst for each level of intensity of designer's concern over the deviations. Results of example problems demonstrate the sensitivity of the approach to the level of intensity of designer's concern and also provides useful insights on optimal trade-offs among the alternative materials.

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