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FREE CONVECTION AND MASS TRANSFER FLOW THROUGH A POROUS MEDIUM WITH VARIABLE TEMPERATURE

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ABSTRACT: *Aim of the paper is to investigate the hydro magnetic effects of a uniform transverse magnetic field on the free convection and mass transfer flow of an electrically conducting fluid past an exponentially accelerated infinite vertical plate through a porous medium with variable temperature. The problem is governed by coupled nonlinear partial differential equations. The dimensionless equations of the problem have been solved by the explicit finite difference method. Here the plate temperature is increasing linearly with time and the concentration level near the plate is increased. Among the effects of various parameters in going into the problem, the velocity and skin friction is broadly discussed with the help of graph.*

KEYWORDS*: Vertical plate, Unsteady, Explicit finite difference method, Porous medium andelectrically conducting fluid.*

I. INTRODUCTION

Magneto hydrodynamic (MHD) is the branch of continuum mechanics which deals with the flow of electrically conducting fluids in electric and magnetic fields. Many natural phenomena and engineering problems are important being subjected to an (MHD) analysis. Moreover, Magneto hydrodynamic (MHD) has drawn the attention of a broad number of scholars due to its variant applications. In engineering it finds its application in (MHD) pumps, (MHD) bearings etc. Freeconvection flows are of a great attention in a number of industrial applications like as fiber and granular insulation, geothermal systems etc. Convection in porous media has application on geothermal energy recovery, oil extraction, thermal energy storage and flow through filtering devices. The occurrences of mass transfer are also very general in theory of stellar structure and remarkable effects are detectable, at least on the solar surface. The study of influences of magnetic field on free convection flow is important in liquid metal, electrolytes and ionized gases. Also the study of flows through porous media became of great attention due to its wide application in many scientific and engineering problems.

Gupta et al. [9] (Gupta et al. 1979) studied free convection on flow past a linearly accelerated vertical plate in the arrival of viscous dissipative heat using perturbation method. Free convection flow past an accelerated infinite plate discussed by Pop, I. and. Soundalgekar [1](Pop, I. and. Soundalgekar 1980). Kafousias and Raptis [4] (Kafousias and Raptis 1981) expanded the problem of Gupta et al. to involve mass transfer effects subjected to variable suction and injection. Sing and Naveen Kumar [3] (Sing and Naveen Kumar 1984) was studied free convection effects on flow past an exponentially accelerated vertical plate. Hossain and Shayo [7] (Hossain and Shayo 1986) discussed skin friction for accelerated vertical plate analytically. BasantkumarJha [2] (BasantkumarJha 1991) studied MHD free convection and mass transform flow through a porous medium. LatterlyCombined heat and mass transfer effects on MHD free convection flow past an oscillating plate embedded in porous medium have analyzed by R.C. Chaudhary and Arpita Jain [5] (R.C.Chaudhary et al. 2007). Recently Muthukumaraswamy et al. [6](Muthukumaraswamy et al. 2008) discussed mass transfer effects on exponentially accelerated isothermal vertical plate.

In current years, the problems of free convective and heat and mass transfer flows through porous medium under the effects of magnetic field have drawn the attention of a large number of researchers due to their applications in many branches of science and technology such as transportation cooling of re-entry vehicles and rocket boosters and film vaporization in combustion chambers. On the other hand in case of power generation MHD is receiving considerable attention due to the possibilities it offers for much higher thermal efficiencies in power plants. In view of these applications, the aim of the paper is to study on the transverse magnetic field on the free convection and mass transfer flow past an exponentially accelerated vertical plate with variable temperature through a porous medium. Dimensionless governing equations are solved by using the explicit finite difference method.

In our present work, we have studied about free convection and mass transfer flow through a porous medium with variable temperature. The governing equations for the unsteady case are also studied. Then these governing equations are transformed into dimensionless momentum, energy and concentration equations are solved numerically by using explicit finite difference technique with the help of a computer programming language Compaq visual FORTRAN 6.6. The obtained results of this problem have been discussed for the different values of well-known parameters with different time steps. The tecplot 9.0 is used to draw graph of the flow.

II. MATHEMATICAL FORMULATION

$$\frac{\partial \bar{u}}{\partial \bar{t}} = g\beta(\bar{T}-\bar{T}_\infty)+g\beta'(\bar{C}-\bar{C}_\infty)+\nu\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}-\frac{\sigma B_0^2}{\rho}(\bar{u}-u_0e^{\bar{a}\bar{t}})-\frac{\nu\bar{u}}{\bar{K}} \quad (1)$$

$$\rho C_p\frac{\partial \bar{T}}{\partial \bar{t}} = \kappa\frac{\partial^2 \bar{T}}{\partial \bar{y}^2} \quad (2)$$

$$\frac{\partial \bar{C}}{\partial \bar{t}} = D\frac{\partial^2 \bar{C}}{\partial \bar{y}^2} \quad (3)$$

The boundary conditions related with the problem are,

$$\begin{aligned}
&\bar{t}\le 0: \bar{u}=0, \quad \bar{T}=\bar{T}_\infty, \bar{C}=\bar{C}_\infty \quad \text{for all } \bar{y}\\
&\bar{t}>0: \bar{u}=u_0e^{\bar{a}\bar{t}}, \bar{T}=\bar{T}_\infty+(\bar{T}_w-\bar{T}_\infty)A\bar{t}, \bar{C}=\bar{C}_w \text{ for all } \bar{y}=0 \\
&\bar{u}=0,\ \bar{T}\to\bar{T}_\infty, \quad \bar{C}=\bar{C}_\infty, \quad \text{as } \bar{y}\to\infty
\end{aligned} \quad (4)$$

where $\bar{u}$ is the velocity of the fluid in $\bar{x}$ direction, $\bar{T}$ is the temperature and $\bar{C}$ is the concentration component of the fluid respectively, g is the acceleration due to gravity, $\bar{C}_\infty$ is the concentration in the fluid far away from the plate, $\bar{C}_w$ is the concentration of the plate, $\bar{y}$ is coordinate axis normal the plate, B_0 is the external magnetic field, C_p is specific heat at constant pressure, $\bar{T}_\infty$ is the temperature of the fluid far away from the plate, $\bar{T}_w$ is the temperature of the plate, a is the acceleration parameter, u_0 is the velocity of the plate, D is the chemical molecular diffusivity, K is the permeability parameter, ρ is the density, ν is kinematic viscosity and $\bar{t}$ is the corresponding time, β is the volumetric coefficient of thermal expansion and β' is the volumetric coefficient of expansion with concentration respectively.

Here, $A=\frac{u_0{}^2}{\nu}$,

Since the solutions of the governing equations under the boundary conditions will be based on the finite difference method so it is necessary to make the equation dimensionless. For this reason now we introduce the following dimensionless quantities,

$$\begin{gathered}
u=\frac{\bar{u}}{u_0},\ y=\frac{\bar{y}u_0}{\nu},\ t=\frac{\bar{t}u_0{}^2}{\nu},\ \theta=\frac{\bar{T}-\bar{T}_\infty}{\bar{T}_w-\bar{T}_\infty},\ G_r=\frac{g\beta\nu(\bar{T}_w-\bar{T}_\infty)}{u_0{}^3},\ a=\frac{\bar{a}\,\nu}{u_0{}^2},\\
G_m=\frac{g\beta'\nu(\bar{C}_w-\bar{C}_\infty)}{u_0{}^3},\ P_r=\frac{\mu C_p}{\kappa},\ S_c=\frac{\nu}{D},\ M=\frac{\sigma B_0{}^2\nu}{\rho u_0{}^2},\ K=\frac{u_0{}^2\bar{K}}{\nu^2}
\end{gathered} \quad (5)$$

Then equation (1)-(3) and boundary conditions (4) leads to,

$$\frac{\partial u}{\partial t} = G_r\theta + G_m C + \frac{\partial^2 u}{\partial y^2} - M(u - e^{at}) - \frac{u}{K} \tag{6}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{P_r}\frac{\partial^2 \theta}{\partial y^2} \tag{7}$$

$$\frac{\partial C}{\partial t} = \frac{1}{S_c}\frac{\partial^2 C}{\partial y^2} \tag{8}$$

With the initial and boundary conditions;

$$\begin{aligned} &t \le 0: \quad u = 0, \quad \theta = 0, \quad C = 0, && \text{for all} \quad y \\ &t > 0: \quad u = e^{at}, \; \theta = t, \; C = 1 && \text{for all} \quad y = 0 \end{aligned} \tag{9}$$

Skin friction and Nusselt number

From the velocity field, the effects of different parameters on the Skin friction and Nusselt number have been studied. Skin friction is defined as, $\tau = \frac{1}{2\sqrt{2}} Gr^{-\frac{3}{4}} (\frac{\partial u}{\partial y})_{y=0}$ and Nusselt number is defined as,

$$Nu = \frac{1}{\sqrt{2}} Gr^{-\frac{3}{4}} (\frac{\partial T}{\partial y})_{y=0}.$$

III. NUMERICAL SOLUTION

Many physical phenomena in applied science and engineering when formulated into mathematical models fall into a category of systems known as non-linear coupled partial differential equations. Most of these problems can be formulated as second order partial differential equations. A system of non-linear coupled partial differential equations with the boundary conditions is very difficult to solve analytically. For obtaining the solution of such problems we adopt advanced numerical methods. The governing equations of our problem contain a system of partial differential equations which are transformed by usual transformations into a non-dimensional system of non-linear coupled partial differential equations with initial and boundary conditions. Hence the solution of the problem would be based on advanced numerical methods. The finite difference Method will be used for solving our obtained non-similar coupled partial differential equations.

From the concept of the above discussion, for simplicity the explicit finite difference method has been used to solve from equations (6) to (8) subject to the conditions given by (9).To obtain the difference equations the region of the flow is divided into a grid or mesh of lines parallel to *X* and *Y* axis is taken along the plate and *Y*-axis is normal to the plate.

Here the plate of height $X_{\max}(=20)$ i.e. X varies from 0 to 20 and regard $Y_{\max}(=50)$ as corresponding to $Y \to \infty$ i.e. Y varies from 0 to 50. There are m=100 and n=200 grid spacing in the X and Y directions respectively.

It is assumed that ΔX and ΔY are constant mesh sizes along X and Y directions respectively and taken as follows,

$$\Delta X = 0.20(0 \le x \le 20)$$

$$\Delta Y = 0.25(0 \le x \le 50)$$

with the smaller time-step, Δt=0.005.

Using the explicit finite difference approximation, the following appropriate set of finite differenceequations are obtained as;

$$\frac{U_j' - U_j}{\Delta\tau} = G_r\theta_j + G_m C_j + \frac{U_{j+1} - 2U_j + U_{j-1}}{(\Delta y)^2} - M(U_j - exp(a.j.\Delta t)) - \frac{U_j}{K} \tag{10}$$

$$\frac{\theta_j' - \theta_j}{\Delta\tau} = \frac{1}{P_r}\left(\frac{\theta_{j+1} - 2\theta_j + \theta_{j-1}}{(\Delta y)^2}\right) \tag{11}$$

$$\frac{C_j' - C_j}{\Delta\tau} = \frac{1}{S_c}\left(\frac{C_{j+1} - 2C_j + C_{j-1}}{(\Delta y)^2}\right) \tag{12}$$

Initial and boundary condition takes the following forms,

When $t \le 0$ then $U_j^0 = 0,\ \theta_j^0 = 0,\ C_j^0 = 0,$ for all y

When $t > 0$ then $U_j^0 = \exp(a.j.\Delta t),\ \theta_j^0 = t,\ C_j^0 = 1,$ for all $y = 0.$ (13)

$U_j^n = 0,\ \theta_j^n = 0,\ C_j^n = 0,$ as $y \to \infty$

IV. RESULTS AND DISCUSSION

It is very difficult to study the influence of all governing parameters involved in the present problem " Free convection and mass transform flow through a porous medium with variable temperature".Therefore,this study is noticed on the effects of governing parameters on the transient velocity,temperature as well as on the concentration profiles.To have a physical feel of the problem we exhibit results to how the material parameters of the problem affect the velocity and skin friction.The velocity profiles for various parameters are presented in the figure (1) to (12) for the cases of heating and cooling of the plate.

The effects of Magnetic parameter (M), Permeability parameter (K), Schmidt number (S_c), Thermal grashof number (G_r), Mass grashof number (G_m) and accelerated parameter (a) on the velocity profiles and skin frictions are discussed below:

Figure (1) and (2) discuss about the effects of M (Magnetic parameter) in case of cooling and heating of the plate at t=0.2 and t=0.4. It is found that the velocity decreases with increasing magnetic parameter (M) for the case of cooling of the plate. It is because that the presence of transverse magnetic field produces a resistive force similar to drag force which results to resist the fluid flow and that is the reason for reducing its velocity.The alternative effects is noticed in the case of heating of the plate.Figure (3) and (4) reveal the velocity variations with K (Permeability parameter) in case of cooling and heating of the plate at t=0.2 and t=0.4 respectively. It is found that the velocity increases with increasing permeability parameter (K) for the case of cooling of the plate.Because of the fact that the presence of porous medium increases the resistive flow. The reverse effects is noticed in the case of heating of the plate.

Figure (5) and (6) express the velocity variations with S_c (Schmidt number) in case of cooling and heating of the plate at t=0.2 and t=0.4 respectively. It is noticed that the velocity decreases with increasing Schmidt number (S_c) for the case of cooling of the plate.But the reverse effects is found in the case of heating of the plate.It is also found that for the case of heating of the plate the velocity profiles increases near the surface of the plate and become maximum and decreases far away from the surface.But the reverse effects is noticed in the case of cooling of the plate.

Figure (7) and (8) discussed the effects of G_r (Thermal grashof number) and G_m (Mass grashof number) in case of cooling and heating of the plate at t=0.2 and t=0.4.It is found that for the increasing of G_r (Thermal grashof number) and G_m (Mass grashof number) obtained the increases of velocity for the both cooling and heating of the plate.Figure (9) and (10) reveal the velocity variations with a (accelerated parameter) in case of cooling and heating of the plate at t=0.2 and t=0.4. It is observed that the velocity increases with increasing accelerated parameter (a) for the case of cooling and heating of the plate.

The skin friction is displayed in the figure (11) and (12). From the figure (11) it is clear that for the increasing of M (Magnetic parameter) skin friction decreases but for the incresing of K (Permeability parameter) skin friction increases. From the figure (12) it is observed that the increasing of a (accelerated parameter) resulting the decreases of skin friction.

In our present paper we have solved our problem by using explicit finite difference method as the solution tool whereas Rajesh V,[8] used Laplace transform method as the solution tool. We see that there is almost same agreement between our numerically calculated results and the pervious results of Rajesh, V [8].

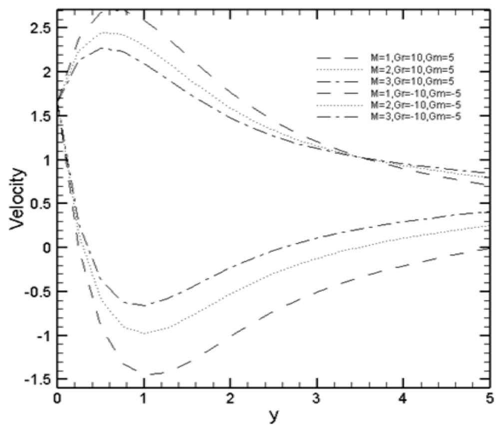

Figure 1: Velocity profiles for different values of M against y when, K=0.5, a=0.5, S_c=0.60, P_r=7.0 and t=0.2.

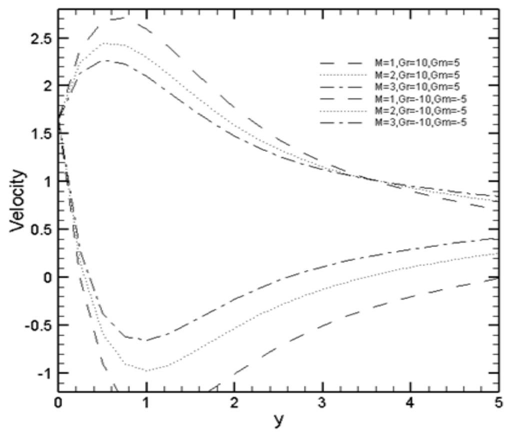

Figure 2: Velocity profiles for different values of M against y when, K=0.5, a=0.5, S_c=0.60, P_r=7.0 and t=0.4.

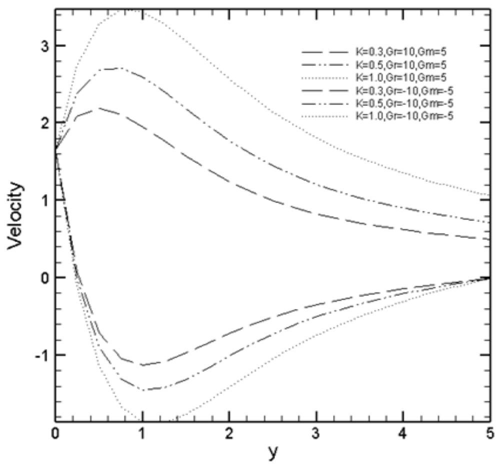

Figure 3: Velocity profiles for different values of K against y when, M=1.0, a=0.5, S_c=0.60, P_r=7.0 and t=0.2.

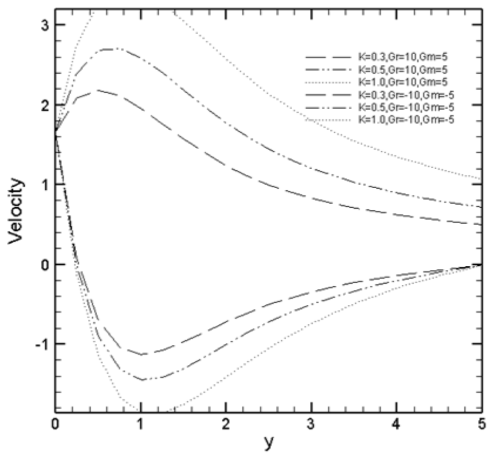

Figure 4: Velocity profiles for different values of K against y when, M=1.0, a=0.5, S_c=0.60, P_r=7.0 and t=0.4.

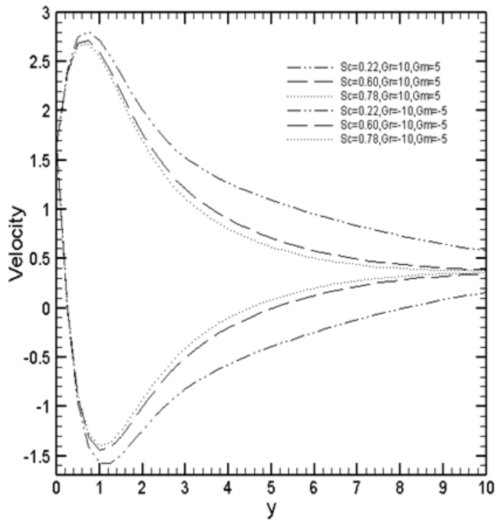

Figure 5: Velocity profiles for different values of S_c against y when, M=1.0, K=0.5, a=0.5, P_r=7.0 and t=0.2.

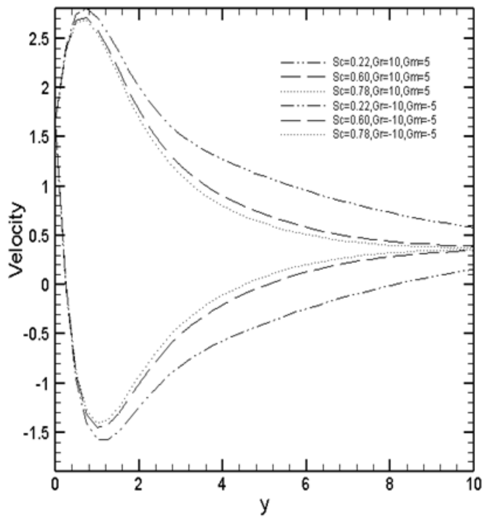

Figure 6: Velocity profiles for different values of S_c against y when, M=1.0, K=0.5, a=0.5, P_r=7.0 and t=0.4.

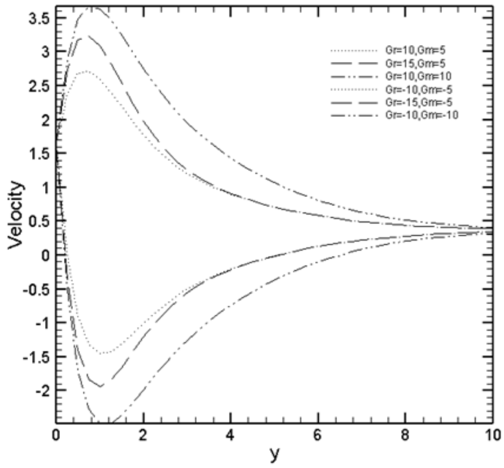

Figure 7: Velocity profiles for different values of G_r when, M=1.0, K=0.5, a=0.5, S_c=0.60, P_r=7.0 and t=0.2.

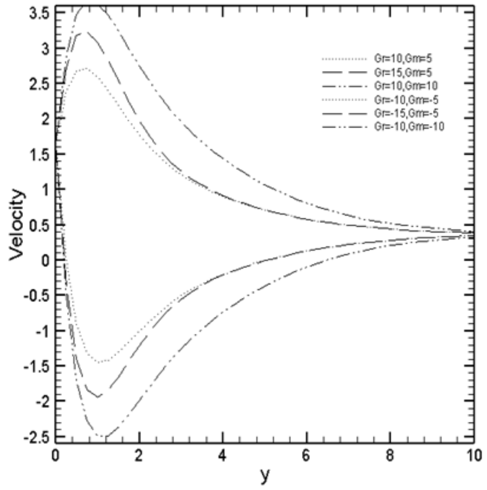

Figure 8: Velocity profiles for different values of G_r against y when, M=1.0, K=0.5, a=0.5, S_c=0.60, P_r=7.0 and t=0.4.

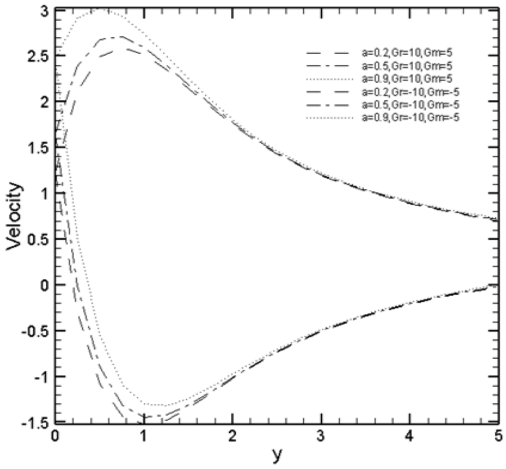

Figure 9: Velocity profiles for different values of a against y when, M=1.0, K=0.5, S_c=0.60, P_r=7.0 and t=0.2.

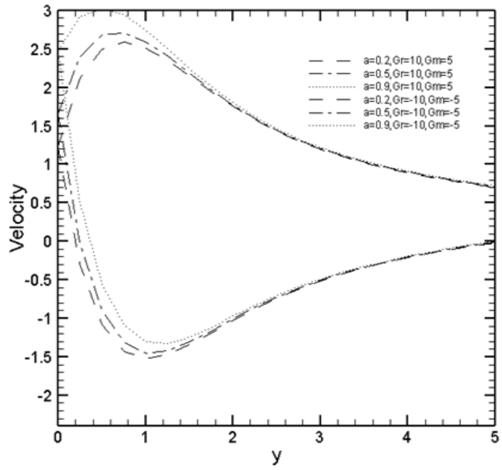

Figure 10: Velocity profiles for different values of a against y when, M=1.0, K=0.5, S_c=0.60, P_r=7.0 and t=0.4.

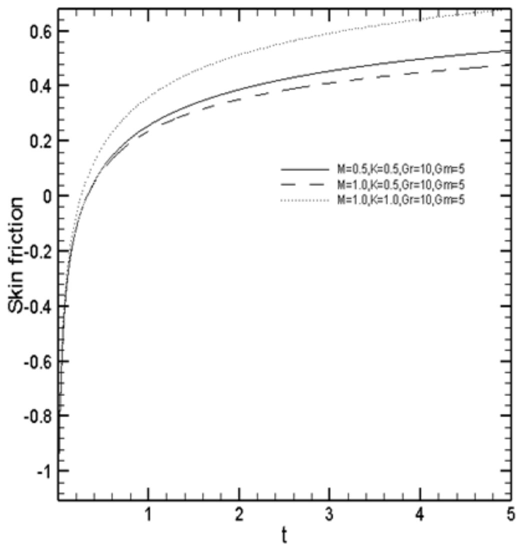

Figure 11: Skin friction for different values of M and K against t when, a=0.5, S_c=0.60 and P_r=7.0.

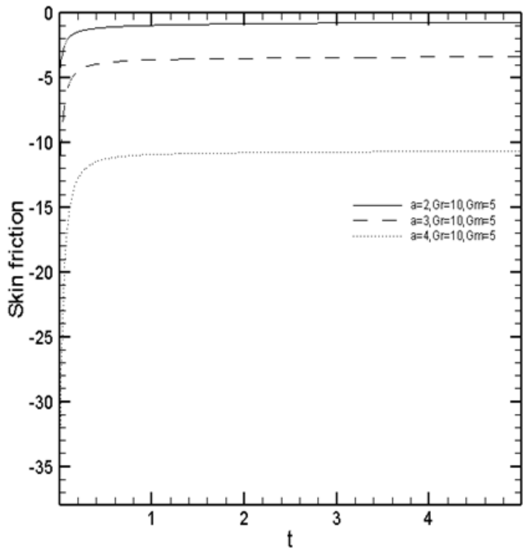

Figure 12: Skin friction for different values of a against t when, M=1, K=0.5, S_c=0.60 and P_r=7.0.

V. CONCLUSION

In this paper we have been studied the free convection and mass transform flow through a porous medium with variable temperature. Here we discussed the various effects of the physical parameter on the velocity. We are restricted in case of temperature and concentration. Because there is no effects of temperature and concentration on the fluid. Here we also discussed the effects of skin friction for different parameters.

- The velocity decreases with increasing Magnetic parameter (M) and Scmidth number (Sc) whereas the velocity profiles increases with increasing thePermeability parameter (K),Thermal grashof number (G_r), Mass grashof number (G_m) and accelerated parameter (a) in case of cooling of the plate.The reverse effect occurs in case of heating plate.
- The skin friction decreases with increasing of Magnetic parameter (M) and accelerated parameter (a)whereasskin friction increases with the increasing value of Permeability parameter (K).

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