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Influence Of Thermal Radiation On Magnetohydrodynamic (Mhd) Boundary Layer Flow Of A Viscous Fluid Over An Exponentially Stretching Sheet

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ABSTRACT: *Radiation on magnetohydrodynamic (MHD) boundary layer flow of a viscous fluid over an exponentially stretching sheet was considered together with it's effects. The new technique of homotopy analysis method (nHAM) was used to obtain the convergent series expressions for velocity and temperature, where the governig system of partial differential equations has been transformed into ordinary differential equations. The interpretation to these expressions is shown physically through graphs. We observed that the effects of Prandtl and Magnetic number acts in opposite to each other on the temperature.*

KEYWORDS: *Boundary-layer; heat transfer; MHD; radiation; stretching sheet*

I. INTRODUCTION

In many engineering processes today incompressible boundary layer flow due to an exponentially stretching sheet is useful in a good number of applications. Such applications includes the industrial manufacturing in the aerodynamic extrusion of plastic sheets, hot rolling, the boundary layer along a liquid film condensation process, cooling process of metal plate in a bath and in the polymer industries. It is seen that kinematics of stretching with both the simultaneous heating or cooling during this processes has great influence on the quality of the end products(Magyari and Keller, (1999)). The work of Sakiadis, (1961) was able to look into the stretching flow problem where from there Crane, (1970) became the first to study the boundary layer flow caused by a stretching sheet which accelerates with a velocity varying linearly with the distance from a fixed point. Carragher and Crane, (1982) investigated the heat transfer area under this problem, with the conditions that the temperature difference between the surface and the ambient fluid is proportional to a power of the distance from a fixed point. The steady boundary layer on an exponentially stretching continuous surface with an exponential temperature distribution was also discussed by Magyari, et al. (1999) . Effect of viscous dissipation on the mixed convection heat transfer from an exponentially stretching surface was studied by Partha, et al. (2005) . Sajid and Hayat, (2008) in recent time considered the radiation effects on the flow over an exponentially stretching sheet, where the problem was solved analytically using the homotopy analysis method. To deal with the problems such as cooling of nuclear reactors by liquid sodium and induction flow meter, which depends on the potential difference in the fluid in the direction perpendicular to the motion and to the magnetic field(Ganesan and Palani, (2004)), MHD has important applications. Various processes in engineering areas occur at high temperature where radiation heat transfer becomes of great importance in the design of partinent equipments(seddeek, (2002)). Anuar, (2011) studied the MHD boundary layer flow on an exponentially stretching sheet taking the velocity gradient in the energy equation to be zero. The motivation to this present work is the variation of velocity gradient on the problem of MHD boundary layer flow over an exponentially stretching sheet in the presence of radiation where the velociy gradient in the energy equation not zero, which has not been studied.

II. PROBLEM FORMULATION

Consider the two-dimensional flow of an incompressible, steady viscous fluid bounded by a stretching sheet and conducted ellectrically which is placed in a fluid of uniform temperature T_∞, given in Fig 1. with magnetic field B(x) applied normal to the sheet and the induced magnetic field neglected,which is justified for MHD flow at small magnetic Raynold number.Under the usual boundary layer approximations,the flow and heat transfer with the radiation effects are governed by the following equations:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_o^2 u}{\rho} \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\kappa}{\rho C_p}\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho C_p}(\frac{\partial u}{\partial y})^2-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} \tag{3}$$

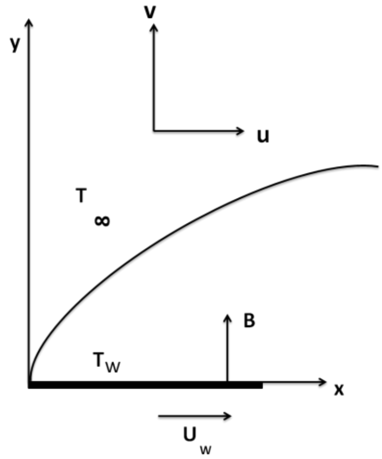


Fig 1: physical Model and Coordinate System

where u and v are the velocities in the x- and y-directions respectively, ρ is the fluid density, ν the kinematic viscosity, μ is the dynamic viscosity, κ the thermal conductivity, C_p the specific heat,T the fluid temperature in the boundary layer and q_r is the radiation heat flux.The boundary conditions are given by:

$$\left.\begin{array}{c} u=U_w=U_\circ exp(\frac{x}{l}),\ v=0, \\ T=T_w=T_\infty+T_\circ exp(\frac{x}{2l})\ \ at\ \ y=0, \\ u\to 0, T\to T_\infty\ \ as\ \ y\to\infty \end{array}\right\} \tag{4}$$

where $U_\circ$ is the reference velocity, $T_\circ$ and T_∞ are respectively the temperature at and far from the plate and L the reference length. Understanding fluid radiation is devoted to the derivation of reasonable simplifications(Aboeldahab and El Gendy(2002)).One of these simplifications was made by Cogley, et al. (1968) who assumed that the fluid does not absorb it's own radiation,but,it only absorbs radiation emitted by the boundaries.Hence,the problem can be simplified by using the Rosseland approximation (Rosseland 1936 ;Siegel and Howell 1992 ;Sparrow and Cess 1978)which simplifies the radiation heat flux as:

$$q_r = -\frac{4\sigma^*}{3\kappa^*}\frac{\partial T^4}{\partial y} \qquad (5)$$

where σ^* and κ^* are the Stefan-Boltzmann constant and the mean absorption coefficient respectively.This approximation is valid at points optically far from the boundary surface and it is good only for intensive absorption which is far for an optically thick boundary layer(Bataller 2008 ;Siegel and Howell 1992 ;Sparrow and Cess 1978). Assumed that the temperature differences within the flow such that the term T^4 may be expressed as a linear function of temperature.Expanding T^4 in a Taylor series about T_∞ and neglecting higher order terms gives:

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4 \qquad (6)$$

Using Equations (5) and (6), Equation (3) reduces to:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho C_p}(\frac{\partial u}{\partial y})^2 + \frac{16T_\infty^3\sigma^*}{\rho C_p 3k^*}\frac{\partial^2 T}{\partial y^2} \qquad (7)$$

To get the similarity solutions, we assumed that the magnetic field $B(x)$ is of the form:

$$B = B_\circ exp(\frac{x}{2L}) \qquad (8)$$

where $B_\circ$ is the constant magnetic field.

Equation (1) is satisfied by introducing a stream function ψ such that:

$$u = \frac{\partial \psi}{\partial y} \;\; and \;\; v = -\frac{\partial \psi}{\partial x} \qquad (9)$$

Equation (2) and Equation (3) are transformed into the corresponding ordinary differential equations by the following transformation(Sajid and Hayat 2008):

$$\left.\begin{aligned} u = U_\circ exp(\frac{x}{L}) f'(\eta),\; v = -(\frac{\nu U_\circ}{2L})^{\frac{1}{2}} exp(\frac{x}{2L})(f(\eta) + \eta f'(\eta)) \\ T = T_\infty + T_\circ exp(\frac{x}{2L})\theta(\eta),\; \eta = (\frac{U_\circ}{2\nu L})^{\frac{1}{2}} exp(\frac{x}{2L}) y \end{aligned}\right\} \qquad (10)$$

where η is the similarity variable, $f(\eta)$ is the dimensionless stream function, $\theta(\eta)$ is the dimensionless temperature and prime denotes differentiation with respect to η. The transformed ordinary differential equations are:

$$f''' + ff'' - 2f'^2 - Mf' = 0 \qquad (11)$$

$$[1+\frac{4K}{3}]\theta''(\eta)+P_r[\theta'(\eta)f(\eta)-\theta(\eta)f'(\eta)+Ef''^2(\eta)]=0 \qquad (12)$$

Where $M=\frac{2\sigma B_\circ^2 L}{\rho U_\circ}$, $K=\frac{4\sigma^* T_\infty^3}{kk^*}$, $P_r=\frac{\mu C_p}{k}$ and $E=\frac{U_\circ^2}{T_\circ C_p}$ are repectively the Magnetic, Radiation, Prandtl and Eckert parameter respectively.

The transformed boundary conditions are:

$$\left.\begin{array}{l} f(0)=0,\ f'(0)=1,\ \theta(0)=1 \\ f'(\eta)\to 0,\ \theta(\eta)\to 0\ \ as\ \ \eta\to\infty \end{array}\right\} \qquad (13)$$

nHAM SOLUTION

In order to solve Equations (11) - (13) using nHAM, assume that $f''(0)=\alpha$ and $\theta'(0)=\beta$, we construct system of differential equations, as follows:

(14)

$$f'(\eta)=v$$

$$v'(\eta)=w$$

$$w'(\eta)=2f'^2-ff''+Mf'$$

}

with intial approximations

$$f_\circ(\eta)=0, v_\circ(\eta)=1, w_\circ(\eta)=\alpha \qquad (15)$$

the auxiliary linear operators are

$Lf(\eta)=\frac{\partial f}{\partial \eta}, Lv(\eta)=\frac{\partial v}{\partial \eta}, Lw(\eta)=\frac{\partial w}{\partial \eta}$,where L is an auxiliary linear operator.

and

(16)

$$\theta'(\eta)=C$$

$$C'(\eta)=-\frac{P_r}{1+\frac{4K}{3}}[f\theta'-f'\theta+Ef''^2]$$

}

using initial approximations

$$\theta_\circ(\eta)=1,\ C_\circ(\eta)=\ \beta \qquad (17)\backslash$$

and the auxiliary linear operators are

$$L\theta(\eta) = \frac{\partial \theta}{\partial \eta}, \ LC(\eta) = \frac{\partial C}{\partial \eta} \tag{18}$$

we then have

(19)

$$f_1(\eta) = \hbar_1 \int_0^\eta [-v_\circ(\eta)] d\eta$$

$$v_1(\eta) = \hbar_1 \int_0^\eta [-w_\circ(\eta)] d\eta$$

$$w_1(\eta) = \hbar_1 \int_0^\eta [-Mv_\circ - 2v_\circ^2 + f_\circ w_\circ] d\eta$$

}

and

(20)

$$\theta_1(\eta) = \hbar_2 \int_0^\eta [-C_\circ(\eta)] d\eta$$

$$C_1(\eta) = \frac{P_r \hbar_2}{1 + \frac{4K}{3}} \int_0^\eta [f_\circ C_\circ - f_\circ' \theta_\circ + E f_\circ''^2]$$

}

For $m \geq 2$,

(21)

$$f_m(\eta) = (1+\hbar_1) f_{m-1}(\eta) + \hbar_1 \int_0^\eta [-v_{m-1}(\eta)] d\eta$$

$$v_m(\eta) = (1+\hbar_1) v_{m-1}(\eta) + \hbar_1 \int_0^\eta [-w_{m-1}(\eta)] d\eta$$

$$w_m(\eta) = (1+\hbar_1) w_{m-1}(\eta) + \hbar_1 \int_0^\eta [-Mf_{m-1}' + \sum_{i=0}^{m-1} (-2 f_{m-1-i}' f_i' + f_{m-1-i} f_i'')] d\eta$$

}

and

(22)

$$\theta_m(\eta) = (1+\hbar_2) \theta_{m-1}(\eta) + \hbar_2 \int_0^\eta [-C_{m-1}(\eta)] d\eta$$

$$C_m(\eta) = (1+\hbar_2)C_{m-1}(\eta) + \frac{P_r\hbar_2}{1+\frac{4K}{3}}\int_0^{\eta}\sum_{i=0}^{m-1}[f_{m-1-i}\theta_i' - f_{m-1-i}'\theta_i + Ef_{m-1-i}''f_i']d\eta$$

}

The systems of Eqs. (19) - (22) have been solved using symbolic computation software MAPLE. It is found that

(23)

$$f_1(\eta) = -\hbar_1\eta$$

$$v_1(\eta) = -\hbar_1\alpha\eta$$

$$w_1(\eta) = \hbar_1(-2\eta - M\eta)$$

}

(24)

$$f_2(\eta) = -(1+\hbar_1)\hbar_1\eta + \frac{1}{2}\hbar_1^2\alpha\eta^2$$

$$v_2(\eta) = -(1+\hbar_1)\hbar_1\alpha\eta - \frac{1}{2}\hbar_1^2(-2-M)\eta^2$$

$$w_2(\eta) = (1+\hbar_1)\hbar_1(-2\eta - M\eta) + \frac{1}{2}(M\hbar_1\alpha + 3\hbar_1\alpha)\eta^2$$

}

and

(25)

$$\theta_1(\eta) = -\hbar_2\ \beta\eta$$

$$C_1(\eta) = -\frac{P_r\hbar_2\eta}{1+\frac{4K}{3}}$$

}

(26)

$$\theta_2(\eta) = -(1+\hbar_2)\hbar_2\eta\ \ \beta + \frac{1}{2}\frac{\hbar_2^2P_r\eta^2}{1+\frac{4K}{3}}$$

$$C_2(\eta) = -\frac{(1+\hbar_2)P_r\hbar_2\eta}{1+\frac{4K}{3}} + \frac{0.50P_r\hbar_2(0.70\ \beta+\hbar_2\ \beta)\eta^2}{1+\frac{4K}{3}}$$

}

$f_m(\eta,\alpha;\hbar_1)(m=3,4,5,...)$ and $\theta_2(\eta,\beta;\hbar_2)(m=3,4,5,...)$ can be calculated similarly.Then the series solution expressions by nHAM can be written in the form

(27)

$$F_Q(\eta,\alpha,\hbar_1) = \sum_{m=0}^{Q} f_m(\eta,\alpha,\hbar_1)$$

(28)

$$\theta_N(\eta,\beta,\hbar_2) = \sum_{m=0}^{N} \theta_m(\eta,\beta,\hbar_2)$$

we note that analytic expressions (27) and (28) cantains two auxiliary parameters $\hbar_1$, and $\hbar_2$ as suggested by Liao (1992) and (2003)one can choose the values of $\hbar_1$ and $\hbar_2$ properly for $\hbar$ -curves which ensure the convergence of the series solutions.Using the boundary condition $f' \to 0\ as\ \eta \to \infty$ and $\theta \to 0\ as\ \eta \to \infty$ we get $\alpha = 0\ for\ M = 0$ and also $\beta = 0\ for\ K = 0, M = 0, P_r = 0, E = 0$.

fig 2: The $\hbar_1$ curve

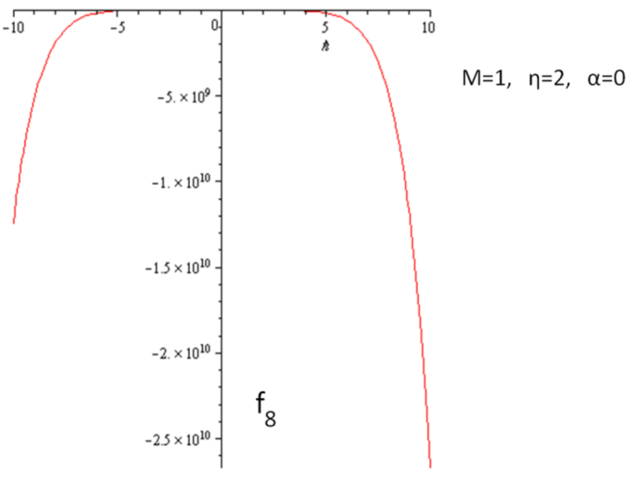
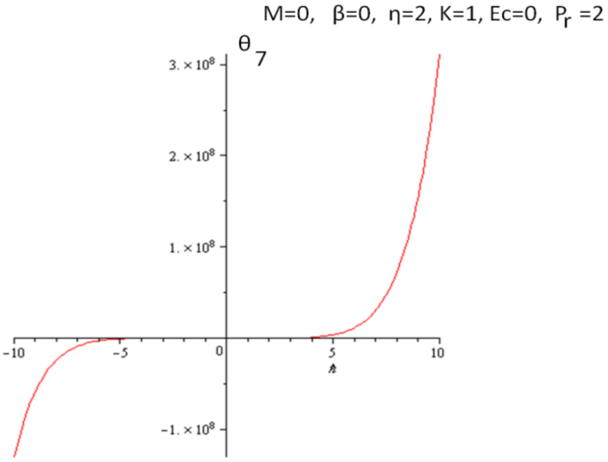


fig 3: The $\hbar_2$ curve



In figs. 2 and 3 the $\hbar$-curves are shown for the range of admissible values of $\hbar_1$ and $\hbar_2$. Figs. 2 and 3 clearly indicate that the ranges for the admissible values of $\hbar_1$ and $\hbar_2$ are $[-5.6,4.2]$ and $[-5.1,3.1]$.Our calculations show that the series solution (27) and (28) converge in the whole region of η, when $\hbar_1 = -0.7$, $\hbar_2 = -1$.

III. RESULTS AND DISCUSSION

The system of ordinary differential equations (11)-(13) whigh have been solved numerically using nHAM as described by Hassan and El-tawil (2012).This method has been used to solve several boundary layer problems,we show the graphical results of velocity and temperature.Attention has been focused to the variations of P_r, M, E *and* K. For this purpose Figs. $4-8$ have been displayed. Fig. 4 shows the effect of magnetic number on velocity $f'(\eta)$. Figs. $5-8$ elucidate the influence of Radiation number K, Magnetic number M, Prandtl number P_r and Eckert number E on the temperature $\theta(\eta)$. From the present study, the main findings can be summarized as follows:

- Increase in magnetic parameter M have accelerating effect on velocity of the flow field.
- Increase in radiation parameter K,magnetic parameter M and Eckert number Ec retard the magnitude of temperature of the flow field and the thickness of thermal boundary layer.
- Increase in the prandtl number P_r shows that there is a rise in the magnitude of temperature of the flow field and the thickness of thermal boundary layer.

fig 4: The Effect of Magnetic Parameter, M on Velocity, f' for $\alpha = 0, \hbar = -0.7$

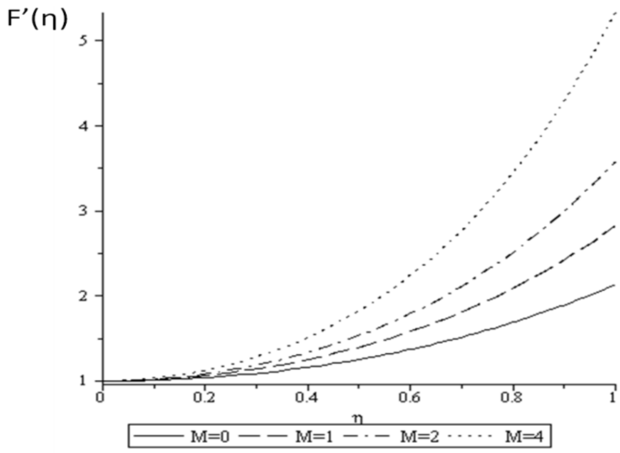


fig 5: The Effect of Radiation Parameter, K on Temperature, θ for $P_r = 1, Ec = 0.2, M = 1$

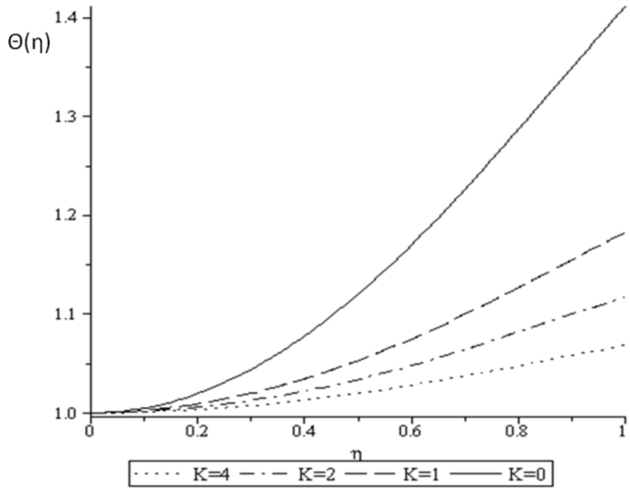


fig 6: The Effect of Magnetic parameter , M on Temperature, θ for $P_r = 1, Ec = 0.2, K = 1$

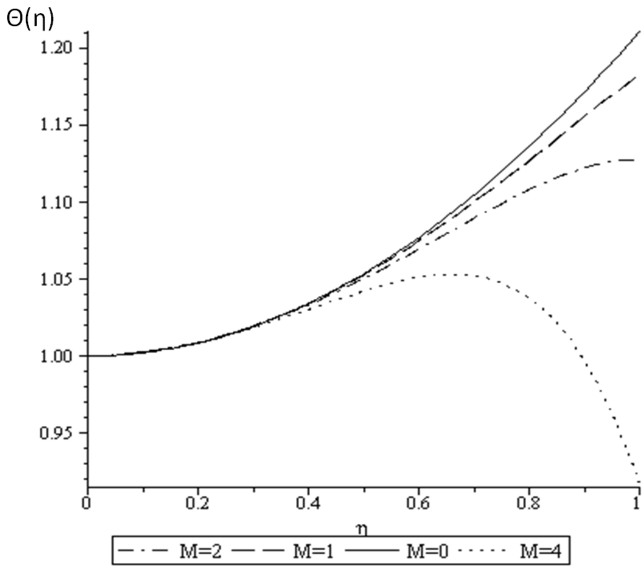


fig 7: The Effect of Prandtl Number, P_r on Temperature, θ for $K=1, Ec=0.2, M=1$

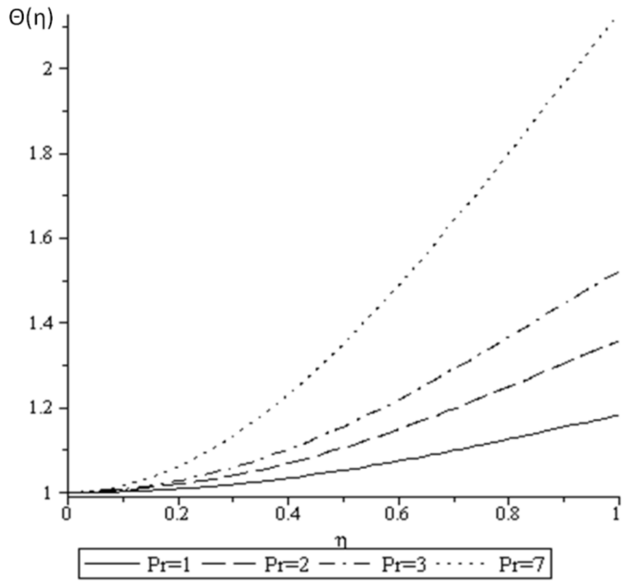
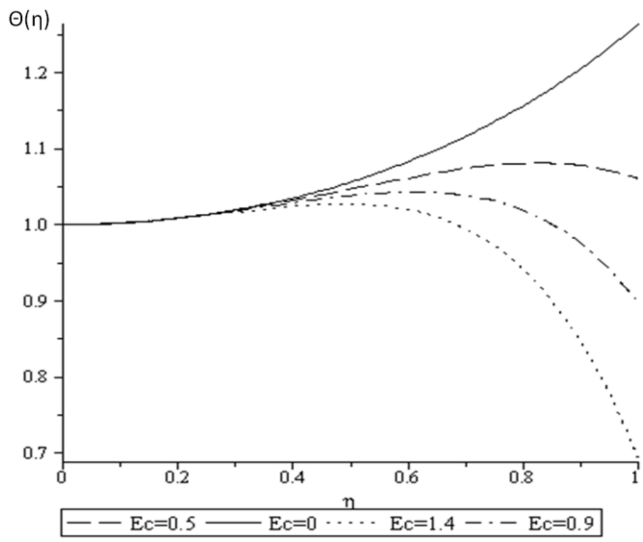


fig 8: The Effect of Eckert Number, Ec on Temperature, θ for $K=1, P_r=1, M=1$



IV. CONCLUSIONS

Radiation on steady MHD boundary layer flow over an exponentially stretching sheet was investigated and it's effects observed. The similarity transformations are used to reduce the partial differential equations into ordinary differential equations. Analytical solutions for the velocity and temperature distributions are obtained using a nHAM. It was found that the heat rate increases with prandtl number P_r, but decreases with both magnetic parameter M and radiation parameter K. Thus Magnetic and Radiation parameter brought about cooling effect on the sheet, because the higher the increase in the parameter reduces the heat rate.

REFERENCES

[1]. Anuar I. (2011): MHD Boundary Layer Flow Due to an Exponentially Stretching Sheet with Radiation Effect.Sains Malaysiana. $40(4):391-395$.

[2]. Bataller, R.C. (2008): Similarity solutions for boundary layer flow and heat transfer of a FENE-P fluid with thermal radiation.Phys. Lett.A $372:2431-2439$.

[3]. Bidin, B. and Nazar, R. (2009): Numerical solution of the boundary Layer Flow over an Exponentially Stretching Sheet with Thermal Radiation.Euro.J.Sci. $33(4):710-717$.

[4]. Carragher, P. and Crane, L.J. (1982): Heat Transfer on a Continuous Streeting Sheet. Z.Angew.Math.Mech. $62:564-573$.

[5]. Crane, L.J. (1970): Flow Past a Stretching Plate",Z.Angew.Math. mech. $21:645-647$.

[6]. Ganesan, P. and Palani, G. (2004): finite Difference Analysis of Unsteady Natural Convection MHD Past an Inclined Plate with variable Surface Heat and Mass Flux. Int.J.Heat Mass Transfer. $47:4449-4457$.

[7]. Hany, N.H. and Magdy, A.E. (2012): A new Technique of Using Homotopy Analysis Method for Second Order Nonlinear Differential Equation. Appl.Math.Comput. $219:708-728$.

[8]. Hany, N.H. and Magdy, A.E. (2010): A new Technique of Using Homotopy Analysis Method for Solving High-Order Nonlinear Differential Equations. Wiley online Libery.

[9]. Hayat, T. and Sajid, M. (2008) Analyti Solution for Axisymmetric Flow and Heat transfer of a Second Grade Fluid Past a Stretching Sheet. Int.J.Heat Mass Transfer. $50:75-84$.

[10]. Liao, S.J. (1992): The proposed Homotopy Analysis Technique for the Solution of Nonliear Problems. Ph.D. Thesis.

[11]. Liao, S.J. (2003): Beyond Perturbation:Introduction to Homotopy Analysis Method. Chapman and Hall/CRC Press.

[12]. Magyari, E. and Keller, B. (1999): Heat and Transfer in the Boundary Layer on an Exponentially Stretching Continuous Surface. J.phys.D.Appl.Phys. $32:577-585$.

[13]. Partha, M.K.,Murthy, P.V.S.N. and Rajasekhar, G.P. (2005): Effect of Viscous Dissipation on the Mixed Convection Heat transfer from an Exponentially Stretching Surface. Heat Mass Transfer. $41:360-366$.

[14]. Raptis, A. (2004): Effect of Thermal Radiationon MHD Flow. Appl.math.Comput. $32:577-585$

[15]. Sajid, I. and Hayat, T. (2008): Influence of Thermal Radiation on the Boundary Layer Flow Due to an Exponentially Stretching Sheet. Int. Comput. in Heat Mass Transfer. $35:347-356$.

[16]. Sakiadis, B.C. (1961): Boundary Layer bahaviour on Continous Solid Surface:I Boundary Layer Equation for two Dimensional and Axisymmetric Flow. AIChE J. $7:26-28$.

[17]. Seddeed, M.A. (2002): Effect of Radiation and Variable Viscousity on a MHD Free Convection Flow Past a Semi-Infinite Flat Plate with an Aligned Magnetic Field in the Case of Unsteady Flow. Int.J.Heat Mass Transfer. $45:931-935$.

[18]. Siegel, R. and Howell, J.R. (1992): Thermal Radiation: Heat Transfer $3rd$ ed. Washington DC: Hemisphere.

[19]. Sparrow, E.M. and Cess, R.D. (1978): Radiation Heat Transfer Washinton DC: Hemisphere.