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Investigating the transitional state between circular plates and shallow spherical shells

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ABSTRACT: *The stiffness of circular plates can be increased by inducing a rise at the center of these plates; this rise converts the circular plates from two-dimensional stiffness elements into three-dimensional stiffness elements. This slight change in the geometry shifts the state of stresses from mainly bending stresses to tensile-compressive stresses. The rise at the center of a circular plate is increased gradually to the point where a shell element is formed. This paper focuses on this particular transition between the plate elements to the shell element which is called the transitional rise. Several finite element models were used to identify the transitional rise given fixed parameters. Stresses and deflections are also studied for each case. An optimized approach was used to minimize the cost and improve the serviceability of the structural elements. In this present study, numerous analyses were conducted using the finite element methodology on shell model. Finite element mesh was established for each different rise value starting from zero (plate). The rise is increased gradually to the point where shell stiffness becomes insensitive to the increase in the rise. An empirical relationship was established between the transition state of the plate/shell elements and relevant geometrical parameters. Parametric study is also conducted using several loading cases.*

Keywords: *Spherical shells, circular plates, transition state, optimization*

I. INTRODUCTION

Plates and shells have several important civil engineering applications. They are used as structural elements, roofs, domes, and water tanks. Also for industrial purposes plates and shells are used extensively in many applications including machines, gas vessels and cans. Introducing a rise at the center of a circular plate will increase the stiffness of the plate. As the rise becomes large enough, the plate will change into a shallow spherical shell, and consequently the strength of the plate increases substantially to a point where the rate of increase in strength becomes insignificant. This particular rise (apex) is referred to here as the Transitional Rise ɳ. This study also focused on the other parameters that may have any effect on the transitional rise ɳ.

Many researchers explored the difference between plates and shells, but most of their work produced solutions for stresses and deformations for symmetrically loaded plates and shells. The finite element method nowadays becomes widely used and much commercial software became available in the market. The new advances in computer hardware especially the processing power made it much easier to use the finite element method in engineering applications.

AlNasra conducted several experimental tests on reinforced concrete square plates loaded at the center. The central deflection is measured at each load level until failure. The plates showed concaved shapes before failure. The plates eventually failed by punching shear [1, 2]

In order to build a circular roof, there are two major choices; a circular plate or a spherical shell in the shape of a dome. Taking into consideration many factors one may go for a plate which might be characterized as an element of large thickness that uses more material, exhibiting large deflection under loads, and consequently generating cracks. The other available choice is to use a full-height spherical shell which is characterized by relatively thin element covering large space. Nevertheless, the construction cost of a spherical roof is higher than the plate. Developing a small rise in the center of the plate, one can take advantage of both plates and shells. A small rise at the middle of a circular plate can be induced making a concaved shape. This rise can change the state of stresses and the serviceability of the structural element. The finite element method can be used to determine the values of stresses and deflection at a given value of the rise and under given load case. A gradual increase in the rise makes the plate behaves like a shell. The value of the rise that makes the plate becomes a full blast shell is called the transitional rise based on the calculated values of stresses and deformations

Circular plates and spherical shells were studied intensively in the past using either numerical methods or experimental techniques subjected to different cases of loading. The bending solution of simply supported circular plates with uniformly distributed load produces the max deflection at the middle of the plate as follows [3, 4, 5, 6, 7]:

$W_{max} = (P_o a^4 (5+\nu)) / (64 D (1 + \nu))$ (1)

Where,

P_o = uniformly distributed load

a= radius of plate

ν = Poison's ratio

D = flexural rigidity of plate.

$D = Et^3 / (12 (1 - \nu^2))$ (2)

Where,

E = modulus of elasticity

t = thickness of plate

Then

$Po = [(64 D (1+\nu)) / (a^3 (5 + \nu))] (w_{max}/a)$ (3)

Equation 3 relates the uniformly distributed load with the deflection at the center of a plate in what is known as small deflection theory. Several researchers also worked on the derivation of the large deflection theory of plates. The derivation is based on simply supported circular plates which in turns helped in understanding the post bucking behavior of circular plates. Equation (4) relates the uniformly distributed load on a simply supported circular plate with the total deflection at the center of the plate [8, 9, 10].

$$P_o = \frac{8}{3}\frac{E}{(1-\nu)}\frac{t}{a}\left(\frac{W_{max}}{a}\right)^3 + \frac{64D\,(1+\nu)}{a^3(5+\nu)}\frac{W_{max}}{a} \quad (4)$$

As a case study, a plot of the applied uniformly distributed load Po and the deflection equations is shown in Fig. (1) for the following constants:

E= 200,000 MPa

ν = 0.3

a=1000 mm

t=10.0mm

Figure (1) shows that the first portion of the curves can be used satisfactorily to calculate the central deflection by the small deflection theory (SDT). As the central deflection increases the small deflection theory becomes inappropriate while the large deflection theory (LDT) can give satisfactory results at any central deflection range. The early portion of this curve will be used to study the plate behavior. The remainder of the curve shows rapid divergence between the large deflection theory (LDT) and the small deflection theory (SDT). Another point than can be made out of these curves, that the stiffness of the plate increases at a higher rate as the maximum central deflection increases, a preparation for shell behavior. Figure (2) shows the relationship between $1/P_o$ and the maximum central deflection $W_{max.}$ The first portion of the curve represents the plate behavior, and the rest represent the shallow shell behavior. A tangent can be drawn out of the first portion of the curve, which intersects the tangent drawn out of the second portion of the curve at a point denoted by ɳ as shown. The intersection of these two tangents is taken as the point of shifting behavior between plate and shallow shell, at which the rise is called the transition rise (TR). .

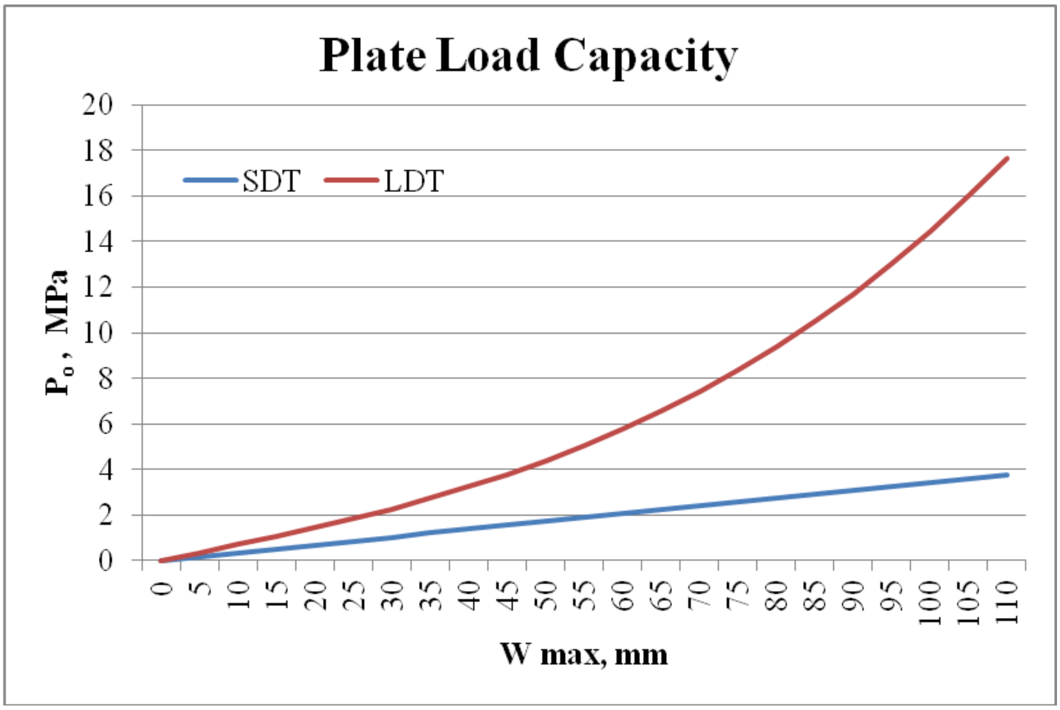

Figure 1: Load deflection relationship of simply supported plate.

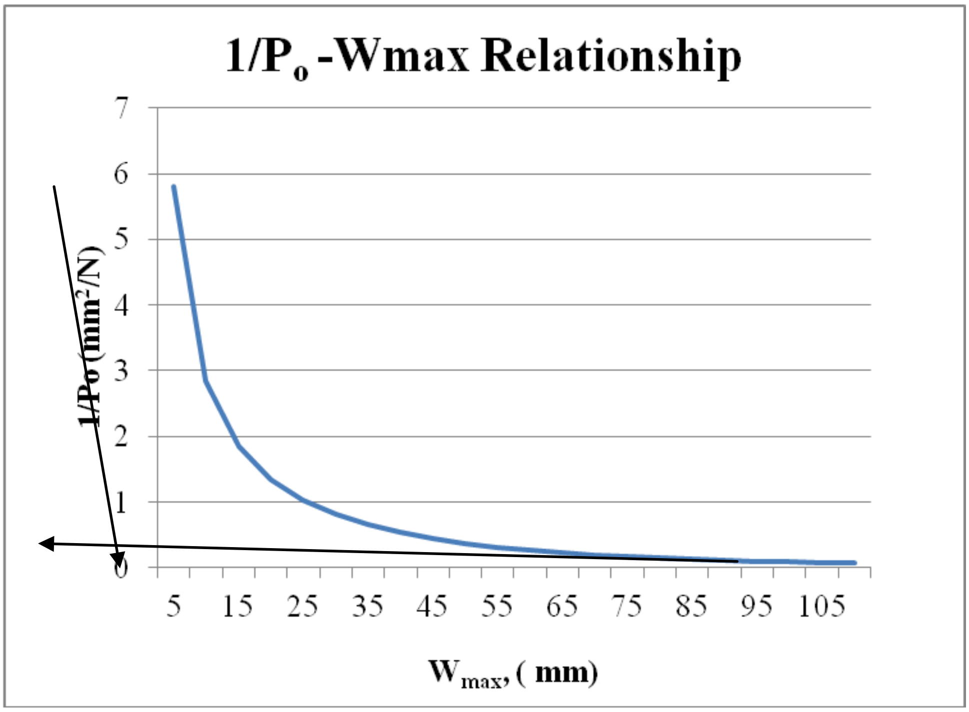

Figure 2: Locating the transition rise

II. PLATE-SHELL THEORY

The finite element method is used to measure the deflection as the applied load increases. This method measures the stresses and deflection with appropriate accuracy. Plates and shells elements can be brick eight elements, rectangular elements, triangular elements or trapezoidal elements. At all cases the equilibrium and compatibility must always be satisfied in the analysis of the finite element model. Several finite element software are widely available and can be easily used with acceptable accuracy. For this study STAAD program was used. Figure (3) shows typical finite element model for plates and shells used. The plate/shell is divided into circles of equal spacing. The number of circles and sectors were selected proportionally to give meaningful outputs. The accuracy of the output depends on the number of segments used. Common rules that are widely acceptable among researchers when using this type of elements are; element aspect ratio should not be excessive, and each individual element should not be distorted. The preferable aspect ratio is 1:1 but it should not exceed 4:1. Also angles between any two adjacent sides within any element should be around 90 degrees but less than 180 degrees [11].

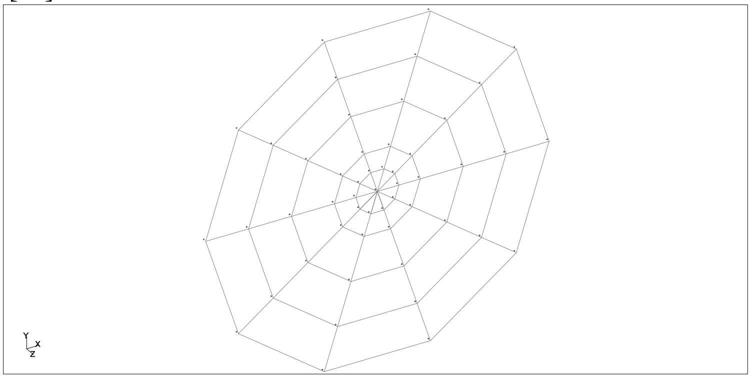

Figure 3: Finite element typical mesh

The types of supports used in this research are the ones that allow horizontal translation and rotation, while the vertical movement is restrained. This type of support suits the boundary condition of the membrane element, which is responsible of carrying the major part of the load in the large deflection theory. For analytical purposes, plates and shells used in this study are made of concrete, making the output easier to compare with experimental results [12, 13].

It was convenient to start the analysis with plate of zero height of apex. The height of apex is increased gradually by two centimeter increment until the change in deflection due to applied load reached a well specified tolerance range. Models of three, four, five, six, seven and eight meter diameter plates were used. The thickness-diameter ratio in the order of 0.010, 0.012, 0.014, 0.016, 0.018, 0.020, 0.022, and 0.024 were also used for each plate/shell diameter. In other words, for the plate of 3 m diameter, 8 different thicknesses of 6 mm increment were considered (30.0, 36.0, 42.0, 48.0, 54.0, 60.0, 66.0, and 72.0 mm).

Four different type of loading were used in this study, including uniformly distributed pressure, concentrated load at the center of the plate/shell element, ring loads, and hydrostatic pressure load as shown in Fig.(4). Each type of loading is used within the finite element model. Loads were increased gradually in some specific cases.

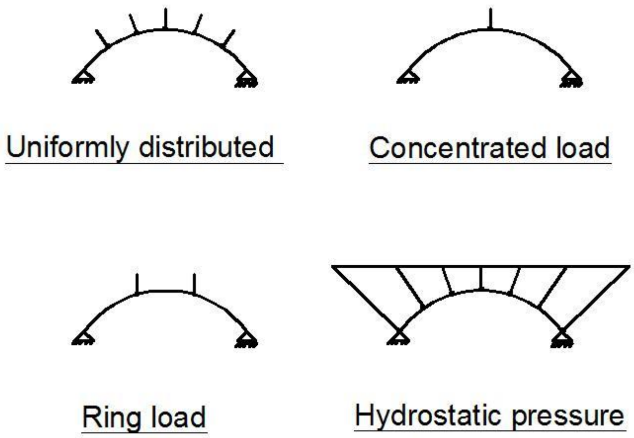

Figure 4: Load cases used in this study

For analytical purposes, the level of stress in the plate/shell element was controlled and bounded by the modulus of rupture of concrete. The concrete compressive strength is taken here to be 30 MPa, and the modulus of rupture for this type of concrete is taken as 4.3 MPa according to the local codes and confirmed by experimental data. In the case of the uniformly distributed load, the maximum stress can be calculated according to the following formula:-

$$\sigma_{r-max} = \sigma_{\theta-max} = \frac{3(3+\vartheta)}{32} Po\left(\frac{D}{t}\right)^2 \qquad (5)$$

Large number of data were generated and analyzed. Several graphs were generated too. Only small portion of the results will be presented in this study. Some typical results will be shown. Figure 5 shows a typical rise – central deflection relationships. This graph shows also that the transition rise to be 66.1 mm at which the plate element will shift behavior to shell element. This graph is developed for uniformly distributed load, diameter of 3 m, and thickness – diameter ratio of 0.010.

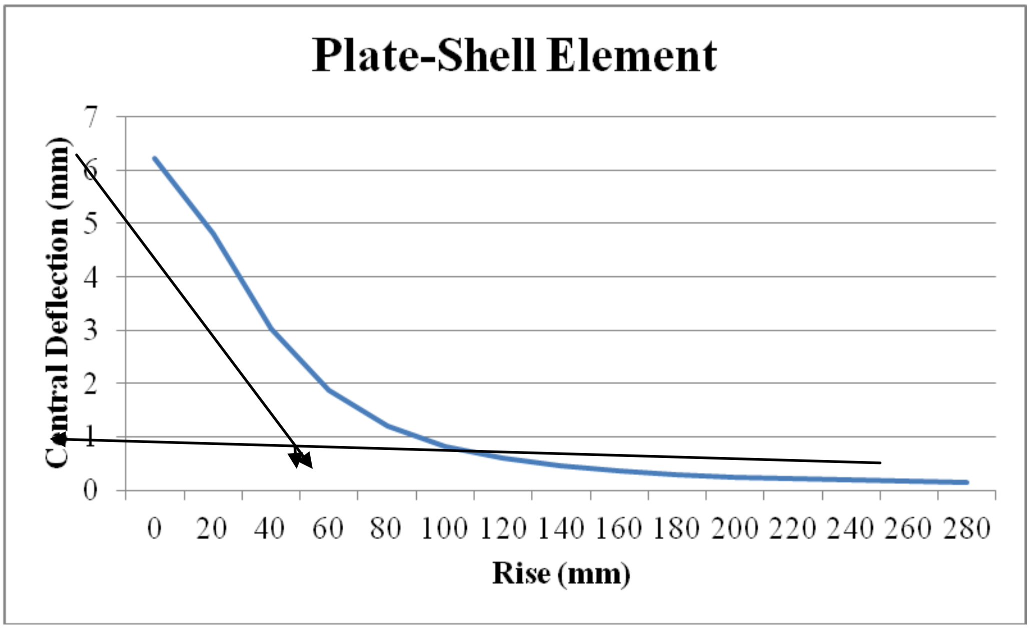

Figure 5: Typical rise – central deflection relationship

III. EFFECT OF LOAD INTENSITY ON THE TRANSITION RISE

Table 1 shows the effect of the applied load on the transition rise. The other parameters are kept constant in order to draw a meaningful conclusion. All of the plates/shells used in this particular part are of 3 m diameter, and thickness of 30 mm. The table also shows that the increase in the applied load has no effect on the value transition rise. Each unique load type has its own value of transition rise.

Table 1: Transition rise values

Load (KN)	Load Type		
	Concentrated Load	Ring Load	Water Pressure Load
1.0	66.8	64.6	63.4
1.5	66.8	64.6	63.4
2.0	66.8	64.6	63.4
2.5	66.8	64.6	63.4
3.0	66.8	64.6	63.4
3.5	66.8	64.6	63.4

IV. EFFECT OF THE PLATE/SHELL THICKNESS ON THE VALUE OF THE TRANSITION RISE

Several hundreds of cases were studied and analyzed in order to draw a meaningful conclusion that relates the variation of the plate/shell thickness with the value of the transition rise. Table (2) shows a summary of these findings. Table (2) shows only a sample of the data collected for a given plate/shell diameter of 3.0 m loaded by uniformly distributed load type. The same type of data were collected for other 5 different diameters considered in this study for each different load type, a total of 24 similar tables of 192 different load cases.

Table 2: Effect of plate/shell thickness on the value of the transition rise

Thickness-Diameter ratio (t/D)	Thickness (mm)	Transition Rise (mm)	Transition Rise – Diameter ratio (TR/D)	Transition Rise – Thickness ratio (TR/t)
0.010	30.0	66.1	0.022033	2.203333
0.012	36.0	77.9	0.025967	2.163889
0.014	42.0	88.1	0.029367	2.097619
0.016	48.0	102.4	0.034133	2.133333
0.018	54.0	115.1	0.038367	2.131481
0.020	60.0	131.8	0.043933	2.196667
0.022	66.0	144.5	0.048167	2.189394
0.024	72.0	159.5	0.053167	2.215278

Table (2) also shows that the value of the transition rise – plate/shell thickness ratio does not vary significantly, and considered almost constant oscillating around the value of 2.12. This conclusion takes into consideration all cases considered. The average value is also found to be 2.12. Among all the cases studied and analyzed the ratio of the value of the transition rise (η) to the plate/shell thickness can be represented by the following formula

$$\eta/t = 2.12\,(1 \pm 5\%) \qquad (6)$$

Figure 6 shows the plate/shell thickness and the value of the transition rise η. The increase in plate/shell thickness increases the value of the transition rise. These values are calculated for the plate/shell thickness of 30 mm through 72 mm.

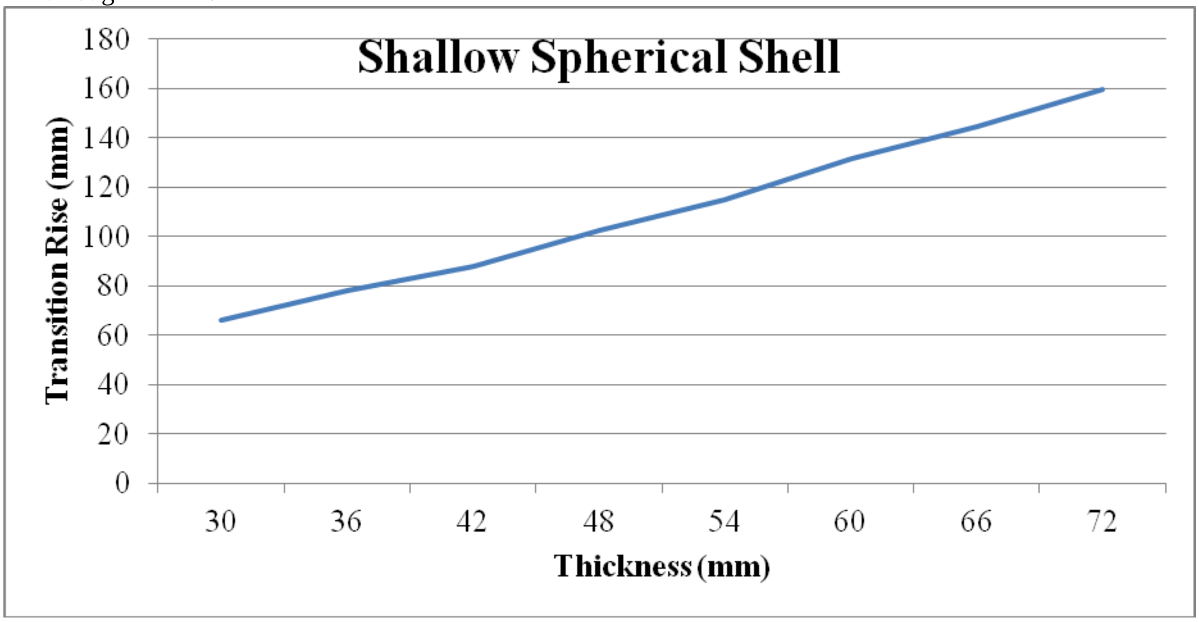

Figure 6: Transition rise-thickness relationship for shallow spherical shell

V. CONCLUSIONS

Large number of cases was studied using the finite element method. The study focused on the shift between plate element behaviors to shallow shell element behavior. Rigorous parametric study was conducted to study the effect of several plate/shell parameters on the value of the transition rise. One of the main conclusions out of this study is that the value of the transition rise – thickness ratio is almost constant with 5% variations and measured to be 2.12. The shallow shell diameter has no effect on the value of the transition rise. The value of the transition rise is not sensitive to either the level of loading nor is it sensitive to the type of loading.

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