

A Review On The Development And Application Of Methods For Estimating Head Loss Components In Water Distribution Pipework

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ABSTRACT: The historical development of the common methods of estimating the frictional loss and the loss through pipe fittings in water distribution systems (respectively, the Hazen-Williams and D'Arcy-Weisbach equations) are briefly reviewed. Furthermore, the methods of applying these equations to index pipe runs are outlined.

KEYWORDS: Hazen-Williams, D'Arcy-Weisbach, Index Pipe Runs

I. INTRODUCTION

The available pressure at any point in a fluid flow conduit is progressively reduced away from the pressure source (such as the elevated storage, or the pump, in a water distribution system) due to frictional losses through conduit fittings (such as elbows, tees and reducers) and valves. Thus, the determination of the required source pressure requires the calculation of the system loss components. This paper outlines the historical development and application of the common methods of estimating the head loss components in water distribution systems.

II. EQUATIONS FOR CALCULATING HEAD LOSS COMPONENTS AND THEIR HISTORICAL DEVELOPMENT

The equations for calculating the head loss components in water distribution systems, namely the friction loss and the loss through pipe fittings are discussed as follows:

Frictional Loss : The empirical Prony equation (Wikipedia, 2013b) was the most widely used equation in the 19th century. It is stated as

$$h_f = \frac{l}{d} \cdot (av + bv^2) \quad \text{-----(1)}$$

where h_f = frictional loss

l = pipe length

d = pipe internal diameter

v = mean flow velocity

and a and b are empirical friction coefficients.

Later empirical developments brought about the D'Arcy – Weisbach equation (D'Arcy, 1857; Weisbach, 1845; Brown, 2000; Haktanir and Ardiclioglu, 2004) which is considered more accurate than several other methods of calculating the frictional head loss in steady flow by many engineers (Giles, 1977; Douglas et al, 1995; Walski,

2001). This equation is expressed as

$$h_f = \frac{4fl}{d} \cdot \frac{v^2}{2g} \tag{2}$$

where f = friction coefficient of the internal pipe wall

g = gravitational acceleration = 9.81m/s²

The major effort in the application of Eqn. 2 is the determination of the pipe friction coefficient which is a function of the flow Reynolds number Re , this number being given as (Reynolds, 1883; Langan, 1988)

$$Re = \frac{\rho v d}{\mu} \tag{3}$$

where ρ = fluid density

μ = fluid dynamic viscosity

For $Re \leq 2000$, which is the laminar flow regime, f is obtained from the Hagen – Poiseuille equation (Poiseuille, 1841; Klabunde, 2008; Wikipedia, 2013a) as

$$f = \frac{16}{Re} \tag{4}$$

For the determination of f in the turbulent flow regime $3000 \leq Re \leq 100000$, Blasius in 1913 proposed through experiments the relation (Blasius, 1913; Kijjarvi, 2011)

$$f = 0.079 Re^{-0.25} \tag{5}$$

Nikuradse later in 1933 showed by experiments the dependence of f on ϵ the average size of the pipe internal surface imperfections, through the relation (Nikuradse, 1933; Yang and Joseph, 2009)

$$f = \phi\left(Re, \frac{\epsilon}{d}\right) \tag{6}$$

where ϕ represents a function.

For all pipes, many engineers consider the Colebrook-White equation (Colebrook and White, 1937; Keady, 1998; Schroeder, 2001; Douglas et al, 1995) more reliable in evaluating f . The equation is

$$\frac{1}{\sqrt{f}} = -4 \log_{10} \left\{ \frac{\epsilon}{3.7d} + \frac{1.26}{Re\sqrt{f}} \right\} \tag{7}$$

Equation 7 is difficult to solve as f appears on both sides of the equation. Typically, it is solved by iterating through assumed values of f until both sides become equal. The hydraulic analysis of pipelines and water distribution systems, using the equation, often involves the implementation of a tedious and time-consuming iterative procedure that requires the extensive use of computers. Empirical head loss equations have a long and honorable history of use in pipeline problems. The use of such empirical equations preceded by decades the development of the Moody diagram (Moody, 1944) which gives the relation between f , Re and relative roughness $\frac{\epsilon}{d}$. Another of such developments are the Hunter Curves due to Hunter Rouse, 1943. The Moody diagram and old empirical equations are still commonly used today.

An alternative method of calculating the frictional head loss to the D’Arcy – Weisbach equation is the Hazen-Williams formula (Hazen and Williams, 1920), expressed in terms of readily measurable variables as (Sodiki, 2002)

$$h_f = \frac{10.62l}{c^{1.85}} d^{-4.967} q^{1.85} \tag{8}$$

where c = Hazen – Williams Coefficient of relative roughness of the pipe material

q = mean flow rate (m³/s)

The Hazen – Williams Coefficient C of Eqn. 8 subsumes the friction factor f of Eqn. 2. Also, the flow rate q subsumes the velocity v of Eqn. 2 as

$$v = \frac{4q}{\pi d^2} \tag{9}$$

For the circular pipe section, values of C for common pipe materials (obtained empirically) are listed in Table 1 (Giles, 1977). It had been noted that C -values obtained from different sources have some differences due to the differing experimental conditions (Keller and Bliesner, 1990).

Applying Eqn. 8, with a particular choice of pipe material, the frictional head loss per metre run of pipe can be calculated from the diameter d and the flow rate q . For instance, for a plastic pipe material ($C = 140$), the loss per metre run is given by Eqn. 8 as

$$\frac{h_f}{l} = 1.1374 \times 10^{-3} d^{-4.967} q^{1.85} \quad \text{-----(10)}$$

The use of the Hazen-Williams formula avoids the use of Eqn. 7 and as pointed out by Larock et al, 2000, many engineers prefer to use it due to the difficulties of determining f . Also, Usman et al, 1998 had noted: “it is easier to apply the Hazen-Williams formula than to obtain f from the Colebrook-White equation and then utilizing f in the D’Arcy -Weisbach equation to obtain the frictional loss”. The Hazen-Williams formula is also accurate over a wide range of Reynolds numbers.

Graphical presentations of the form of Eqn. 10 (the so-called ‘Pipe Sizing Graphs’) (Institute of Plumbing, 1977; Barry, 1984; Mueller, 1987; Fluid Handling Inc, 2008; Construction Knowledge, 2010) are more commonly used in engineering practice than the foregoing equations. In particular, pipe sizes are easily selected with knowledge of the flow rate q and a permissible maximum head loss per metre pipe run, h_f/l . One of such graphs is shown in Fig. 1 (Institute of Plumbing, 1977). Also, nomograms which represent Eqn. 10 (www.heatweb.com, 2010) are sometimes used for pipe sizing.

Furthermore, can (2005) derived model equations for calculating friction head losses in some commercial pipe materials by first creating a dimensional grid of 25 pipe diameters (selected in equal increments in the interval of 0.1m to 1.2m) and 25 flow velocities (selected in equal increments in the interval of 0.5 m/s to 3.1 m/s), and then obtaining f values using the Colebrook-White equation for each pipe material in an iterative process. The f values, so obtained, were then applied in the D’Arcy – Weisbach equation to obtain a set of head loss values. These values were used to develop a model equation for each pipe material in the form

$$h_f = \frac{a q^b}{d^c} \quad \text{-----(11)}$$

where a , b and c are model parameters, values of which were obtained using multivariable regression analysis.

2.2 Head Loss through Pipe Fittings

The loss through fittings h_p is usually expressed in terms of a loss coefficient k of the fitting as (Roberson and Crowe, 1975; Giles, 1977)

$$h_p = k \cdot \frac{v^2}{2g} \quad \text{----- (12)}$$

Substituting for v from Eqn. 9 and writing 9.81 m/s^2 for g yields

$$h_p = 0.08256 k d^{-4} q^2 \quad \text{-----(13)}$$

Values of k (which are empirically determined) are usually listed in tabular form such as Table 2 (Giles, 1977). Graphical presentations are also common (Hydraulic Institute, 1990; Heald, 2002). Furthermore, several correlations had been done to obtain equations useful in predicting losses in pipe fittings (Hooper, 1981; Crane Co., 1991; Darby, 1999; Rahimi, 2011; Yurdem et al, 2008).

It has been observed that k -values obtained from different sources have some differences due to the differing empirical conditions (Ding et al, 2005; Muklis, 2011). Furthermore, experiments performed at the Department of Mechanical Engineering of Indian Institute of Technology, Bombay had shown variations of k with the flow Reynolds number, Re (www.mc.iitb.ac.in, 2013). Variations of k with size of fitting had also been observed (Rahimi, 2011). Thus, the k -value for a particular fitting is not universally constant. It is, however, useful for arriving at a reasonable estimate of the head loss through the pipe fitting.

In consideration of the uncertainties in loss calculations resulting from uncertainties in k -values and the Hazen-Williams C - values, Keller and Bliesner, 1990 recommend a 20% addition to the total head loss in water distribution systems, as a safety margin.

An alternative method of estimating head loss through fittings uses the concept of ‘equivalent length l_e ’ of pipe which would result in the same frictional loss as the loss through the fitting (Muklis, 2011; www.engineeringtoolbox.com, 2012; Schulte, 2010). By this concept, the appropriate form of Eqn. 2 is equated

to Eqn. 12:

$$h_p = \frac{4f l_e}{d} \cdot \frac{v^2}{2g} = k \frac{v^2}{2g}$$

and $l_e = \frac{k}{4f} \cdot d$ -----(14)

The equivalent length l_e of the fitting is, thus, expressed as a number of pipe diameters to be added to the actual pipe length in Eqn. 2 to account for the loss in the fitting. Hence, the total loss (frictional and through the fitting) in a given pipe section is

$$h = h_f + h_p = \frac{4f (l+l_e) v^2}{d 2g}$$
 -----(15)

Values of l_e for common types of fitting are as listed in Table 3 (Barry, 1984).

2.3 Application of the Head Loss Equations to Index Pipe Runs

As the foregoing equations apply to each pipe section along an index pipe run having several branches, the additive forms of the head loss equations, namely Eqns. 2, 8, 10, 11, 12, 13 and 15 should be applied along the index run. Eqns. 8 and 13 would, for instance, then take the respective forms

$$h_f = \frac{10.62}{c^{1.85}} \sum_{j=1}^n l_j d_j^{-4.867} q_j^{1.85}$$
 -----(16)

and $h_p = 0.08256 \sum_{j=1}^n d_j^{-4} q_j^2 \left(\sum_{i=1}^m K_{ij} \right)$ -----(17)

where j denotes the j^{th} pipe section, n is the number of pipe sections in the index pipe run, i denotes the i^{th} fittings in a given pipe section and m is the number of fittings in the section.

III. CONCLUDING REMARKS

The paper outlined the development of the Hazen-Williams and D’Arcy-Weisbach equations which are applicable in the analysis of frictional loss and the loss through pipe fittings in water distribution systems. Their application in the analysis of index pipe runs has also been discussed.

Table 1: Some Values of Hazen-Williams Coefficient C

Types of Pipe	C
Smooth pipes	140
New cast iron pipe	130
Average cast iron, new riveted steel pipes	110
Vitrified sewer pipes	110
Cast iron pipes, some years in service	100
Cast iron pipes, in bad condition	80

Table 2: Typical K values through common fittings

Pipe fitting	K
45° bend	0.35 to 0.45
90° bend	0.50 to 0.75
Tees	1.50 to 2.00
Gate valve	about 0.25
Non-return valve	about 3.0

Table 3: Equivalent lengths of pipe fittings

Pipe fittings	Equivalent length of pipe in pipe diameters
90 elbows	30
Tees	40
Gate valves	20
Globe valves and taps	300

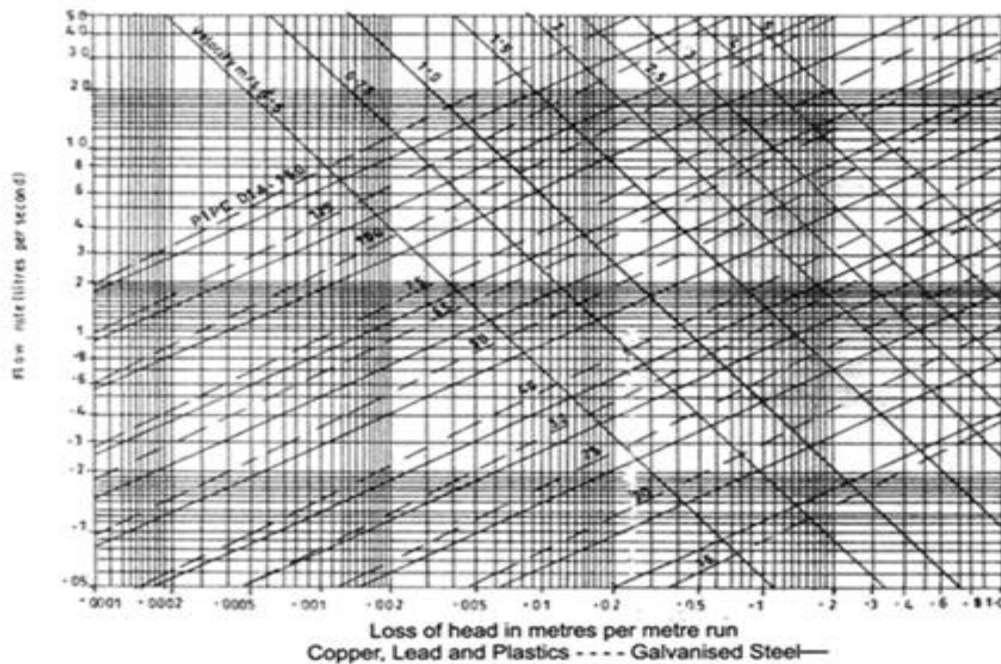


Fig. 1. Pipe Sizing Graph (Institute of Plumbing, 1977)

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