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Stochastic Analysis of Benue River Flow Using Moving Average (Ma) Model.

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Abstract :-Stochastic analysis is an established method of generating data,this was used to analyze 31 years of average monthly stream flowsof Benue River, the popular model known as Moving average (**MA)** was fitted to the data .The Parametersfor the model was estimated accompanied bydiagnostic checks, which showed the model fitness to the dataadequately, also the histogram of the residuals revealed that themodel assumption was correct (i.e. residualsare normally distributed), this was further tested by forecasting of10 years of stream flows at 95% confidence. The resultsdemonstrated the usefulness of the method in generatingadditional data in watersheds where there is paucity of time series data.

Keywords:-*Stream flow, Analysis , Water management*

I INTRODUCTION

Inadequacy and paucity of data has resulted in improper planning. Thus, in order to bridge this gap, stochastic analysis has been introduced into watershed forecasting. Stochasticanalysis is an established method of forecasting in hydrology. The pioneering works of Thomas and Fiering (1962), Matalas (1967) and Box and Jenkins (1976) and several other researchers brought to light, its importance in water resources management, planning and design. Stochastic analysis is the probabilistic analysis of time dependent random variable In which the variable is broken into its components: mean, cyclical or periodic and purely random components The mean and cyclical components are usually removed before application of well-known stochastic models to the purely-random components, for this reason, it is essential that the data be stationary. A data is said to be stationary, if there is no systematic change in variance and the mean is constant. Where the data are non-stationary, method proposed by Box and Jenkins (1976) and Chatfield (1992) can be used to convert the series into stationary one. Stochastic analysis is very useful in understanding the underlying process ofhydrological data and forecasting. This tool is therefore important in situations where there is paucity of data, required for planning and water resources management it is for this reason that we analyzed the monthly average stream flow records of River Benue, in view of its importance in water supply and waste watermanagement of Benue city.

II LITERATURE

There is extensive literature on the application of stochastic modeling in hydrology, Rao (1974) has successfullydeveloped model for monthly river flow analysis, whileClarke (1984) and Hurst and Bowles (1979) developedrainfall model, and water quality models respectively by modifying the theory of Box and Jenkins (1970) Thomasand Fiering (1962), through their pioneering work, have shown that stream flows can be generated using stochasticmodels this was further demonstrated by Okuta (1988), who used the model to generate stream flows In southernGhana, Stochastic analysis had previously been applied to some aspect of stream flow of River Kaduna. Oni (1990)developed a water quality management model for the river, while Harrison (1980) proposed a water supply protectionmodel earlier application was the stochastic modeling of sediment load of the southern water works of River Kadunaby Gleave (1971).

III MATERIALS AND METHOD

The simplest of the stochastic models is the Markov model, which correlates the value of the random variable at a current time to the value of earlier time The first order Markov process is used when serial correlations for lags greater than one are not important and is of the form:

$X_{i+1} = \mu_x + \rho_x(1)(X_i - \mu_x) + \varepsilon_{i+1}(1)$

Where:

X_{i+1}= The value of the process at time I,$\mu = The\ mean\ of\ X.\rho_x(1) = The\ first\ order\ serial\ correlation.$

$\varepsilon_{i+1} = Random\ component\ with\ E(\varepsilon) = 0\ and\ var(\varepsilon) = \sigma_\varepsilon^2$

$\bar{X}_t = \emptyset_{p,1}\bar{X}_{t-1} + \emptyset_{p,2}\bar{X}_{t-2} + \ldots \emptyset_{p,p}\bar{X}_{t-p} + a_t - \emptyset_{p,1}a_{t-1}\emptyset_2 a_{t-2} - \ldots - \emptyset_a a_{t-q}(2)$

The Auto correlation of the process is:

$\rho_1 = \frac{(1\emptyset_{1.1}\theta_{1.1})(\emptyset_{1.1}-\theta_{1.1})}{1+\theta_{1.1}^2-2\emptyset_{1.1}\theta_{1.1}}$

(3)

And $\rho_k = \emptyset_{1.1}\rho_{k,1}$

The stationarity and invertibility constraint to be satisfied are:

$-1 < \emptyset_{1.1} < 1\ and - 1 < \emptyset_{1.1} < 1(4)$

O'Connell (1974) found that the value of $\emptyset and \theta$, used for modeling stream flow time series are in the range where both parameter are positive and $\emptyset_I$ greater than θ. The relation $(\emptyset_{1.1} - \theta_{1.1})$ determines the sign of the lag-one autocorrelation, ρ is always positive.A general Model can be of the form:

$\check{Z}t = a_t - Ø_1 a_{t-1} - Ø_2 a_{t-2} - \ldots - Ø_q a_{t-q}(5)$

This model type is called moving average (MA) process of order q. The name "moving average" is somewhat misleading because the weights 1,- **Ø₁, – Ø₂,...** , - **Øq,** Which multiply the a's, need not total unity nor need they be positive. However, this nomenclature is in common use, and therefore we employ it**.**

If we define moving average operator of order q by

$Ø(B) = 1 - Ø_1 B - Ø_2 B^2 - \ldots - Ø_q B^q(6)$

Then the MOVING AVERAGE MODEL may be written economically as:

$\check{Z}t = Ø(B)a_t$ (7)

It contains q +2 unknown parameters μ, andØ1 ... $Ø_{q\sigma}{}^2{}_a$ which in practice have tobe estimated from the data.

The likelihood estimates of the parameters of MA (p,q) and the conditional sum of square (ss) are obtained using these equations

$L_{\cdot(\emptyset,\theta,r_a)} = nL_\theta r_a - \frac{S.(\emptyset,\theta)}{2r_a^2}8)$

$S.(\emptyset,\theta) = \sum_{t=1}^{n} at\left(\frac{\emptyset,\theta}{w.,a.,w}\right)(9)$

Where:

$L.$and$S.$ are the conditional likely hood and sum of square of the model also:

The auto correlation at lag k is computed as:

$\frac{\sum_{i=1}^{N-K}(x_1-\bar{x})(x_{1+k}-\bar{x})}{\sum_{i=1}^{N}(x_1-\bar{x})^2)}(10)$

K= 0, 1, 2, 3, 4, ………………..k

This model can be applied, and parameters estimated using available statiscal software. For this work MINITAB R14 and STATISTICA.

Stochastic analysis of stream flow requires unbroken record of flow data, collected over a period of time at the gauging station. The monthly mean daily discharge average stream flow was collected for 31 years between 1973 and 2003, with sample size N = 492 from preliminary checks, it was discovered that earlier data between 1973 were collected in ft/s; these were harmonized with later data and analyzed with homogenous units. A plot of the stream flow showed an increasing trend, probably due to urbanization and deforestation in the watershed this would make the data non- stationary, this was corrected by removing the trend and differencing of data with lag 12, this is seasonal differencing of lag 1.

The recommended procedures for Stochastic modeling was adopted which included the following (Box and Jenkins 1970)

(a) Determination of suitability of data for analysis through plotting of series

(b) Identification of models by examination of the autocorrelation function, ACF, and partial autocorrelation function, PACF, plots

(C) Estimation of model parameters

(d) Diagnostic check of the model to verify whether it complies with certain assumptions for the model.

(e) Model Calibration by plotting observed and filled dated.

These steps were carried out for this work using two readily available statistical software MINITAB R 14 and STATISCA.
Moving average MA (1) model was fitted to the data, we performed the time-series plots, model identification and estimation of parameters, diagnostic checking using the normal probability plot of the residuals detrended normal plot and histogram plot of the autocorrelation functions, We also conducted model calibration by comparing original and computed data, which were plotted for all the models Finally ten-year forecasts of stream flows using the models were made.

IV RESULTS AND DISCUSSION

The plots of the original data were shown in fig1 and of transformed data in fig2 the former showed an increasingtrend, which was muffled after logarithmic transformation

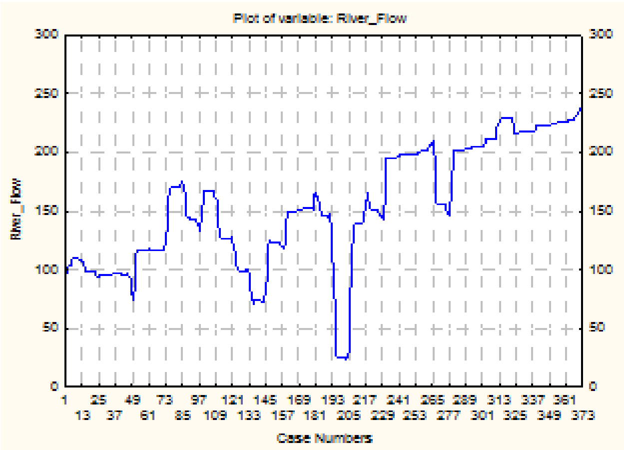

Fig1: The monthly stream flow data Benue River

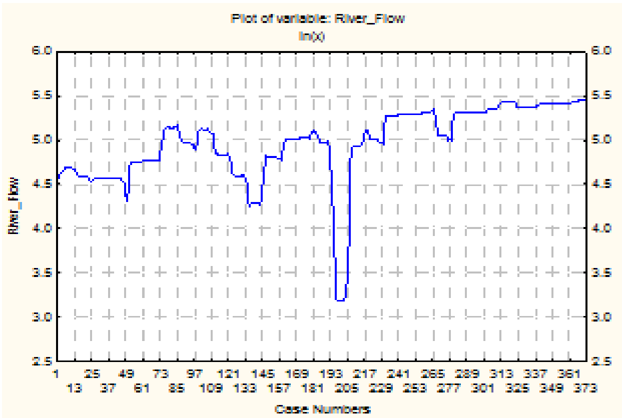

Fig 2: The transformed monthly stream flows of Benue River

In addition, in fig3 and 4 shows the autocorrelation and partial correlation plots of transformed data

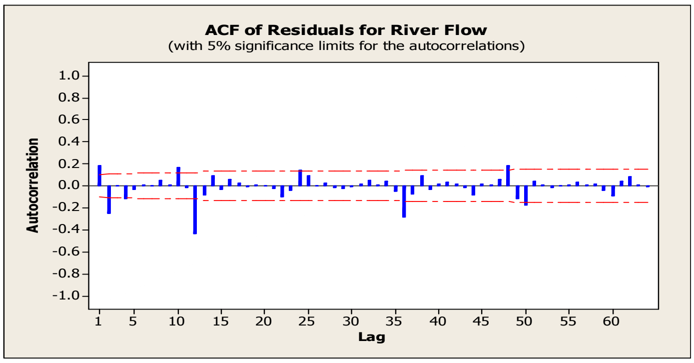

Fig 3: Autocorrelation function of the transformed data

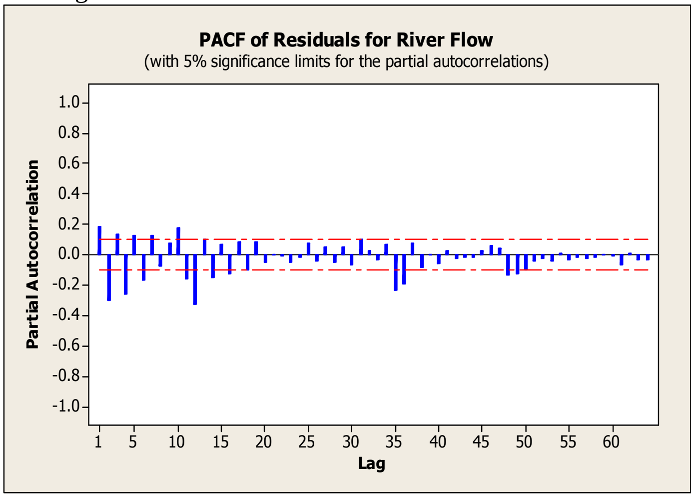

Fig4. Partial autocorrelation function of the transformed data

Diagnostic checks were carried out on the model by plotting the residuals and the order of the data asshown in Fig 5, also Histograph is shown in Fig 6. The Plots of the ACF of the residuals were also made earlier, to test the independence of the residuals, as a necessary condition for acceptance of model.

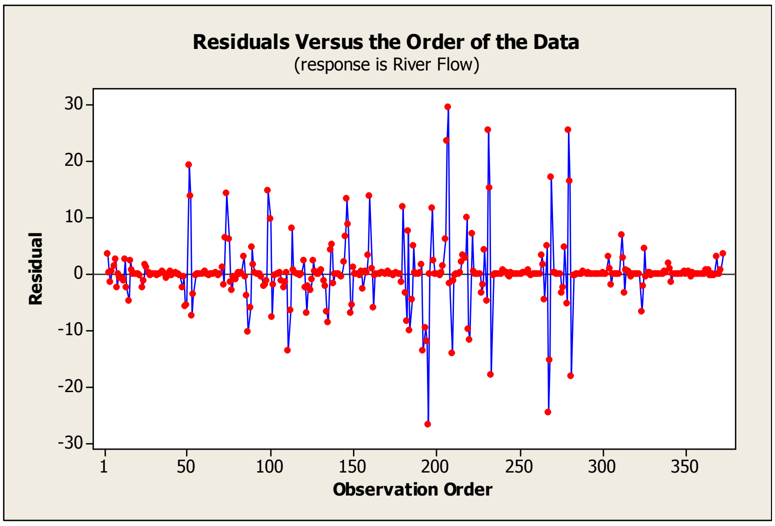

Fig. 5 plot of the residuals versus order of the data

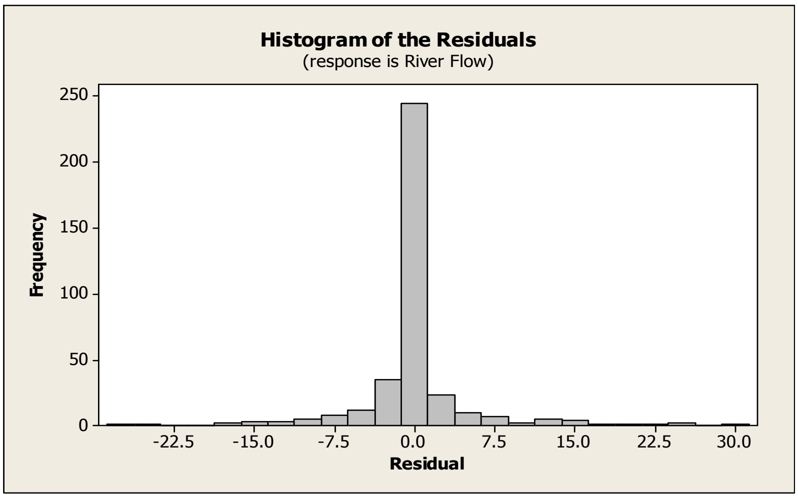

Fig. 6 Histogram plot of the residuals

The plots of the original and computed data are shown in Fig 7, while ten years forecasts are given in Fig 8.

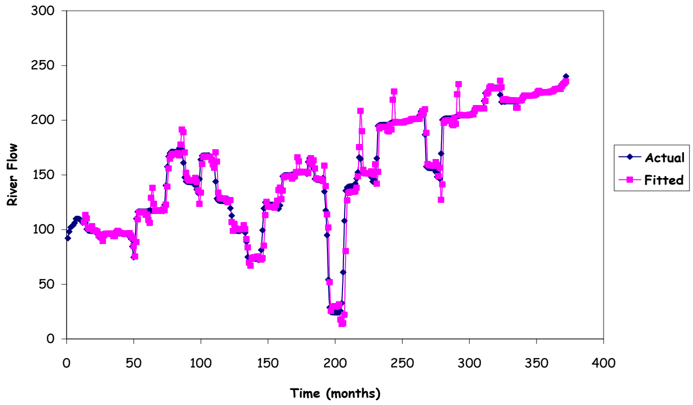

Fig.7 Actual and predicted monthly stream flows of Benue River.

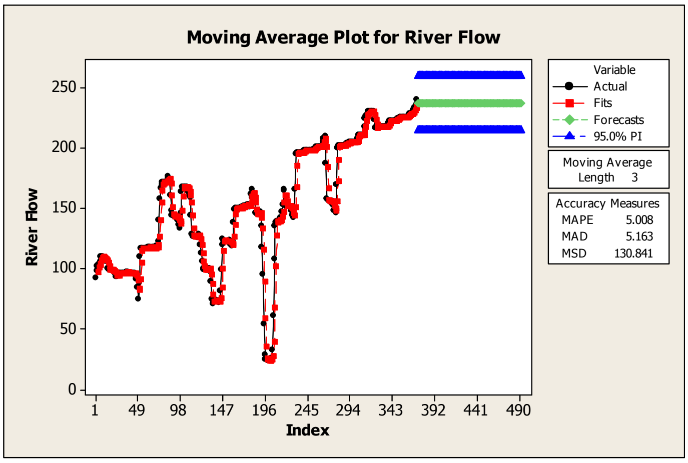

Fig.8: Actual, predicted 10-years forecast of stream flows.

Below is the estimation of parameters yielded the results for MA (1) Model:
Data: River Flow
Length: 372
NMissing: 0
Accuracy Measures:
Mean Absolute Percentage Error (MAPE) =5.008

Mean Absolute Deviation (MAD) = 5.163

Mean Square Deviation (MSD) = 06.6224

The transformation of the data and seasonal differencing reduced the serial autocorrelations. This fulfilled the model
assumptions; this is evident in the normal plots of the residuals, which showed independence. The histograph of the residuals also showed that the assumption was correct for the moving average model. The model calibrations showcased the capability of the model to generate the original data .The increasing flows may be due to increasing urbanization and deforestation in the catchments of the River Benue. Deforestation has the tendency of reducing infiltration and increasing runoff, as noticed in the watershed. There had been significant changes in land use patterns within the period of study i. e 1973 – 2003.

V CONCLUSION

The application of stochastic analysis of stream flows on River Benue was considered. A single model known as Moving Average (MA) was fitted to the data, using standard procedures enunciated by researchers. Diagnostic checks showed the model fitted the data adequately and this was tested by forecasting of 10 years of stream flows. This highlighted the importance and usefulness of the model in areas where we have paucity of data.

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