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Critical Strength of steel Girder Web

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***Abstract:* -** When a member is subjected to combined action of bending moment, shear and axial force, bending moment & axial force is assumed to resist by whole section and shear is resisted by web only. In such case web shall be designed for combined shear and axial force. The present study determines the strength of web of steel girder under the action of pure shear, pure axial force and combination of it. The classical plate buckling theory is already established to determine the critical buckling strength of web panel of the girder under pure axial compression and pure shear. Using Von Mises yield criteria interaction between axial compression and shear is presented. It has been observed that limit strength of web in combined axial compression and shear is governed by buckling of girder rather buckling of web panel.

***Keywords:* -** *buckling, yielding, limit strength, compression, shear, interaction*

I. INTRODUCTION

Nearly all members in a structure are subjected to bending moment, shear and axial load. Particularly, while designing steel structural member, interactions of these actions need to be considered at limit state. Many literatures, codal provisions are available on interaction of Moment & axial compression (M-P) as well as for moment & shear (M-V). As per the revised Indian codal provisions, in case of sections with web susceptible to shear buckling before yielding, strength of member shall be calculated using one of the considerations as

1 The bending moment and axial force assumed to be resisted by flanges and only shear is resisted by web.
2 The bending moment and axial force acting on the section may be assumed to be resisted by the whole section. In such case web shall be designed for combined action of shear and normal stresses.

It is observed that, the provision of interaction for shear and axial compression (V-P) is less attended. The present study determines the strength of web of steel girder under the action of pure shear, pure axial force and combination of it. For the design of web plate two checks need to be applied, check against buckling and check against local yielding. The classical plate buckling theory is applied to determine the critical buckling strength of web of the girder and Von Mises yield criteria is also used to present interaction between axial compression and shear.

II. BUCKLING OF WEB

Web of rolled steel I section or built up plate girder behaves as a plate subjected to inplane uniform axial compression and shear as shown in Fig 1. Buckling strength of web is affected by web panel dimensions, length to depth ratio – called as aspect ratio (a/b), depth to thickness ratio (d/t) and boundary conditions.

Depending upon whether the web panel is from rolled steel section or from built up plate girder, boundary conditions vary. Web panel of rolled steel section is taken as simply supported alongside ‘b’ and built-in or fixed at the junction of web and flange abbreviated as SFSF, whereas web panel of plate girder is taken as simply supported along all edges abbreviated as SSSS. Elastic buckling strength of such plates is known by classical plate theories.

Classical plate theories are established by various researchers [1,3,4] for elastic buckling of plate under inplane action of pure compression, pure shear and combination of shear and axial compression.

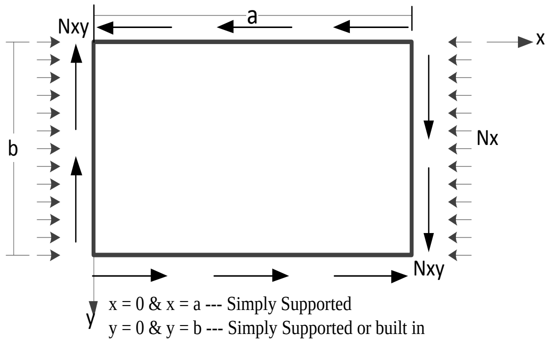

Figure1: Web panel subjected to inplane forces

2.1 Buckling Strength of Web: Classical plate theory approach

2.1.1 Web Panel under Uniform Compression:

When a web panel subjected to uniform compression (Nx) it will buckle in half waves along its longitudinal direction. By gradually increasing Nx and using equilibrium of the compressed plate, critical load at buckling for plate is given by

$$N_{x,cr} = k_a \frac{\pi^2 D}{b^2} \qquad (1)$$

Where, D is the flexural rigidity of plate.

The buckling coefficient 'ka' for all edges simply supported, i.e. SSSS plate is given as [1],

$$k_a = \left(\frac{mb}{a} + \frac{a}{mb} \right)^2 \qquad (2)$$

Where, 'm' is the number of half waves of buckling.

The buckling coefficient as per aspect ratio and transition of the buckling modes can be seen from Fig. 2

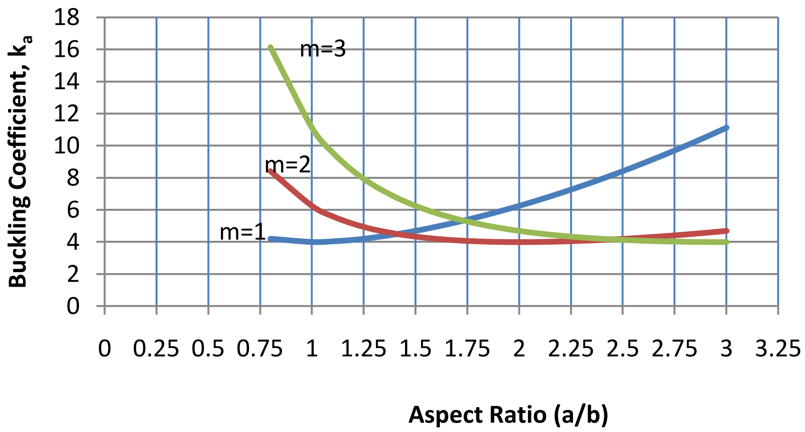

Figure 2: Buckling Coefficient ka as per aspect ratio for SSSS Condition

Rudolph Szilard [3] presented the buckling coefficient 'ka' for SFSF plate

$$k_a = \frac{1}{3}\left[\frac{16}{m}\left(\frac{a}{b}\right)^2 + 3m^2\left(\frac{b}{a}\right)^2 + 8 \right] \qquad (3)$$

The buckling coefficient for this condition as per aspect ratio and transition of the buckling modes can be seen from Fig. 3

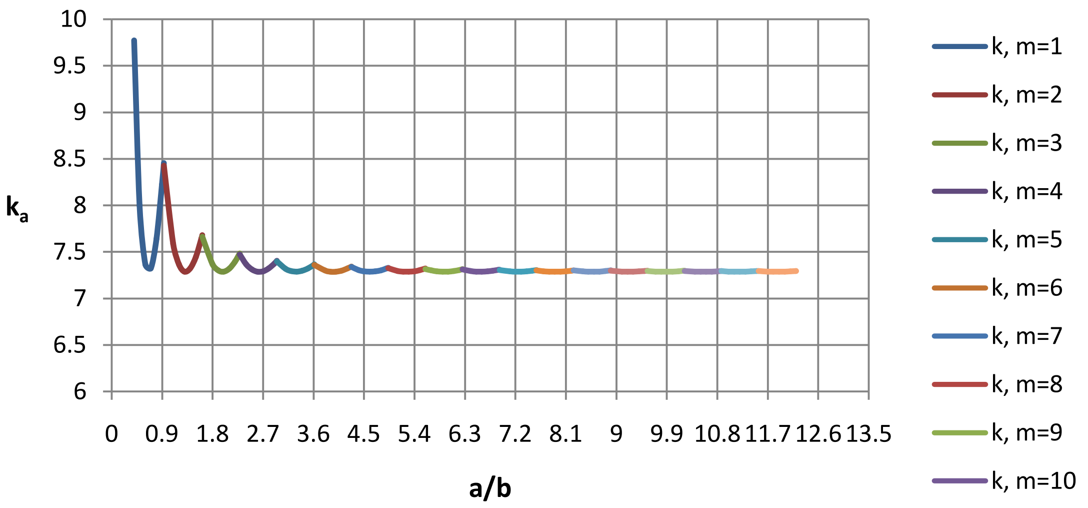

Figure 3: Buckling Coefficient ka as per aspect ratio for SFSF Condition

Considering 't' as the thickness of plate, the critical value of the compressive stress is

$$\sigma_{cr} = \frac{N_{x,cr}}{t} = k_a \frac{\pi^2 E}{12(1-\nu^2)} \frac{t^2}{b^2} \tag{4}$$

2.1.2 Web Panel under Uniform Shear:

When a plate is subjected to the action of shearing forces Nxy uniformly distributed along the edges, the critical value of shearing stress, τcr, at which buckling of the panel occurs is determined using the energy method. [1, 4] Critical shear stress, τcr, at buckling is presented in equation (5)

$$\tau_{cr} = k_v \frac{\pi^2 D}{b^2 t} \tag{5}$$

Where, kv is the shear buckling coefficient and its value with respect to aspect ratio are given in Table 1. [1]

Table 1: Shear Buckling Coefficient

a/b	1.0	1.2	1.4	1.5	1.6	1.8	2.0	2.5	3.0	4.0	∞
K_v , SSSS plate	9.34	8.0	7.34	7.1	7.0	6.8	6.6	6.1	5.9	5.7	-
K_v , SFSF plate	12.28	-	-	11.12	-	-	10.21	9.81	9.61	-	8.99

2.1.3 Web Panel under combined uniaxial compression and Shear:

The interaction equation for determination of buckling strength under combined compression and shear using above plate theories is presented by P S Bulson [4] as under

$$\frac{\sigma_x}{\sigma_{x,cr}} + \left(\frac{\tau}{\tau_{cr}}\right)^2 \leq 1 \tag{6}$$

III. ANALYTICAL VERIFICATION

Verification of results by numerical analysis using ANSYS is carried out. Based on the boundary conditions, two plate models are analyzed using ANSYS for various (a/b) ratios under action of pure compression and pure shear seprately. The boundary conditions considered are SSSS & SFSF.

Following is the data considered for analysis in ANSYS

b = 1.2 m, ν = 0.3, E = 210 GPa, t = 0.01 m

3.1 Web Panel under Uniform Compression

Performing Eigen buckling solution and using element as SHELL93, the plate is checked for buckling resistance in ANSYS. For buckling of SSSS plate under pure axial compression, for minimum buckling load, buckling coefficient, 'ka' is giving close relation with classical solution as shown in Table 2. The results of SFSF plate are tabulated in Table 3.

3.2 Web Panel under Pure Shear

Similarly, using SHELL63, plate is checked for shear buckling resistance by ANSYS. The results for SSSS condition are tabulated in Table 4. In case of web of rolled steel section i.e. plate of SFSF condition, the results of classical plate theory are directly adopted. It is observed that for larger (a/b) ratio, buckling coefficient kv is almost constant to a value around 9. In practice ratio of (a/b) for rolled steel section is around 10, hence assumed value of kv = 9.00 is reasonably acceptable.

Table 2: Buckling Coefficients k_a for SSSS Condition (ANSYS)

a/b	K_a [1]	Nx,cr [1] N/m	Nx,cr (ANSYS) N/m	% error	K_a ANSYS
1	4	75917	75210	-0.94	3.962749
1.1	4.04	76676.17	75911	-1.01	3.999684
1.2	4.13	78384.3	77822	-0.72	4.100373
1.3	4.28	81231.19	80639	-0.73	4.248798
1.4	4.47	84837.24	84235	-0.71	4.438268
1.41	4.49	85216.83	84633	-0.69	4.459239

Table 3: Buckling Coefficients k_a for SFSF Condition (ANSYS)

a/b	K_a [1]	Nx,cr [1] N/m	Nx,cr (ANSYS) N/m	% error	K_a ANSYS
0.4	9.44	179164.1	169413	5.44	8.93
0.5	7.69	145950.4	135243	7.34	7.13
0.6	7.05	133803.7	122084	8.76	6.43
0.7	7	132854.7	119808	9.82	6.31
0.8	7.29	138358.7	124189	10.24	6.54
0.9	7.83	148607.5	133160	10.39	7.02
1	7.69	145950.4	145370	0.40	7.66
2	-	-	162982	-	8.59
3	-	-	118931	-	6.27
8	-	-	126826	-	6.68
10	-	-	124987	-	6.59
11	-	-	124683	-	6.57
12	-	-	129567	-	6.83

Table 4: Buckling Coefficients k_v for SSSS Condition (ANSYS)

a/b	k_v [1]	Nxy,cr [1] N/m	Nxy,cr (ANSYS) N/m	% error	k_v ANSYS
1	9.34	1231064	1.47E+06	-19.0478	11.11906
1.2	8	1054445	1059000	-0.43199	8.034559
1.4	7.34	967453.2	966471	0.101525	7.332548
1.5	7.1	935819.9	955360	-2.08802	7.24825
1.6	7	922639.3	939040	-1.77759	7.124431

IV. YIELDING OF WEB

Distortion energy theory of failure arrives at expression for equivalent stress (σ_e), which is also referred to as Von Mises stress which considers failure by yielding. A yield criterion defines the limit of elastic behavior under any possible combination of stresses at a point in a given material.

Equation based on the Von Mises yield criterion is expressed as

$$\sigma_e = \sqrt{\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2}{2}} = f_y \qquad (7)$$

Where, σ_1, σ_2, σ_3 are the principal stresses and f_y is the yield stress.
Considering only σ_1, σ_2 the principal stresses acting at a point in the web panel and further web panel under uniaxial compressive stress σ_x and shear stress τ_{xy}, above expression (7) reduces knowing $\sigma_3 = 0$

$$\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 = f_y^2$$

In case of 2D elastic system, σ_1 and σ_2 can be expressed in terms of σ_x & τ_{xy}.

Hence, $$\sigma_x^2 + 3\tau_{xy}^2 = f_y^2$$

Converting this relation in nondimensional form, equation (8) is arrived as,

$$\frac{\sigma_x^2}{f_y^2} + 3\frac{\tau_{xy}^2}{f_y^2} = 1 \qquad (8)$$

V. DESIGN CONSIDERATIONS

A conclusion section must be included and should indicate clearly the advantages, limitations, and possible applications of the paper. Although a conclusion may review the main points of the paper, do not replicate the abstract as the conclusion. A conclusion might elaborate on the importance of the work or suggest applications and extentions. (10)

5.1 Interaction equation presented by Bulson [4], as per equation (6) is in the form of stress. Modifying the equation (6) in nondimensional form of compression & shear capacities, equation (9) is arrived,

$$\frac{P_w}{P_{cr,w}} + \left(\frac{V}{V_{cr}}\right)^2 \leq 1 \qquad (9)$$

Where, Pw and V are the actual actions in the web & Pcr,w and Vcr are buckling capacities in axial compression and shear respectively for the web panel.

5.2 Interaction equation based on Von Mises yield criterion defines the limit of elastic behavior under any possible combination of stresses at a point in a given material. Thus the interaction gives the section capacity at a point. Modifying the equation (8) in nondimensional form of compression and shear capacities, equation (10) is arrived,

$$\left(\frac{P_w}{N_{dw}}\right)^2 + \left(\frac{V}{V_d}\right)^2 = 1 \qquad (10)$$

Where Pw and V are the actual actions and Ndw & Vdw are the section capacities of the web in axial compression & shear respectively. Section capacity or section strength as governed by material failure by yielding for individual actions as per IS 800:2007 are

1. Member subjected to pure axial compression

$$N_d = A_g f_y / \gamma_{mo}$$

Where, Ag is the gross area of cross section and γmo is the partial safety factor for material.

2. Member subjected to pure Shear

$$V_d = A_v f_y / \sqrt{3}\, \gamma_{mo}$$

Where, Av is the area of the web = b.t

5.3 Indian Standard Codal provisions / Limit state of strength in axial compression is given as under

$$P_{dw} = A_e f_{cd}$$

Where, Ae is the effective cross sectional area of the member and fcd is the stress which accounts for the overall member buckling. In the case of girder subjected to axial compression, overall member buckling stress 'fcd' is based on governing slenderness ratio of the member and relevant buckling class of the section. Using this stress buckling capacity of web is evaluated as

$$P_{dw} = A_v f_{cd}$$

Where, Av is the area of the web = b.t

Thus , interaction equation considering overall buckling of web is

$$\frac{P_w}{P_{dw}} + \left(\frac{V}{V_{cr}}\right)^2 \leq 1 \quad (11)$$

VI. PHILOSOPHY OF INTERACTION EQUATIONS

Above proposed interaction equations (9), (10) & (11) are used for webs of various Indian Standard rolled steel section and plate girder. The critical buckling strength is determined based on plate buckling theory applicable for web panel in equation (9). Taking stiffened web of plate girder, a panel is considered as web portion between two adjacent stiffeners. Using practical spacing of stiffeners, panel aspect ratio (a/b) are taken as 0.9, 1.0, 1.2, 1.4 for plate girder. For unstiffened web of rolled steel beam ratio (a/b) is span to depth of web & it is much larger than that of plate girder. In practice this ratio is around 10 for rolled steel beam. Shear buckling resistance of the web of rolled steel section is determined considering shear buckling coefficient as 9.00 (refer Table 1).

Equation (10) based on Von Mises yield criteria. The strength in denominator is section strength at yielding of section which indicates failure at a point.

The buckling strength determined in equation (11) is based on overall member buckling in case of axial compression and panel dimensions in case of shear.

Fig 4, Fig. 5 & Fig. 6 shows the interaction relationship as expressed by equation (9), (10), (11)

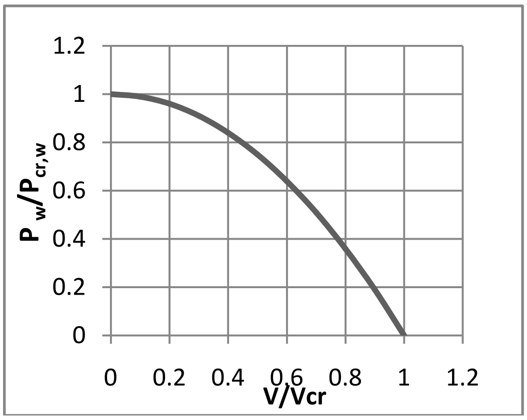

Figure 4: Interaction Curve using Eq (9)

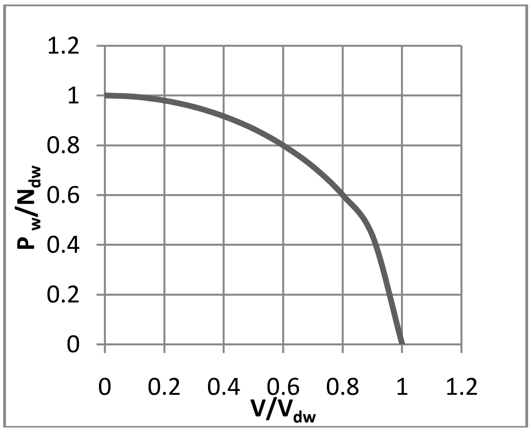

Figure 5: Interaction Curve using eq. (10)

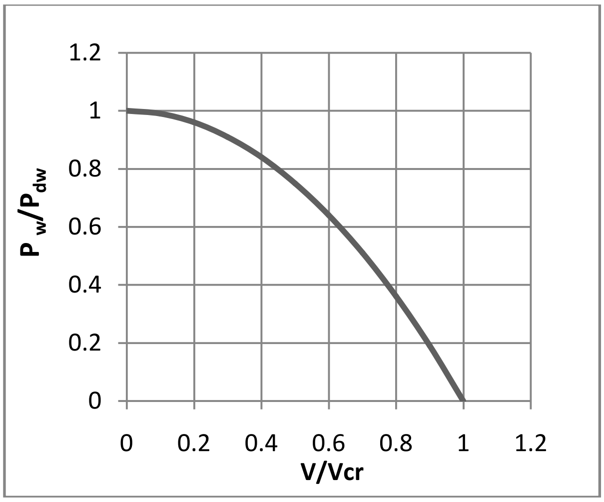

Figure 6: Interaction Curve using Eq (11)

VII. APPLICATION OF INTERACTION EQUATIONS

Some of the cases where the member is subjected to combined action of shear and axial compression are hinged bases of column, hinged bases of arch bridge and at simply supported end of plate girder of railway bridge under longitudinal traction.

The above proposed interaction equations are applied to various configurations of plate girder by varying (a/b) and for the ratio of (Pw/V). Parametric studies are performed based on following data is tabulated here,

Web panel dimensions (a/b) = (1500/1200) = 1.25, thickness of panel = 10 mm, modulus of elasticity = E = 200 GPa, ν = 0.3, fy = 250 MPa

Table 5: Comparison of Interaction Equations

		Axial Compression Capacities			Shaer Capacities		Interaction Equations		
a/b	P_w/V	Pcr	Nd	Pd	Vcr	Vd	Eq. (9)	Eq. (10)	Eq. (11)
1.25	1	527.22	2727.27	407.35	1004.23	1574.59	0.419	0.022	0.588
1.25	2	527.22	2727.27	407.35	1004.23	1574.59	0.389	0.009	0.552
1.25	3	527.22	2727.27	407.35	1004.23	1574.59	0.384	0.007	0.545
1.25	4	527.22	2727.27	407.35	1004.23	1574.59	0.382	0.006	0.543
1.25	5	527.22	2727.27	407.35	1004.23	1574.59	0.381	0.006	0.542

VIII. CONCLUSION

- Interaction equation proposed by P S Bulson [4] for combined action of P & V is linear-quadratic which uses classical plate theory. Buckling strength under individual action of P & V are evaluated on the basis of web panel dimensions and boundary condition of panel edges.
- Von Mises energy distortion theory which is known for yielding at the point of failure, when used for web is not a function of web panel dimensions. The equation basically indicates section strength at yielding rather buckling. The interaction for combined action of P & V is quadratic-quadratic.
- Considering buckling of member subjected to combined action of P & V, the value of P when acting alone & at which member buckles is dependent on Iyy and effective length of the member. Thus, strength under compression is determined based on overall member buckling. However, buckling strength of web under pure shear depends upon web panel dimensions & its boundary condition. Interaction for combined action is linear-quadratic.
- At limit state, interaction of P & V is governed by the limit strength in axial compression on the basis of member strength and that of shear on the basis of panel strength. Interaction values of P & V under three different limit states such as web panel buckling, local yielding, & overall buckling of the member, it is observed that overall buckling of the member governs the limit state.

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