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Developing a Mathematical Model to Dampen the Effect of Chromatic Dispersion in Optic Fibre Carrying-Capacity Due To External Pressure

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Abstract: Telecommunications requires a medium to propagate the signals from source to destination without signal impairment .The optical fibre has been rated the best medium ,but for long haulage ,wavelength multiplex (WDM) has to be installed which introduces dispersion The information carving capacity of a fibre is then limited by the phenomenon called dispersion .This introduces small change in the transmit time for signals travelling through the fibre The arrival of different wavelength at different time and the interspersing of the signal bits causes signal broadening at the destination ,resulting degradation of blurred images for data and noisy audio output, A mathematical model is developed to counter the effect of external pressure in aiding the fast spreading of these signals, known as -p compensation method.

Keywords: Telecommunication, impairment, signal broadening, long-haulage , carrying capacity

I. INTRODUCTION

Telecommunication is simply the coding and decoding of information from source to destination by electronic means. A medium is required for the transportation of this information from source to destination. The available media that can be used to carry the signals include but not limited to wired cables of copper, coaxial, optic fibre, and wireless medium waves infrared, Bluetooth, radio signals, and very small aperture terminal (V-sat) satellite. This work concentrates solely on optic fibre medium that relies on the principles of light rays for signal transmission. It consists of isolated medium which contains strands of glass fibre that carries more information over long distance than electrical signals being routed over orthodox copper cable, coaxial cable, or radio waves. Consequently, the increase in the demand for broadband backbone has led to concerted efforts on studies, analysis, planning, installation and maintenance of optic fibre network. When the optic fibre network is well articulated, planned and designed its transmission capabilities by principles of light ray propagation, is over hundreds of kilometres with large bandwidth potentials, unattended. Telecommunications service providers have to face continuously growing bandwidth demands in all networks areas, from local area (subscriber/end user terminal to long-haul to access network. Sequel to the fact that installing new communication links would require huge Investments, telecommunications carriers prefer to increase the capacity of their existing fibre links by using possible expandable method (multiplexing) which means introducing wavelength division multiplex (WDM) thereby bringing in dispersion into the system. The major problem of the wave length division multiplex (WDM) is how to avoid the interference/ distortion that normally arises from transmitting the signals belonging to different operators into one fibre as this will lead to over lapping bearing in mind that this will cause signal broadening (chromatic dispersion) in the channel.

II. MATERIALS AND METHODS

In view of developing protection against pressure for future installations and to satisfy the academic benefits of this research, a mathematical modelling was developed which will apply negative compensation to the effect of external pressure on the fibre optics cable. The model takes cognisance of the fact that wave length (λ),aligns with wave parameters, pressure aligns with force the modelling is as follows: Since the propagation

constant and phase velocity are involved in fibre transmission context, Fourier and Taylor series (Power series) will be adopted in the modelling formulations of this research. This will further help in characterizing the system. In Fibre optics, the velocity of propagation of light or the speed of signal propagation depends on the wavelength of the fibre. Optic fibre is affected by two kinds of dispersion, intermodal and intramodal dispersion, each of them leads to pulse broadening inside the fibre. This is the reason why multimode fibres are not very popular in optic fibre communication systems. Intramodal dispersion or group velocity dispersion (GVD), chromatic dispersion (CD) or fibre dispersion will usually exists in both single mode fibre and multimode fibre.
But in our model, let the two intramodal dispersion be material M_d and waveguide dispersion W_d.
M_d essentially depends on the source spectral width S_w and is less at longer wavelengths λ_l. The reason for W_d. is that propagation constant β is a function of fibre core size S_c and wave length λ. So, W_d results because the propagation constant (β) is a function of S_c and λ. Moreover the refractive indexes R_i of the core and the cladding could be different, so light propagates differently in them because, the core and cladding R_i are different so light propagates differently in each. At higher bit rates (>10Gb/s), chromatic dispersion (CD) becomes so serious that it degrades information capacity and fibre transmission distance due to pulse broadening, i.e. travel at different group velocities (v_g). Now, given that pulse broadening stems from frequency dependence of β, Using Taylor's series expansion of propagation constant, $\beta(\omega)$ around the carrier frequency, ω_c, we now obtain:

$$\beta(\omega)=\beta_0+\beta_1(\omega-\omega_c)+{}^1/_{2!}\,\beta_2(\omega-\omega_c)^2+{}^1/_{3!}\,\beta_3(\omega_c)(\omega-\omega_c)^3+ \quad \ldots\ldots\ldots(1)$$

From equation (1),β(ω) can be used to estimate CD for arbitrary modulation formats impaired by arbitrary amounts of CD and noise based on an analytical formula for the CD in terms of the phase of the optical signal at four frequencies. The model could be used to monitor performance in reconfigurable heterogeneous optical networks and in the design of general purpose coherent receiver systems.
If the signal is impaired only by CD, then the Fourier transform, $\hat{u}$, of the coherently-received signal, u, is given by

$$\hat{u}(\omega) = \hat{v}(\omega)\left(\sum_{n=0}^{N-1} b_n \exp\,(i\omega_n T)\exp\,\left({}^{i\beta\omega^2}/_2\right)\exp\,(i\theta_0 + i\omega\,\theta_0)\right) \quad \ldots\ldots\ldots(2)$$

Where ω is angular frequency, $v = v(t)$ is the pulse shape at the transmitter, b_n is the *n-th* data symbol, T is the symbol period, b is the total dispersion, and t_0 and θ_0 are unknown time and phase offsets.
Just like in equation (2).applying z-transform: let *f(z)* be analytic in the domain D (dispersion) and applying z-transform, let $z = a$; be any point in D ,Then there exist precisely one power series with centre a, which represents *f(z)*, this series then is of the form

$$f(z) = \sum_{n=0}^{\infty} b_n\ (q-a))^n \quad (3)$$

Applying binomial expansion in $f(z)$ and taken necessary factors into consideration i,e, have a non zero radius convergence R then it represents some analytical function in the disk as in [1]:
For $|z-a| < R$, let

$$f(z) = b_0 + b_1(z-a) + b_2(z-a)^2 + b_3(z-a)^3 + \cdots + b_{n+1}(z-a)^{n+1} \quad (4)$$

$$\beta(w) = \beta(w_o) + \beta_1(w_0)(w-w_0) + \frac{1}{2}\beta_2(w_0)(w-w_0)^2 + \frac{1}{6}\beta_3(w_0)(w-w_0)^3 \quad (5)$$

where $\beta_m(w_0)$ denotes the nth derivative β with respect to w evaluated at $w - w_0$, that is,

$$\beta_m = \left(\frac{\delta^m\beta}{\delta w^m}\right)_{w-w_0} \quad (6)$$

The first term causes a shift, and the product $[\beta_0(w_0) - \beta_{0y}(w_o)].z$, (i.e., z times the difference in the x and y components of (β_0) describes the polarization evolution of the optical wave.
In the second term of equation (1), the factor $\beta_1(w_0)z$ produces a group delay $\tau_g = {}^z/_{V_g}$, where z is the distance travelled by the pulse and $V_g = {}^1/_{\beta_1}$ is the group velocity.
Using the delay equation as stated in equations (4) and (5) according to[1],

$$\frac{\tau_g}{L} = \frac{1}{V_g} = \frac{1}{c}\frac{d\beta}{dk} = -\frac{\lambda^2}{2\pi c}\frac{d\beta}{d\lambda} \quad (7)$$

where L is the distance travelled by the pulse, β is the propagation constant along the fibre axis, $k = 2\pi/\lambda$, and the *group velocity*

$$V_g = c\left(\frac{d\beta}{dk}\right)^{-1} = \left(\frac{\partial\beta}{\partial w}\right)^{-1} \quad (8)$$

Is the velocity at which the energy in a pulse travels along a fibre.
The phase velocity, V_p, of a wave in a given uniform medium is given by:

$$V_p = \frac{c}{n} = \frac{\omega}{k} \tag{9}$$

Where c is the speed of light in a vacuum and n is the refractive index of the medium, wavelength λ, ω is the optical frequency and k is the propagation constant.

For visible light:

$$1 < n(\lambda_{red}) < n(\lambda_{yellow}) < n(\lambda_{blue}) \tag{10}$$

or

$$\frac{dn}{d\lambda} = 0 \tag{11}$$

The group velocity Vg is related to the phase velocity by

$$V_g = c\left(n - \lambda\frac{dn}{d\lambda}\right)^{-1} \tag{12}$$

Phase velocity can also be represented as

$$V_p = \left(\frac{dk}{d\omega}\right)^{-1} = \frac{1}{k} \tag{13}$$

Where k = propagation constant and ω for optical frequency. The term $\frac{dk}{d\omega}$ represents the first order coefficient. Due to the different in velocity experienced by various optical frequencies the output pulse is scattered and dispersed in the time domain. The delay experienced by the optical frequency ω over a fibre length *L* is as given by equation (13) which is equivalent to phase velocity.
Let $\Delta\omega$ be the optical bandwidth, the extent of pulse broadening $\Delta\tau$ is given in the following equation.

$$\tau = \frac{L}{V_g} \tag{14}$$

Chromatic dispersion defined with reference to wavelength, hence the dispersion coefficient D expressed in unit ps/(nm-km) is defined as

$$D = \frac{\Delta\tau}{\Delta\omega\lambda}\frac{d\omega}{d\lambda} = k\frac{d\omega}{d\lambda} = -\frac{2\pi c}{\lambda^2}k'' \tag{15}$$

Second order dispersion parameter

$$S = \frac{dD}{d\lambda} = \left(\frac{2\pi c}{\lambda^2}\right)^2 k''' + \left(\frac{4\pi c}{\lambda^3}\right)k'' \tag{16}$$

Where λ is the operating wavelength, n is the refractive index, c is the free speed of light. The dispersion parameter D is known for a fibre and is 17ps/(km-nm) for a single mode fibre. The GVD parameter k is expressed in ps^2/km [3].

III. DESIGN MODEL FOR FIBRE DEPLOYMENT

Sequel to the above wave derivative the value of v_g is the group phase velocity, then the need to find the relationship between velocity and pressure (force) plays a great role to solving this problem. Ordinarily, the pressure equation *(p);* is given by Pressure (P) = force (f)/area (a) And force *(f)*= mass*(m)*×acceleration*(a)* Where *m* = mass of the earth above the fibre and a= v_g = group velocity, equation (8) becomes *F* = weight of the earth(ω)×V_g Following the above derivative where V_g is the group phase velocity, we seek to establish the relationship with velocity and force-pressure.
Basically, pressure *P= force/Area*
Where force *F* = *ma* where *m* = mass of the soil domain for the fibre.
F =*ma* =Weight of the soil $[c(\frac{d\beta}{dk})^{-1})]$ Figure 1 shows a fibre pipeline deployment layout carrying signal *x* with a density *dx* that modulates an optical source. The design of the pipeline involves the determination of the inside diameter of the pipe *D* and its wall thickness *t* for the signal flow. We shall assume that the modulated optical signal excites all modes equally at the input end of the fibre hence carrying equal amount of energy.

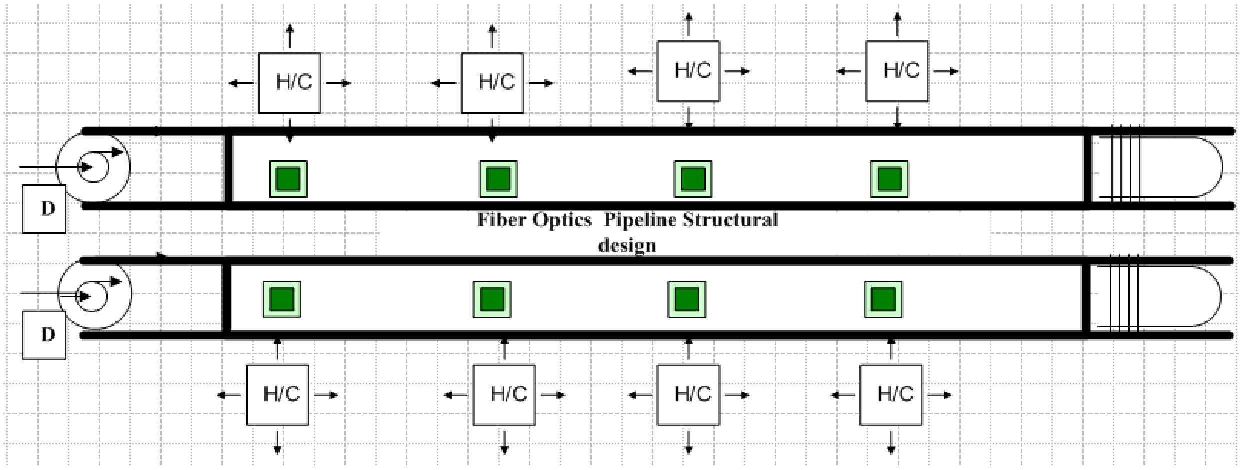

Fig. 1a Structural Pipeline Deployment and Design

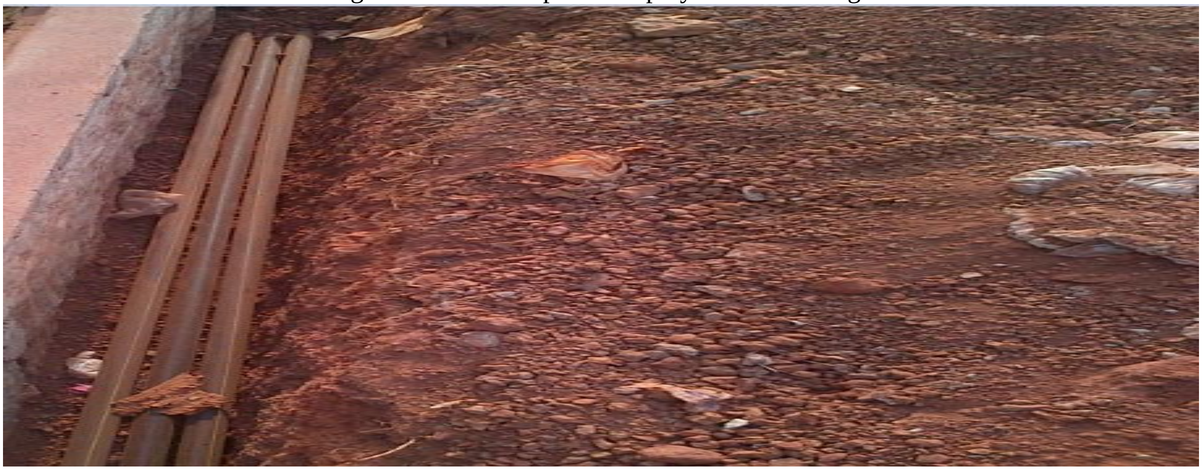

Fig. 1b Structural Pipeline Deployment and Design

On the fibre trench, the forces that are prevalent on the fibre is developed below. In the fibre, the stress on the wall cannot be equally distributed as the load varies, hence the tangential and radial forces are developed from engineering point of view.

Let the tangential stress maximum be at the inner surface and minimum at the outer surface, while the radial stress will be maximum at the inner surface and zero at the outer surface. The lames equation [2], remains significant in this context.

i. **Equation for Inside Diameter of the Fibre Pipe**: This depends on the quantity of the signal to be delivered, as such, Let,

A represent the cross sectional Area of the fibre pipe
D represents the diameter of the fibre pipe
V represents the velocity of fibre pipeline signal per minute
Q represents Quantity of fluid carried per minute
But the Quantity Q of signal flowing per minute is given by:

$$\mathbf{Q = A*V = \frac{\pi}{4}*D^2*V} \tag{17}$$

Solving for D yields

$$\mathbf{D = \sqrt{\frac{4}{\pi}*\frac{Q}{V}} = 1.13\sqrt{\frac{Q}{V}}} \tag{18}$$

ii. Wall Thickness of the fibre Pipeline *t*

From figure 1a , t is considered next in order to withstand the internal signal fluid pressure p in the thin or thick cylindrical fibre pipeline.

Essentially, the thin cylindrical equation will be applied when:

i. Stress across the fibre pipeline section is uniform
ii. The internal diameter of the fibre pipeline section D is >20t; i.e D/t>20

iii. The allowable stress σ_t is more than six times the pressure inside the fibre pipe *P* ie. $\sigma_t > 6$

According to the thin cylindrical equation 19, wall thickness *t* of the fibre pipeline is given by

$$\mathbf{t} = \frac{P.D}{2Q_t} = \frac{P.D}{2Q_t \eta_t} + \mathbf{C} \qquad (19)$$

Where, η_t is the Efficiency of fibre pipeline longitudinal joint and ***C*** is the Weishack constant

IV. Design Model For fibre Pipeline Stress

Consider a cylindrical shell of a pressure vessel carrying signal which was subjected to a high internal pressure *p*, the wall of the cylinder must be made extremely thick ***t*.** The factors under consideration are the pressure (forces) and the cylindrical fibre with core and cladding surfaces.

Assuming that the tensile stresses are uniformly distributed over the section of the walls, let,

r_0 represent outer radius of the cylindrical shell

r_i represent inner radius of the cylindrical shell

t represent thickness of the cylindrical shell = $r_0 - r_i$

p represents intensity of internal pressure

μ represent Poisson's ratio

σ_t represents tangential stress

σ_r represents radial stress

Using lame's law which states that assuming that the longitudinal fibres of the cylindrical shell are strained, the tangential stress at any radius *x* is give by

$$\sigma_t = \frac{P_i (r_i)^2 - P_0 (r_0)^2}{(r_0)^2 - (r_0)^2} + \frac{(r_i)^2 (r_0)^2}{x^2} \left[\frac{p_i - p_0}{(r_0)^2 - (r_i)^2}\right] \qquad (20)$$

Now, radial stress at any radius *x* is given by

$$\sigma_r = \frac{P_i (r_i)^2 - P_0 (r_0)^2}{(r_0)^2 - (r_0)^2} - \frac{(r_i)^2 (r_0)^2}{x^2} \left[\frac{p_i - p_0}{(r_0)^2 - (r_i)^2}\right] \qquad (21)$$

Considering the internal pressure only ($P_i = P$) which assists the spreading of the pulse very fast. In this case, let external pressure, $P_0 = 0$,

From Equation 20, the tangential stress at any radius *x* is give by.

$$\sigma_t = \frac{P_i (r_i)^2}{(r_0)^2 - (r_0)^2} \left[1 + \frac{(r_0)^2}{x^2}\right] \qquad (22)$$

$$\sigma_r = \frac{P_i (r_i)^2}{(r_0)^2 - (r_0)^2} \left[1 + \frac{(r_0)^2}{x^2}\right] \qquad (23)$$

From equation (22) and (23), the tangential stress is a tensile stress whereas the radial stress is a compressive stress.

Again, the tangential stress is Maximum at the inner surface of the pipeline ie. $x = r_i$ and it is minimum at the outer surface of the shell ie. $x = r_0$

By taking the value of $\boldsymbol{x = r_i}$ and $\boldsymbol{x = r_0}$ in Equation (21) and (22), the Maximum tangential stress at the inner surface of the pipeline is given by,

$$\sigma_{(t_{Max})} = \frac{P[(r_0)^2 + (r_i)^2]}{(r_0)^2 - (r_i)^2} \qquad (24)$$

While the minimum tangential stress at the outer surface of the shell is given by,

$$\sigma_{(t_{Min})} = \frac{2P[(r_i)^2]}{(r_0)^2 - (r_i)^2} \qquad (25)$$

$\sigma_{(t_{Max})}$ **= -p (Compressive) and at** $\sigma_{(t_{Min})}$ **=0** **(26)**

Therefore, Equations (25) and (26) becomes the major maximum and minimum stress equation for the fibre pipeline design. Now, the tangential stress is always maximum at the inner surface of the core ie when $x = r_1$ and is minimum at the outer surface where $x = r_o$

V. CONCLUSION AND RECOMMENDATION

Conclusively, this model, gives adequate impedance matching to any external force which would have led to signal degradation leading to various dispersion failures (group velocity dispersion- GVD, polarisation mode dispersion- PMD, material dispersion and chromatic dispersion- CD). When implemented, it will ensure that the intelligent signals being propagated are decoded accurately.

Hence, it is recommended that this neutrality of external pressure can be carried using sensor simulation by subsequent researchers.

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