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Computation of Modulus of Elasticity of Concrete

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Abstract: - In this presentation, a computer based method which uses a set of algebraic equations and statistical data, were used to compute concrete mixes for prescribeable elastic concrete modulus, and vice versa. The computer programs based on Simplex and Regression theories can be used to predict several mix proportions for obtaining a desired modulus of elasticity of concrete made from crushed granite rock and other materials. The modulus of elasticity of concrete predicted by these programs agreed with experimentally obtained values. The programs are easy and inexpensive to use, and give instant and accurate results. For example, if the modulus of elasticity is specified as input, the computer instantly prints out all possible concrete mix ratios that can yield concrete having the specified elastic modulus. When the concrete mix ratio is specified as input, the computer quickly prints out the elastic modulus of the concrete obtainable from a given concrete mix ratio.

Keywords: - *Modulus of elasticity, Concrete, Computer programs, Simplex theory, Regression theory.*

I. INTRODUCTION

The predictions of the values of modulus of elasticity of concrete for the design of concrete structures have always required extensive sampling and testing. This has always limited design to only specific concrete mixtures whose values are known. The existing empirical methods of obtaining specific values of elastic modulus of concrete are cumbersome, labor intensive and time consuming with high degree of errors (Simon,2003).

In this work, simplex design method (1958) and a modified regression design method (2003) that use the theory of statistics and experimental results to obtain response functions, were employed in the development of computer programs for the computation of elastic modulus of concrete obtainable from a specified mix proportion and vice versa. The advantages of the developed programs, are that the programs eliminate the need for trial mixes, save time and cost, and give more accurate results.

II. NUMERICAL ANALYSIS

The computer programs are developed from response functions based on two statistical theories, namely, simplex theory and regression theory for determining properties of concrete mixes. The main advantage of this statistical approach, is that computer programming can be applied to the response functions derived from theories.

2.1 Simplex Response Function

The first response function for the computation of the elastic modulus of a normal concrete, is derived from simplex theory (1958), which uses practical data from laboratory tests, to formulate expression for determining the modulus of elasticity. The response function, Y_i, gives the relationship between the response (i.e. the property sought) and the proportions of concrete constituents.

In simplex theory, the general response function applicable to a mixture with ‘q’ constituents is as follows:

$$Y_i = b_0 + \sum_{i=1}^{q} b_i x_1 + \sum_{i=1}^{q} \sum_{j}^{q} b_{ij} x_i x_j \quad \text{.......} \quad (1)$$

where b_i and b_{ij} are constants

X_i and X_j are pseudo constituents

Two conditions must be satisfied for Eqn(1) to hold, namely,
(i) The first condition is that the i^{th} component, X_i, must be greater than zero and or less than or equal to one. .i.e.

$$0 < X_i \leq 1 \quad \ldots\ldots(2)$$

where $i = 1, 2, \ldots q$

(ii) And the second condition is that the sum of the components must be equal to one, i.e.

$$X_1 + X_2 + X_3 + \ldots\ldots X_q = 1 \quad \ldots\ldots (3)$$

For a four- component mixture,
X_1 = proportion of water-cement (w/c) ratio
X_2 = proportion of cement
X_3 = proportion of sand
X_4 = proportion of crushed rock

The response function, Y_i, developed for a mixture with four pseudo components and two degree reaction, is given by Eqn(4).

$$Y_i = b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4 + b_{12}X_1X_2 + b_{13}X_1X_3 + b_{14}X_1X_4 + b_{23}X_2X_3 + b_{24}X_2X_4 + b_{34}X_3X_4 \quad \ldots\ldots (4)$$

And the final response function formulated for the determination of elastic modulus of concrete, using simplex theory and experimental results, is as follows:

$$Y = 41.31X_1 + 50.04X_2 + 25.21X_3 + 19.24X_4 - 95.58X_1X_2 - 28.64X_1X_3 + 22.10X_1X_4 - 55.76X_2X_3 - 25.96X_2X_4 + 14.26X_3X_4 \quad \ldots\ldots (5)$$

The details of the derivation of the function is given by Egbulonu (2011).

The actual components, Z_i, used in the mixture experiment are transformed from the Eqn (6)

$$[Z] = [A][X] \quad \ldots\ldots(6)$$

Conversely,

$$[X] = [B][Z] \quad \ldots\ldots(7)$$

where,
[Z] = matrix of real component proportions
[X] = matrix of pseudo component proportions
[A] = matrix of arbitrary mix proportions based on past experience.
[B] = inverse of matrix A

The values of the real components, Z_i, are used for conducting the laboratory tests for the determination of experimental values of the elastic modulus Y_i and Y_{ij}

2.2 REGRESSION RESPONSE FUNCTION

The second computer program is based on a continuous regression function which can be differentiated with respect to its variables, Z_i. For a mixture such as concrete whose responses (desired properties), are dependent on the proportions of its constituents, its response function is as follows:

$$Y = \sum_{i=1}^{4} a_i Z_i + \sum_{i=1}^{4} \sum_{j=1}^{4} a_{ij} Z_i Z_j \quad \ldots\ldots (8)$$

where
Z_i and Z_j = Predictors or fractional portion of the ith and jth components respectively.
a_i and a_j = coefficients of the regression function

Since the property of interest in this work, is the the modulus of elasticity, E, of concrete, the response function developed for it using Eqn(8) is as follows:

$$Y = a_1Z_1 + a_2Z_2 + a_3Z_3 + a_4Z_4 + a_{12}Z_1Z_2 + a_{13}Z_1Z_3 + a_{14}Z_1Z_4 + a_{23}Z_2Z_3 + a_{24}Z_2Z_4 + a_{34}Z_3Z_4 \quad \ldots\ldots(9)$$

At the 'n^{th}' observation point, the response function, $Y^{(n)}$ corresponding with the predictor, Z_i, is given by Eqn(10)

$$[Y^{(n)}] = [Z^{(n)}]\{a\} \quad \ldots\ldots(10)$$

where
$[Y^{(n)}]$ = matrix of the response functions
$[Z^{(n)}]$ = matrix of Predictors
$[a]$ = matrix of constant coefficients

The elements of the response matrix [$Y^{(n)}$], are obtained from the laboratory tests while the elements of the matrix of predictors, [$Z^{(n)}$] are obtained from the matrix of actual proportions [$S^{(n)}$] . And the values of the coefficient matrix, [a] are determined from Eqn(10) and substituted into Eqn(9) to obtain the final regression response function i.e Eqn(11)) on which the second program is based.

$$\mathbf{Y} = 5351667.6400\mathbf{Z_1} + 888151.9143\mathbf{Z_2} + 1835.219102\mathbf{Z_3} + 2392.479301\mathbf{Z_4} - 10609392.05\mathbf{Z_1Z_2} - 5791804.077\mathbf{Z_1Z_3} - 5620199.635\mathbf{Z_1Z_4} - 699734.4294\mathbf{Z_2Z_3} - 786415.528\,\mathbf{Z_2Z_4} + 12085.08274\,\mathbf{Z_3Z_4} \quad \ldots\ldots (11)$$

Thus, the modulus of elasticity of a four- component concrete mixture, can be determined from Eqn(11). Full details of its derivation are contained in the works of Egbulonu(2011).

III. COMPUTER PROGRAMS

Computer programs based on these models, were developed for easy, quick, cheap and precise computation of mix proportions of concrete given its elastic modulus, E. The computer programs can also determine modulus of elasticity, E, from a given concrete mix proportion. The programs based on simplex theory and regression theory are as presented in Appendix A and Appendix B respectively. And Tables 1 and 2 contain the outputs of the computer programs.

IV. RESULTS AND ANALYSIS

The inputs required to run the programs are either modulus of elasticity or mix proportions. If the desired modulus of elasticity is fed into the computer, the computer either:

(a) Prints out all possible combinations of actual mix proportions (Z_i) and the corresponding pseudo proportions (X_i) that match the elastic modulus, Y.
(b) Prints out the maximum elastic modulus that can be predicted from program.
(c) Notify the user if there is no matching combination.

On the other hand, if the mix proportion is the input, the computer prints out the elastic modulus obtainable from the given mix proportion.
These outputs are summarized in Tables 1 and 2.
The result of computer programs for specified modulus of elasticity of 36N/mm^2, is presented in Tables 1 and 2.

Table 1: Output of simplex mix ratios corresponding to an input of elastic modulus, Y, of 36N/mm^2

S/N	X_1	X_2	X_3	X_4	Y	Z_1	Z_2	Z_3	Z_4
1	0.08	0.82	0.09	0.01	36.00	0.55	1.00	1.93	2.99
2	0.10	0.79	0.01	0.10	36.00	0.50	1.00	2.00	3.06

where

X_1 = pseudo values of water — Z_1 = actual values of water
X_2 = pseudo values of cement — Z_2 = actual values of cement
X_3 = pseudo values of sand — Z_3 = actual values of sand
X_4 = pseudo values of coarse aggregate — Z_4 = actual values of coarse aggregate

Table 2: Output of Regression- based mix ratios corresponding to an input of elastic modulus, Y, of 36N/mm^2

S/N	Z_1	Z_2	Z_3	Z_4	Y	S_1	S_2	S_3	S_4
1	0.06	0.10	0.25	0.59	36.00	0.64	1.00	2.58	5.98
2	0.08	0.15	0.30	0.46	36.00	0.56	1.00	2.00	3.07
3	0.10	0.20	0.32	0.37	36.00	0.51	1.00	1.62	1.85
4	0.11	0.23	0.34	0.32	36.00	0.50	1.00	1.52	1.42
5	0.13	0,26	0.34	0.34	36.00	0.49	1.00	1.30	1.07
6	0.13	0.27	0.26	0.34	36.00	0.47	1.00	0.95	1.24

where

S_1 =Actual portion of water
S_2 = Actual portion of cement
S_3 = Actual portion of sand
$\mathbf{S_4}$ = Actual portion of coarse aggregate

Table 3: Computer output of modulus of Elasticity, Y, corresponding to input of Various mix proportions

S/N	Simplex function					Regression functionl				
	Given mix ratios				MOE,Y N/mm^2	Given mix ratios				MOE
	Z_1	Z_2	Z_3	Z_4		S_1	S_2	S_3	S_4	
1	0.5600	1.0	1.7	3.05	**19.40**	0.5600	1.0	1.7	3.05	**20.35**
2	0.5650	1.0	2.0	3.30	**30.88**	0.5650	1.0	2.0	3.30	**30.17**
3	0.5825	1.0	2.1	3.65	**23.64**	0.5825	1.0	2.1	3.65	**24.80**
4	0.5850	1.0	2.1	3.60	**23.77**	0.5850	1.0	2.1	3.60	**24.54**
5	0.5900	1.0	2.2	3.70	**24.64**	0.5900	1.0	2.2	3.70	**24.53**
6	0.6400	1.0	2.8	4.80	**22.72**	0.6400	1.0	2.8	4.80	**22.07**

4.1 COMPARISON OF RESULTS

The sets of results (i.e elastic modulus) obtained from Simplex and regression functions, are comparable. Table 4 indicates that the maximum percentage difference between the modulus of elasticity obtained from Simplex function and those obtained from the Regression function, is 4.15%. Thus, the percentage differences is negligible. And so, the results are in agreement.

Table 4: Comparison of Simplex-based and Regression-based elastic moduli

S/N	Modulus of Elasticity (N/mm^2)		Difference	Percentage Difference
	Simplex Values	Regression Values		
1	19.40	20.35	0,95	4.91
2	30.88	30.17	0.71	2.30
3	23.64	24.80	1.16	4.91
4	23.77	24.50	0.79	3.24
5	24.64	24.53	0.11	0.45
6	22.72	22.07	0.65	2.95

V. CONCLUSIONS

Based on the analysis of the results, the following conclusions were reached:

- Either of the programs based on Simplex theory or Regression theory, can be used in determining elastic modulus of normal concrete if the mix ratios are given as input.
- In addition, it can be used to determine the mix proportions that can produce concrete of desired elastic modulus.
- The maximum elasticity modulus that can be determined from the program , is 50.04 N/mm^2.
- The programs are easy and inexpensive to use, and give instant and accurate results.

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APPENDIX A:PROGRAM FOR COMPUTATION MODULUS OF ELASTICITY OF CONCRETE USING SIMPLEX MODEL

```
CLS
 REM A Q-BASIC PROGRAMM THAT OPTIMISES CONCRETE MIX PROPORTIONS
 REM THIS WAS WRITTEN BY BEN EGBULONU
 REM X1,X2,X3,X4,Z1,Z2,Z3,Z4,Ymax,Yout,Yin
 REM MODEL USED ELASTIC MODULUS: BY SCHEFFE'S MODEL
 REM MAIN PROGRAMM BEGINS
     REM TO DEFINE WHAT IS GIVEN AND WHAT REQUIRED TO DETERMINE.
     REM WHEN MIX IS GIVEN G=1 0R G=2 IF E IS GIVEN
     PRINT "DEFINE WHAT IS GIVEN AS G="; G;
     INPUT G
     IF G = 1 THEN  ELSE GOTO 5
     REM MIX GIVEN AS Z1,Z2,Z3,Z4.
     PRINT "ENTER VALUE OF W/C=Z1"; Z1
     INPUT Z1
     PRINT "ENTER VALUE OF CEMENT=Z2"; Z2
     INPUT Z2
     PRINT "ENTER VALUE OF SAND=Z3"; Z3
     INPUT Z3
     PRINT "ENTER VALUE OF AGG=Z4"; Z4
     INPUT Z4
     LET X1 = 40 * Z1 - 16 * Z2 + 0 * Z3 - 2 * Z4
     LET X2 = -60 * Z1 + 26 * Z2 + 1 * Z3 + 2 * Z4
     LET X3 = -20 * Z1 + 9 * Z2 - 2 * Z3 + 2 * Z4
     LET X4 = 40 * Z1 - 18 * Z2 + 1 * Z3 - 2 * Z4
     LET E = 41.31 * X1 + 50.04 * X2 + 25.21 * X3 + 19.24 * X4 - 95.58 * X1 * X2 - 28.64 * X1 * X3 + 22.1 *
X1 * X4 + -55.76 * X2 * X3 - 25.96 * X2 * X4 + 14.26 * X3 * X4
     REM PRINT VALUE E
     PRINT "VALUE OF E="; E; "X1"; X1, "X2"; X2; "X3"; X3; "X4"; X4;
     GOTO 7
5 LET COUNT = 0
 CLS
 GOSUB 10
7 END
 REM END OF MAIN PROGRAMM
10 REM PROCEDURE BEGINS
 LET Ymax = 0
 PRINT
 PRINT
 REM A COMPUTER MODEL FOR COMPUTING CONCRETE MIX PROPORTIONS
 PRINT "CORRESPONDING TO A REQUIRED ELASTIC MODULUS"
 PRINT
 INPUT "ENTER DESIRED ELASTIC MODULUS"; Yin
 GOSUB 70
  FOR X1 = O TO 1 STEP .01
    FOR X2 = O TO 1 - X1 STEP .01
      FOR X3 = O TO 1 - X1 - X2 STEP .01
```

```
    LET X4 = 1 - X1 - X2 - X3
LET Yout = 41.31 * X1 + 50.04 * X2 + 25.21 * X3 + 19.24 * X4 - 95.58 * X1 * X2 - 28.64 * X1 * X3 + 22.1 *
X1 * X4 - 56.1 * X2 * X3 - 25.96 * X2 * X4 + 14.26 * X3 * X4
    GOSUB 80
IF (ABS(Yin - Yout) <= .001) THEN 20 ELSE 30
20   LET COUNT = COUNT + 1
    GOSUB 90
30   NEXT X3
   NEXT X2
 NEXT X1
 PRINT
 IF (COUNT > 0) THEN GOTO 40 ELSE GOTO 50
40 PRINT "THE MAXIMUM ELASTIC MODULUS PREDICTABLE"
 PRINT "BY THIS MODEL IS"; Ymax; "N/sq.mm"
 SLEEP (2)
 GOTO 60
50 PRINT "SORRY! DESIRED ELASTIC MODULUS OUT OF RANGE OF MODEL"
 SLEEP 2
60 RETURN
70 REM PROCEDURE PRINT HEADING
 REM
 PRINT "COUNT X1 X2 X3 X4 Y Z1 Z2 Z3 Z4"
 REM
 RETURN
80 REM PROCEDURE CHECK MAX
 IF Ymax < Yout THEN Ymax = Yout ELSE Ymax = Ymax
 RETURN
90 REM PROCEDURE OUT RESULTS
 LET Z1 = .5 * X1 + .55 * X2 + .6 * X3 + .65 * X4
 LET Z2 = X1 + X2 + X3 + X4
 LET Z3 = X1 + 2 * X2 + 2 * X3 + 3 * X4
 LET Z4 = 1.5 * X1 + 3 * X2 + 4 * X3 + 5 * X4
 PRINT TAB(1); COUNT; USING "###.##"; X1; X2; X3; X4; Yout; Z1; Z2; Z3; Z4
 RETURN
```

APPENDIX B:PROGRAM FOR COMPUTATION OF MODULUS OF ELASTICITY OF CONCRETE USING REGRESSION MODELCLS

```
REM ** 100% CORRECT** OK ** NO MAXIMUM** RUN TIME =60sec TO 5mins ***
REM A Q-BASIC PROGRAMM THAT OPTIMISES CONCRETE MIX PROPORTIONS
REM THIS WAS WRITTEN BY BEN EGBULONU
REM VARIABLES USED ARE : Z1,Z2,Z3,Z4,S1,S2,S3,S4,Ssum,Ymax,Yout,Yin
REM MODEL USED ELASTIC MODULUS
REM PROGRAM PRINTING SPLIT IN 1ST-17, THEN 20s TO ENTER PRINTING WINDOW.
REM MAIN PROGRAMM BEGINS
    REM WHEN MIX IS GIVEN G=1 0R G=2 IF E IS GIVEN
    PRINT "DEFINE WHAT IS GIVEN AS G="; G;
    INPUT G
    IF G = 1 THEN  ELSE GOTO 5
    REM MIX GIVEN AS ACTUAL MIX RATIOS, S1.S2.S3.S4.
    PRINT "ENTER VALUE OF W/C=S1"; S1
    INPUT S1
    PRINT "ENTER VALUE OF CEMENT=S2"; S2
    INPUT S2
    PRINT "ENTER VALUE OF SAND=S3"; S3
    INPUT S3
    PRINT "ENTER VALUE OF AGG=S4"; S4
    INPUT S4
    LET Ssum = S1 + S2 + S3 + S4
    REM CALCULATING FRACTIONAL PARTS
```

```
    LET Z1 = S1 / Ssum: Z2 = S2 / Ssum: Z3 = S3 / Ssum: Z4 = S4 / Ssum
REM       ****** CEFFICIENTS OS REGRESSION *******
    A1 = 5351667.64#: A2 = 888151.9142999999#: A3 = 1835.219102#
    A4 = 2392.479301#: A5 = -10609392.05#: A6 = -5791804.077#
    A7 = -5620199.635#: A8 = -699734.4294#: A9 = -786415.528#
    A10 = 12085.08274#
REM CALCULATING ACTUAL MODULUS OF ELASTICITY
    LET MOE = A1 * Z1 + A2 * Z2 + A3 * Z3 + A4 * Z4 + A5 * Z1 * Z2 + A6 *
    Z1 * Z3 + A7 * Z1 * Z4 + A8 * Z2 * Z3 + A9 * Z2 * Z4 + A10 * Z3 * Z4
    REM PRINT VALUE OF MOE
    PRINT "VALUE OF MOE="; MOE; "S1"; S1; "S2"; S2; "S3"; S3; "S4"; S4;
    "Z1"; Z1; "Z2"; Z2; "Z3"; Z3; "Z4"; Z4
    GOTO 7
5 LET COUNT = 0
 CLS
 GOSUB 10
7 END
 REM END OF MAIN PROGRAMM
10 REM PROCEDURE BEGINS
 LET Ymax = 0
 PRINT
 PRINT
 REM A COMPUTER MODEL FOR COMPUTING CONCRETE MIX PROPORTIONS
 PRINT "CORRESPONDING TO A REQUIRED MODULUS OF ELASTICITY"
PRINT
 INPUT "ENTER DESIRED MODULUS OF ELASTICITY"; Yin
 GOSUB 70
 FOR Z1 = O.035 TO .25 STEP .001
 FOR Z2 = .07 TO .28 STEP .001
 FOR Z3 = .22 TO .35 STEP .001
 LET Z4 = 1 - Z1 - Z2 - Z3
REM       ****** CEFFICIENTS OS REGRESSION *******
    A1 = 5351667.64#: A2 = 888151.9142999999#: A3 = 1835.219102#
    A4 = 2392.479301#: A5 = -10609392.05#: A6 = -5791804.077#
    A7 = -5620199.635#: A8 = -699734.4294#: A9 = -786415.528#
    A10 = 12085.08274#
LET Yout = A1 * Z1 + A2 * Z2 + A3 * Z3 + A4 * Z4 + A5 * Z1 * Z2 + A6 * Z1 * Z3 + A7 * Z1 * Z4 + A8 *
Z2 * Z3 + A9 * Z2 * Z4 + A10 * Z3 * Z4
 IF ABS(Yin - Yout) <= .001 THEN 20 ELSE GOTO 30
20 LET COUNT = COUNT + 1
 GOSUB 90
30 NEXT Z3
 NEXT Z2
 NEXT Z1
 PRINT
 IF (COUNT > 0) THEN GOTO 60 ELSE GOTO 50
50 PRINT "SORRY! DESIRED MOE OUT OF RANGE OF MODEL"
  SLEEP 1
60 RETURN
70 REM PROCEDURE PRINT HEADING
 REM
 PRINT "COUNT Z1 Z2 Z3 Z4 Y S1 S2 S3 S4"
 REM
 RETURN
90 REM PROCEDURE OUT RESULTS
 LET S1 = Z1 / Z2: S2 = Z2 / Z2: S3 = Z3 / Z2: S4 = Z4 / Z2
 REM ************** 1ST PRINTING 0-18,NEXT 17-37,ETC.**********
 PRINT TAB(1); COUNT; USING "###.##"; Z1; Z2; Z3; Z4; Yout; S1; S2; S3; S4
100 RETURN
```