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Mhd Unsteady mixed convective flow between twoin finite Vertical Parallel Plates through Porous Medium in Slip flow Regime with Thermal Diffusion

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Abstract: - In this paper we have studied a free and forced convective flow of a viscous incompressible fluid through a vertical porous channel bounded by two vertical plates moving with same velocity but in opposite directions, with slip parameters. The temperature and concentration of the plate at y = 0 is considered to be oscillating. Expressions for velocity, temperature, concentration along with skin friction and Nusselt number are obtained and comparative study is made to analyze the effects of different parameters. We observe that increase in velocity slip parameter (h_1) decreases the skin friction. Also, it is noteworthy that Nusselt number is higher for water (Pr = 7) as compared for air (Pr = 0.71).

Keywords: - *Mixed convection, Porous medium, Suction velocity, Thermal diffusion, Unsteady.*

I. INTRODUCTION

MHD free convection flows are of great interest in a number of industrial applications such as fiber and granular insulation, geothermal systems etc. The science of magneto hydrodynamics (MHD) was concerned with geophysical and astrophysical problems for a number of years. In recent years, the possible use of MHD is to affect a flow stream of an electronically conducting fluid for the purpose of thermal protection, breaking, propulsion, control etc. We also study the mechanism of electronically conducting fluids for example, magma, highly salted water, liquid metals etc. MHD plays an important role in many engineering and industrial problems such as liquid metal cooling, in nuclear reactors, plasma confinement, control of molten iron flow and many others. Mbeledogu et. al. [1] studied an unsteady MHD free convection flow of a compressible fluid past a moving vertical plate. Moreover, influence of viscous dissipation and radiation on unsteady MHD free convection flow past an infinitely heated vertical plate has been studied by Cookey et. al. [2]. On the other hand Singh and Paul [3] studied natural convection between two vertical walls.

During the last decade many research workers have studied mixed convection in channels, which is a phenomenon in many technological processes, such as designs of solar collectors, thermal designs of buildings, air conditioning etc. Barelletta and Celli [4] investigated a mixed convection MHD flow in a vertical channel where as Rajput and Sahu [5] studied a transient free convection MHD flow between two long vertical parallel plates with variable temperature and uniform mass diffusion in a porous medium. Narahari et. al. [6] discussed a transient free convection flow between two long vertical parallel plates with constant heat flux at one boundary. Working on a horizontal channel, Brown and Lai [7] studied correlations for combined heat and mass transfer from an open cavity.

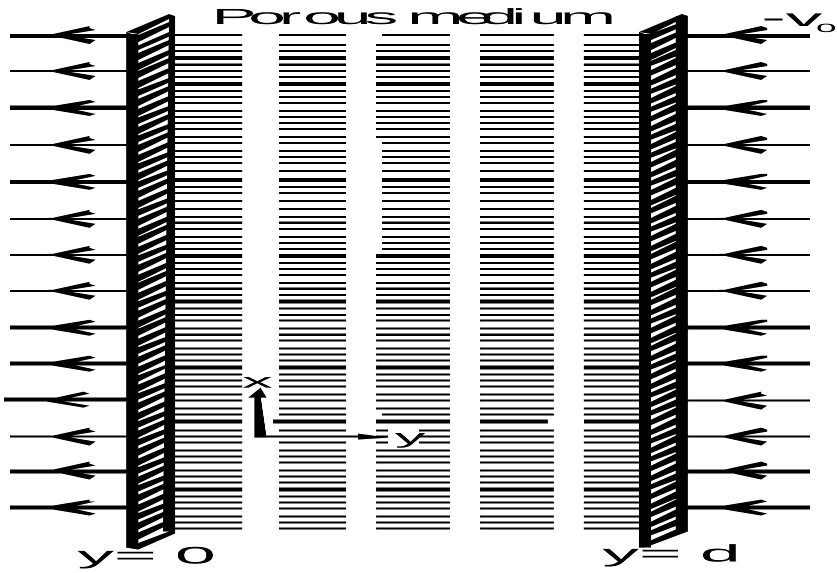
The study of flows through porous medium holds importance in many scientific and engineering applications such as for filteration and purification process, to study the movement of natural gas, water and oil through the oil reservoirs. In view of these applications a series of investigation have been made by Raptis et. al. [8, 9, 10]. Also Geindreau and Auriault [11] studied MHD flows in porous media. On the other hand, Alagoa et. al. [12] investigated radiation and free convection effects of a MHD flow through porous medium between infinite parallel plates. Moreover, Farhad et. al. [13] discussed an accelerated MHD flow in a porous medium with slip condition.

It the present paper, we have analyzed a problem on unsteady free convection MHD flow with mixed convection heat and mass transfer, in a channel filled with porous material, bounded by two vertical parallel plates moving in opposite direction with respect to each other, in slip flow regime. The temperature and mass concentration of the upward moving plate are considered to be oscillating with time. Effects of different parameters entering into the problem are shown graphically on velocity, temperature, concentration, skin friction and Nusselt number. We clearly observe that decrease in velocity slip parameter (h_1) increases the sinusoidal skin friction but it causes the sinusoidal rate of heat transfer to drop.

In many problems like thin film rarefied fluid, fluid containing concentrated suspension, the no slip boundary conditions fails to work. Mankinde and Osalusi [14] have made studies on MHD steady flow in a channel with slip at the permeable boundaries. Moreover, Taneja and Jain [15] discussed MHD flow with slip effects and temperature dependent heat source in a viscous incompressible fluid confined between a long vertical wavy walls and a parallel flat wall.

II. FORMULATION OF THE PROBLEM

In two dimensional rectangular Cartesian Coordinate system, we consider an unsteady free convective flow of an incompressible fluid through a vertical channel formed by two parallel plates moving with equal velocity ‘U’ but in opposite directions, at a distance d apart. The temperature and mass concentration of plate at y = 0, oscillates about a constant non zero mean T_0 and C_0. The suction velocity v_0 and permeability K of the porous medium are taken to be constant.



(Figure 1 : Schematic diagram.)

Hence under these conditions, using Boussinesq’s approximation, equations governing the flow in the presence of magnetic field of uniform strength B_0, are given by:

Momentum Equation

$$\frac{\partial u}{\partial t} - v_0 \frac{\partial u}{\partial y} = g\beta(T - T_d) + g\beta^*(C - C_d) + \nu \frac{\partial^2 u}{\partial y^2} - \frac{\nu}{K} u - \frac{\sigma B_0^2 u}{\rho} \qquad \ldots(1)$$

Energy Equation

$$\frac{\partial T}{\partial t} - v_0 \frac{\partial T}{\partial y} = \frac{\kappa}{\rho C_p} \frac{\partial^2 T}{\partial y^2} + \frac{S}{\rho C_p}(T - T_d) \qquad \ldots(2)$$

Concentration Equation

$$\frac{\partial C}{\partial t} - v_0 \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} + D_\ell \frac{\partial^2 T}{\partial y^2} \qquad \ldots(3)$$

where u and v are the components of the velocity in the x and y direction, g is the acceleration due to gravity, β and β* are the coefficient of volume expansion and species concentration expansion respectively, D is the chemical molecular diffusivity, S is the coefficient of heat source, D_ℓ is coefficient of thermal diffusivity, ρ,

ν, □ and C_p respectively the density, kinematic viscosity, thermal conductivity and specific heat of the fluid at constant pressure. T_d and C_d are the temperature and concentration of the plate at y = d.
We assume that the magnetic Reynolds number is small so that the induced magnetic field is negligible.
The relevant boundary conditions are:

$$\left.\begin{aligned} u &= U + L_1\frac{\partial u}{\partial y},\ T = T_0 + \in (T_0 - T_d)e^{i\omega t}, C = C_0 + \in (C_0 - C_d)e^{i\omega t} \text{ at } y = 0 \\ u &= -U + L_1\frac{\partial u}{\partial y},\ T = T_d, \qquad C = C_d \qquad \text{at } y = d \end{aligned}\right\} \quad \ldots(4)$$

where $L_1\left(\frac{2-m_1}{m_1}\right)L$, L being mean free path and m_1 the Maxwell's reflection coefficient.

On introducing the following non-dimensional quantities:

$$u^* = \frac{u}{U}, \qquad t^* = t\omega, \qquad y^* = y/d$$

$$\omega^* = \frac{\omega d^2}{\nu}\text{(frequency)}, \qquad \theta^* = \frac{T-T_d}{T_0-T_d},$$

$$C^* = \frac{C-C_d}{C_0-C_d}, \qquad K^* = \frac{K\,v_0^2}{\nu^2}\text{(Permeability parameter)},$$

$$Pr = \frac{\mu C_p}{\kappa}\ \text{(Prandtl number)}, \qquad \lambda = \frac{v_0 d}{\nu}\ \text{(Suction parameter)},$$

$$M = B_0 d\left(\frac{\sigma}{\mu}\right)^{1/2}\ \text{(Hartmann number)}, \qquad S^* = \frac{\nu^2 S}{\kappa\,v_0^2}\ \text{(Heat source parameter)},$$

$$Gr = \frac{\nu g\beta(T_0 - T_d)}{U\,v_0^2}\text{(Thermal Grashof number)},$$

$$Gc = \frac{\nu g\beta^*(C_0 - C_d)}{U\,v_0^2}\text{(Mass Grashof number)},$$

$$So = \frac{D_\ell(T_0 - T_d)}{\nu(C_0 - C_d)}\ \text{(Soret number)}, \qquad Sc = \frac{\nu}{D}\ \text{(Schmidt number)},$$

$$h_1 = \frac{L_1}{d}\ \text{(Velocity slip parameter)},$$

in equations (1) to (3), after dropping the asterisks over them they reduce to:

$$\frac{\omega}{\lambda}\frac{\partial u}{\partial t} - \frac{\partial u}{\partial y} = Gr\,\lambda\theta + Gc\,\lambda C + \frac{1}{\lambda}\frac{\partial^2 u}{\partial y^2} - \frac{\lambda}{K}u - \frac{M^2}{\lambda}u \quad \ldots(5)$$

$$\frac{\omega}{\lambda}\frac{\partial \theta}{\partial t} - \frac{\partial \theta}{\partial y} = \frac{1}{Pr\,\lambda}\frac{\partial^2 \theta}{\partial y^2} + \lambda S\theta \quad \ldots(6)$$

$$\frac{\omega}{\lambda}\frac{\partial C}{\partial t} - \frac{\partial C}{\partial y} = \frac{1}{Sc\,\lambda}\frac{\partial^2 C}{\partial y^2} + \frac{So}{\lambda}\frac{\partial^2 \theta}{\partial y^2}. \quad \ldots(7)$$

The corresponding boundary conditions reduce to:

$$\left.\begin{aligned} u &= 1 + h_1\frac{\partial u}{\partial y},\quad \theta = 1 + \in e^{it},\quad C = 1 + \in e^{it};\ \text{at } y = 0 \\ u &= -1 + h_1\frac{\partial u}{\partial y},\quad \theta = 0, \qquad C = 0 \qquad ;\ \text{at } y = 1 \end{aligned}\right\} \quad \ldots(8)$$

III. SOLUTION OF THE PROBLEM

Since the amplitude $\in(<<1)$, we represent the velocity, temperature and concentration as:

$$f(y,t) = f_0(y) + \in e^{it} f_1(y) \quad \ldots(9)$$

where f stands for u, θ and C. With the help of equation (9), the equations (10) to (11) reduces to the following ordinary differential equations by equating like powers of $\in$, neglecting those of $\in^2$ and higher orders:

$$u_0'' + \lambda u_0' - \left(\frac{\lambda^2}{K} + M^2\right)u_0 = -Gr\,\lambda^2\theta_0 - Gc\,\lambda^2 C_0 \quad \ldots(10)$$

$$u_1'' + \lambda u_1' - \left(\frac{\lambda^2}{K} + M^2 + i\omega\right)u_1 = -Gr\,\lambda^2\theta_1 - Gc\,\lambda^2 C_1 \quad \ldots(11)$$

$$\theta_0'' + \Pr\lambda\,\theta_0' + \Pr\lambda^2 S\theta_0 = 0 \quad \ldots(12)$$

$$\theta_1'' + \Pr\lambda\,\theta_1' + (\Pr\lambda^2 S - i\Pr\omega)\theta_1 = 0 \quad \ldots(13)$$

$$C_0'' + Sc\,\lambda\,C_0' = -So\,Sc\,\theta_0'' \quad \ldots(14)$$

$$C_1'' + Sc\,\lambda\,C_1' - i\omega Sc\,C_1 = -So\,Sc\,\theta_1'' \quad \ldots(15)$$

where the prime denotes differentiation with respect to y.
The corresponding boundary conditions becomes:

$$\left.\begin{aligned} &u_0 = 1 + h_1 u_0',\ u_1 = h_1 u_1';\theta_0 = 1, \theta_1 = 1;\ C_0 = 1, C_1 = 1;\ \text{at } y = 0 \\ &u_0 = -1 + h_1 u_0', u_1 = h_1\ u_1';\theta_0 = 0, \theta_1 = 0; C_0 = 0, C_1 = 0;\ \text{at } y = 1 \end{aligned}\right\} \quad \ldots(16)$$

By solving the equations (10) to (15) under boundary conditions (16), we get:

$$\theta_0 = \frac{1}{e^{R_1} - e^{R_2}}\left(e^{R_1+R_2 y} - e^{R_2+R_1 y}\right) \quad \ldots(17)$$

$$\theta_1 = \frac{1}{e^{R_3} - e^{R_4}}\left(e^{R_3+R_4 y} - e^{R_4+R_3 y}\right) \quad \ldots(18)$$

$$C_0 = R_5 e^{R_2 y} + R_6\,e^{R_1 y} + R_7 + R_8\,e^{-Sc\,\lambda y} \quad \ldots(19)$$

$$C_1 = R_{11}e^{R_3 y} + R_{12}e^{R_4 y} + R_{13}\,e^{R_9 y} + R_{14}e^{R_{10} y} \quad \ldots(20)$$

$$u_0 = D_1\,e^{R_{15} y} + D_2\,e^{R_{16} y} - R_{17}e^{R_2 y} + R_{18}\,e^{R_1 y} + R_{19} - R_{20}e^{-Sc\,\lambda y} \quad \ldots(21)$$

$$u_1 = D_3\,e^{R_{23} y} + D_4\,e^{R_{24} y} - R_{25}e^{R_4 y} + R_{26}e^{R_3 y} - R_{27}e^{R_9 y} - R_{28}e^{R_{10} y} \quad \ldots(22)$$

Substituting the equations (17) to (22) into (9) for u, θ and C, we have:

$$\begin{aligned} u(y,t) = {} & D_1\,e^{R_{15} y} + D_2\,e^{R_{16} y} - R_{17}e^{R_2 y} + R_{18}e^{R_1 y} + R_{19} - R_{20}e^{-Sc\,\lambda y} \\ & + \in[D_3 e^{R_{23} y} + D_4 e^{R_{24} y} - R_{25}e^{R_4 y} + R_{26}e^{R_3 y} - R_{27}e^{R_9 y} - R_{28}e^{R_{10} y}]e^{it} \end{aligned} \quad \ldots(23)$$

$$\theta(y,t) = \frac{1}{(e^{R_1} - e^{R_2})}\left(e^{R_1+R_2 y} - e^{R_2+R_1 y}\right) + \in\left[\frac{1}{(e^{R_3} - e^{R_4})}\left\{e^{R_3+R_4 y} - e^{R_4+R_3 y}\right\}\right]e^{it} \quad \ldots(24)$$

$$C(y,t) = \left(R_5\,e^{R_2 y} + R_6 e^{R_1 y} + R_7 + R_8 e^{-Sc\,\lambda y}\right) + \in\left(R_1 e^{R_3 y} + R_{12}e^{R_4 y} + R_{13}e^{R_9 y} + R_{14}e^{R_{10} y}\right)\ e^{it} \quad \ldots(25)$$

With convection that the real parts of complex quantities have physical significance in the problem, we have, the main flow velocity which can now be expressed as:

$$u(y,t) = u_0(y) + \in[U_r\ \cos t - U_i \sin t] \quad \ldots(26)$$

where

$U_r + i\,U_i = u_1(y)$ and

$$\begin{aligned} U_r\Big[& \left(e^{A_9 y}A_{17}\cos B_9 y\text{-}e^{A_9 y}B_{17}\sin B_9 y\right) + \left(e^{A_{10} y}A_{18}\cos B_{10} y - e^{A_{10} y}B_{18}\sin B_{10} y\right) \\ & + \left(e^{A_2 y}B_{11}\sin B_2 y - e^{A_2 y}A_{11}\cos B_2 y\right) + \left(e^{A_1 y}A_{12}\cos B_1 y - e^{A_1 y}B_{12}\sin B_1 y\right) \end{aligned}$$

$$+\left(e^{A_3 y}B_{13}\sin B_3 y-e^{A_3 y}A_{13}\cos B_3 y\right)+\left(e^{A_4 y}B_{14}\sin B_4 y-e^{A_4 y}A_{14}\cos B_4 y\right)\Big]$$

$$U_i=\Big[\left(e^{A_9 y}B_{17}\cos B_9 y+e^{A_9 y}A_{17}\sin B_9 y\right)+\left(e^{A_{10} y}B_{18}\cos B_{10} y+e^{A_{10} y}A_{18}\sin B_{10} y\right)$$

$$-\left(e^{A_2 y}B_{11}\cos B_2 y+e^{A_2 y}A_{11}\sin B_2 y\right)+\left(e^{A_1 y}B_{12}\cos B_1 y+e^{A_1 y}A_{12}\sin B_1 y\right)$$

$$-\left(e^{A_3 y}B_{13}\cos B_3 y+e^{A_3 y}A_{13}\sin B_3 y\right)-\left(e^{A_4 y}B_{14}\cos B_4 y+e^{A_4 y}A_{14}\sin B_4 y\right)\Big]$$

Hence, the expression for the velocity, for $t=\frac{\pi}{2}$, is given by:

$$u\left(y,\frac{\pi}{2}\right)=u_0(y)-\in U_i \qquad \ldots(27)$$

Similarly, the expression for the temperature profiles can now be expressed as:

$$\theta(y,t)=\theta_0(y)+\in\left[M_r\cos t-M_i\sin t\right] \qquad \ldots(28)$$

where

$$M_r+i\,M_i=\theta_1(y)$$

and

$$M_r=e^{A_1+A_2 y}\left[Z_1\left(\cos B_1\cos B_2 y-\sin B_1\sin B_2 y\right)+Z_2\left(\sin B_1\cos B_2 y+\cos B_1\sin B_2 y\right)\right]$$

$$-e^{A_1 y+A_2}\left[Z_1\left(\cos B_2\cos B_1 y-\sin B_2\sin B_1 y\right)+Z_2\left(\sin B_2\cos B_1 y+\cos B_2\sin B_1 y\right)\right]$$

$$M_i=e^{A_1+A_2 y}\left[Z_1\left(\sin B_1\cos B_2 y+\cos B_1\sin B_2 y\right)-Z_2\left(\cos B_1\cos B_2 y-\sin B_1\sin B_2 y\right)\right]$$

$$-e^{A_1 y+A_2}\left[Z_1\left(\sin B_2\cos B_1 y+\cos B_2\sin B_1 y\right)-Z_2\left(\cos B_2\cos B_1 y-\sin B_2\sin B_1 y\right)\right]$$

Hence, the expression for the temperature for $t=\frac{\pi}{2}$ is given by:

$$\theta\left(y,\frac{\pi}{2}\right)=\theta_0(y)-\in M_i \qquad \ldots(29)$$

and the expression for the concentration profiles can now be expressed as:

$$C(y,t)=C_0(y)+\in[N_r\cos t-N_i\sin t] \qquad \ldots(30)$$

Where $N_r+i\,N_i=C_1(y)$

and

$$N_r=\Big[\left(e^{A_1 y}A_5\cos B_1 y-e^{A_1 y}B_5\sin B_1 y\right)+\left(e^{A_2 y}A_6\cos B_2 y-e^{A_2 y}B_6\sin B_2 y\right)$$

$$+\left(e^{A_3 y}A_7\cos B_3 y-e^{A_3 y}B_7\sin B_3 y\right)+\left(e^{A_4 y}A_8\cos B_4 y-e^{A_4 y}B_8\sin B_4 y\right)\Big]$$

$$N_i=\Big[\left(e^{A_1 y}B_5\cos B_1 y+e^{A_1 y}A_5\sin B_1 y\right)+\left(e^{A_2 y}B_6\cos B_2 y+e^{A_2 y}A_6\sin B_2 y\right)$$

$$+\left(e^{A_3 y}A_7\sin B_3 y+e^{A_3 y}B_7\cos B_3 y\right)+\left(e^{A_4 y}A_8\sin B_4 y+e^{A_4 y}B_8\cos B_4 y\right)\Big]$$

Hence, the expression for the concentration for $t=\frac{\pi}{2}$ is given by:

$$C\left(y,\frac{\pi}{2}\right)=C_0(y)-\in N_i \qquad \ldots(31)$$

where

$$R_1=\frac{-\Pr\lambda+\sqrt{\Pr^2\lambda^2-4\Pr\lambda^2 S}}{2}, \qquad R_2=\frac{-\Pr\lambda-\sqrt{\Pr^2\lambda^2-4\Pr\lambda^2 S}}{2}$$

$$R_3 = \frac{-\Pr\lambda + \sqrt{\Pr^2\lambda^2 - 4(\Pr\lambda^2 S - i\Pr\omega)}}{2} = A_1 + iB_1$$

$$R_4 = \frac{-\Pr\lambda - \sqrt{\Pr^2\lambda^2 - 4(\Pr\lambda^2 S - i\Pr\omega)}}{2} = A_2 + iB_2$$

$$R_5 = \frac{-e^{R_1} So\, Sc\, R_2^2}{\left(e^{R_1} - e^{R_2}\right)\left(R_2^2 + Sc\,\lambda\right)}$$

$$R_6 = \frac{e^{R_2} So\, Sc\, R_1^2}{\left(e^{R_1} - e^{R_2}\right)\left(R_1^2 + Sc\,\lambda\right)}, \qquad R_7 = \frac{R_5\left(e^{-Sc\lambda} - e^{R_2}\right) + R_6\left(e^{-Sc\lambda} - e^{R_1}\right) - e^{-Sc\lambda}}{\left(1 - e^{-Sc\,\lambda}\right)}$$

$$R_8 = \frac{R_5\left(e^{R_2} - 1\right) + R_6\left(e^{R_1} - 1\right) + 1}{\left(1 - e^{-Sc\lambda}\right)}, \qquad R_9 = \frac{-Sc\lambda + \sqrt{Sc^2\lambda^2 + 4i\omega Sc}}{2} = A_3 + iB_3$$

$$R_{10} = \frac{-Sc\lambda - \sqrt{Sc^2\lambda^2 + 4i\omega Sc}}{2} = A_4 + iB_4$$

$$R_{11} = \frac{So Sc\, R_3^2 e^{R_4}}{\left(e^{R_3} - e^{R_4}\right)\left(R_3^2 + Sc\lambda R_3 - i\omega Sc\right)} = A_5 + iB_5$$

$$R_{12} = \frac{-So Sc\, R_4^2 e^{R_3}}{\left(e^{R_3} - e^{R_4}\right)\left(R_4^2 + Sc\lambda R_4 - i\omega Sc\right)} = A_6 + iB_6$$

$$R_{13} = \frac{R_{11}\left(e^{R_{10}} - e^{R_3}\right) + R_{12}\left(e^{R_{10}} - e^{R_4}\right) - e^{R_{10}}}{\left(e^{R_9} - e^{R_{10}}\right)} = A_7 + iB_7$$

$$R_{14} = \frac{R_{11}\left(e^{R_3} - e^{R_9}\right) + R_{12}\left(e^{R_4} - e^{R_9}\right) + e^{R_9}}{\left(e^{R_9} - e^{R_{10}}\right)} = A_8 + iB_8$$

$$R_{15} = \frac{-\lambda + \sqrt{\lambda^2 + 4\left(\frac{\lambda^2}{K} + M^2\right)}}{2}$$

$$R_{16} = \frac{-\lambda - \sqrt{\lambda^2 + 4\left(\frac{\lambda^2}{K} + M^2\right)}}{2}$$

$$R_{17} = \frac{\lambda^2 e^{R_1}(Gr + Gc\, R_5) - \lambda^2 e^{R_2} Gc\, R_5}{(e^{R_1} - e^{R_2})\left\{R_2^2 + \lambda R_2 - \left(\frac{\lambda^2}{K} + M^2\right)\right\}}$$

$$R_{18} = \frac{\lambda^2 e^{R_2}(Gr + Gc\, R_6) - \lambda^2 e^{R_1} Gc\, R_6}{(e^{R_1} - e^{R_2})\left\{R_2^2 + \lambda R_2 - \left(\frac{\lambda^2}{K} + M^2\right)\right\}}, \qquad R_{19} = \frac{\lambda^2 Gc\, R_7}{\left(\frac{\lambda^2}{K} + M^2\right)}$$

$$R_{20} = \frac{\lambda^2 Gc\, R_8}{\left\{Sc^2\lambda^2 - Sc\lambda^2 - \left(\frac{\lambda^2}{K} + M^2\right)\right\}}$$

$$R_{21} = 1 + R_{17}(1 - h_1 R_2) + R_{18}(h_1 R_1 - 1) - R_{19} + R_{20}(1 + \lambda h_1 Sc)$$

$$R_{22} = -1 + R_{17}(1 - h_1 R_2)e^{R_2} + R_{18}(h_1 R_1 - 1)e^{R_1} - R_{19} + R_{20}(1 + \lambda h_1 Sc)e^{-Sc\lambda}$$

$$D_1 = \frac{-R_{22} + R_{21}e^{R_{16}}}{\left(e^{R_{16}} - e^{R_{15}}\right)(1 - h_1 R_{15})}, \quad D_2 = \frac{R_{22} - R_{21}e^{R_{15}}}{\left(e^{R_{16}} - e^{R_{15}}\right)(1 - h_1 R_{16})}$$

$$R_{23} = \frac{-\lambda + \sqrt{\lambda^2 - 4\left(\frac{\lambda^2}{K} + M^2 + i\omega\right)}}{2} = A_9 + iB_9$$

$$R_{24} = \frac{-\lambda - \sqrt{\lambda^2 - 4\left(\frac{\lambda^2}{K} + M^2 + i\omega\right)}}{2} = A_{10} + iB_{10}$$

$$R_{25} = \frac{Gr\,\lambda^2 e^{R_3} + Gc\,\lambda^2 R_{12}\left(e^{R_3} - e^{R_4}\right)}{\left(e^{R_3} - e^{R_4}\right)\left\{R_4^2 + \lambda R_4 - \left(\frac{\lambda^2}{K} + M^2 + i\omega\right)\right\}} = A_{11} + iB_{11}$$

$$R_{26} = \frac{Gr\,\lambda^2 e^{R_4} - Gc\,\lambda^2 R_{11}\left(e^{R_3} - e^{R_4}\right)}{\left(e^{R_3} - e^{R_4}\right)\left\{R_3^2 + \lambda R_3 - \left(\frac{\lambda^2}{K} + M^2 + i\omega\right)\right\}} = A_{12} + iB_{12}$$

$$R_{27} = \frac{Gc\,\lambda^2 R_{13}}{\left\{R_9^2 + \lambda R_9 - \left(\frac{\lambda^2}{K} + M^2 + i\omega\right)\right\}} = A_{13} + iB_{13}$$

$$R_{28} = \frac{Gc\,\lambda^2 R_{14}}{\left\{R_{10}^2 + \lambda R_{10} - \left(\frac{\lambda^2}{K} + M^2 + i\omega\right)\right\}} = A_{14} + iB_{14}$$

$$R_{29} = R_{25}(1 - h_1 R_4) + R_{26}(h_1 R_3 - 1) + R_{27}(1 - h_1 R_9) + R_{28}(1 - h_1 R_{10}) = A_{15} + iB_{15}$$

$$R_{30} = R_{25}(1 - h_1 R_4)e^{R_4} + R_{26}(h_1 R_3 - 1)e^{R_3} + R_{27}(1 - h_1 R_9)e^{R_9} + R_{28}(1 - h_1 R_{10})e^{R_{10}} = A_{16} + iB_{16}$$

$$D_3 = \frac{-R_{30} + R_{29}e^{R_{24}}}{\left(e^{R_{24}} - e^{R_{23}}\right)(1 - h_1 R_{23})} = A_{17} + iB_{17}$$

$$D_4 = \frac{R_{30} - R_{29}e^{R_{23}}}{\left(e^{R_{24}} - e^{R_{23}}\right)(1 - h_1 R_{24})} = A_{18} + iB_{18}$$

$$A_1 = \frac{-Pr\lambda}{2} + \frac{1}{2\sqrt{2}}\left[\sqrt{\left(Pr^2\lambda^2 - 4Pr\,\lambda^2 S\right)^2 + 16Pr^2\omega^2} + \left(Pr^2\lambda^2 - 4Pr\lambda^2 S\right)\right]^{1/2}$$

$$B_1 = \frac{1}{2\sqrt{2}}\left[\sqrt{\left(Pr^2\lambda^2 - 4Pr\lambda^2 S\right)^2 + 16Pr^2\omega^2} - \left(Pr^2\lambda^2 - 4Pr\,\lambda^2 S\right)\right]^{1/2}$$

$$A_2 = \frac{-Pr\lambda}{2} - \frac{1}{2\sqrt{2}}\left[\sqrt{\left(Pr^2\lambda^2 - 4Pr\,\lambda^2 S\right)^2 + 16Pr^2\omega^2} + \left(Pr^2\lambda^2 - 4Pr\lambda^2 S\right)\right]^{1/2}$$

$$B_2 = -\frac{1}{2\sqrt{2}}\left[\sqrt{\left(\Pr^2\lambda^2 - 4\Pr\lambda^2 S\right)^2 + 16\Pr^2\omega^2} - \left(\Pr^2\lambda^2 - 4\Pr\lambda^2 S\right)\right]^{1/2}$$

$$A_3 = \frac{-Sc\lambda}{2} + \frac{1}{2\sqrt{2}}\left[\sqrt{\left(Sc^2\lambda^2\right)^2 + 16Sc^2\omega^2} + \left(Sc^2\lambda^2\right)\right]^{1/2}$$

$$B_3 = \frac{1}{2\sqrt{2}}\left[\sqrt{\left(Sc^2\lambda^2\right)^2 + 16Sc^2\omega^2} - \left(Sc^2\lambda^2\right)\right]^{1/2}$$

$$A_4 = \frac{-Sc\lambda}{2} - \frac{1}{2\sqrt{2}}\left[\sqrt{\left(Sc^2\lambda^2\right) + 16Sc^2\omega^2} + \left(Sc^2\lambda^2\right)\right]^{1/2}$$

$$B_4 = -\frac{1}{2\sqrt{2}}\left[\sqrt{\left(Sc^2\lambda^2\right)^2 + 16Sc^2\omega^2} - \left(Sc^2\lambda^2\right)\right]^{1/2}$$

$$A_5 = \frac{P_5P_7 + P_6P_8}{P_7^2 + P_8^2}, \qquad B_5 = \frac{P_6P_7 - P_5P_8}{P_7^2 + P_8^2}$$

$$A_6 = \frac{P_9P_{11} + P_{10}P_{12}}{P_{11}^2 + P_{12}^2}, \qquad B_6 = \frac{P_{10}P_{11} - P_9P_{12}}{P_{11}^2 + P_{12}^2}$$

$$A_7 = \frac{Q_7\left(Q_1 + Q_3 + Q_5\right) + Q_8\left(Q_2 + Q_4 + Q_6\right)}{Q_7^2 + Q_8^2},$$

$$B_7 = \frac{Q_7\left(Q_2 + Q_4 + Q_6\right) - Q_8\left(Q_1 + Q_3 + Q_5\right)}{Q_7^2 + Q_8^2}$$

$$A_8 = \frac{Q_7\left(Q_9 + Q_{11} + Q_{13}\right) + Q_8\left(Q_{10} + Q_{12} + Q_{14}\right)}{Q_7^2 + Q_8^2},$$

$$B_8 = \frac{Q_7\left(Q_{10} + Q_{12} + Q_{14}\right) - Q_8\left(Q_1 + Q_{11} + Q_{13}\right)}{Q_7^2 + Q_8^2}$$

$$A_9 = \frac{-\lambda}{2} + \frac{1}{2\sqrt{2}}\left[\sqrt{\lambda^2 - \frac{4\lambda^2}{K} - 4M^2)^2 + 16\omega^2} + \left(\lambda^2 - \frac{4\lambda^2}{K} - 4M^2\right)\right]^{1/2}$$

$$B_9 = \frac{1}{2\sqrt{2}}\left[\sqrt{\left(\lambda^2 - \frac{4\lambda^2}{K} - 4M^2\right)^2 + 16\omega^2} - \left(\lambda^2 - \frac{4\lambda^2}{K} - 4M^2\right)\right]^{1/2}$$

$$A_{10} = \frac{-\lambda}{2} - \frac{1}{2\sqrt{2}}\left[\sqrt{\left(\lambda^2 - \frac{4\lambda^2}{K} - 4M^2\right)^2 + 16\omega^2} + \left(\lambda^2 - \frac{4\lambda^2}{K} - 4M^2\right)\right]^{1/2}$$

$$B_{10} = \frac{-1}{2\sqrt{2}}\left[\sqrt{\left(\lambda^2 - \frac{4\lambda^2}{K} - 4M^2\right)^2 + 16\omega^2} - \left(\lambda^2 - \frac{4\lambda^2}{K} - 4M^2\right)\right]^{1/2}$$

$$A_{11} = \frac{Q_{15}Q_{17} + Q_{16}Q_{18}}{Q_{17}^2 + Q_{18}^2}, \qquad B_{11} = \frac{Q_{16}Q_{17} - Q_{15}Q_{18}}{Q_{17}^2 + Q_{18}^2}$$

$$A_{12} = \frac{Q_{19}Q_{21} + Q_{20}Q_{22}}{Q_{12}^2 + Q_{22}^2}, \qquad B_{12} = \frac{Q_{20}Q_{21} - Q_{19}Q_{22}}{Q_{21}^2 + Q_{22}^2}$$

$$A_{13} = \frac{Q_{23}Q_{25} + Q_{24}Q_{26}}{Q_{25}^2 + Q_{26}^2}, \quad B_{13} = \frac{Q_{24}Q_{25} - Q_{23}Q_{26}}{Q_{25}^2 + Q_{26}^2}$$

$$A_{14} = \frac{Q_{27}Q_{29} + Q_{28}Q_{30}}{Q_{29}^2 + Q_{30}^2}, \quad B_{14} = \frac{Q_{28}Q_{29} - Q_{27}Q_{30}}{Q_{29}^2 + Q_{30}^2}$$

$$A_{15} = (1 - h_1A_2) + (B_{11}B_2h_1 + A_{12}(h_1A_1 - 1) - B_{12}B_1h_1 + A_{13}(1 - h_1A_3)$$
$$+ B_{13}B_3h_1 + A_{14}(1 - h_1A_4) + B_{14}B_4h_1$$

$$B_{15} = B_{11}(1 - h_1A_2) - A_{11}B_2h_1 + B_{12}(h_1A_1 - 1) + A_{12}B_1h_1 + B_{13}(1 - h_1A_3)$$
$$- A_{13}B_3h_1 + B_{14}(1 - h_1A_4) - A_{14}B_4h_1$$

$$A_{16} = \left(e^{A_2}\cos B_2\right)\left[A_{11}(1 - h_1A_2) + B_{11}B_2h_1\right] - \left(e^{A_2}\sin B_2\right)\left[B_{11}(1 - h_1A_2)\right.$$
$$\left.- A_{11}B_2h_1\right] + \left(e^{A_1}\cos B_1\right)\left[A_{12}(h_1A_1 - 1) - B_{12}B_1h_1\right] - \left(e^{A_1}\sin B_1\right)\left[B_{12}(h_1A_1 - 1)\right.$$
$$\left.+ A_{12}B_1h_1\right] + \left(e^{A_3}\cos B_3\right)\left[A_{13}(1 - h_1A_3) + B_{13}B_3h_1\right] - \left(e^{A_3}\sin B_3\right)\left[B_{13}(1 - h_1A_3)\right.$$
$$\left.- A_{13}B_3h_1\right] + \left(e^{A_4}\cos B_4\right)\left[A_{14}(1 - h_1A_4) + B_{14}B_3h_1\right]$$
$$- \left(e^{A_4}\sin B_4\right)\left[B_{14}(1 - h_1A_4) - A_{14}B_4h_1\right]$$

$$B_{16} = \left(e^{A_2}\sin B_2\right)\left[A_{11}(1 - h_1A_2) + B_{11}B_2h_1\right] - \left(e^{A_2}\cos B_2\right)\left[B_{11}(1 - h_1A_2)\right.$$
$$\left.- A_{11}B_2h_1\right] + \left(e^{A_1}\sin B_1\right)\left[A_{12}(h_1A_1 - 1) - B_{12}B_1h_1\right] - \left(e^{A_1}\cos B_1\right)\left[B_{12}(h_1A_1 - 1)\right.$$
$$\left.+ A_{12}B_1h_1\right] + \left(e^{A_3}\sin B_3\right)\left[A_{13}(1 - h_1A_3) + B_{13}B_3h_1\right] - \left(e^{A_3}\cos B_3\right)\left[B_{13}(1 - h_1A_3)\right.$$
$$\left.- A_{13}B_3h_1\right] + \left(e^{A_4}\sin B_4\right)\left[A_{14}(1 - h_1A_4) + B_{14}B_3h_1\right]$$
$$+ \left(e^{A_4}\cos B_4\right)\left[B_{14}(1 - h_1A_4) - A_{14}B_4h_1\right]$$

$$A_{17} = \frac{Q_{31}Q_{33} + Q_{32}Q_{34}}{Q_{33}^2 + Q_{34}^2}, \quad B_{17} = \frac{Q_{32}Q_{33} - Q_{31}Q_{34}}{Q_{33}^2 + Q_{34}^2}$$

$$A_{18} = \frac{Q_{35}Q_{37} + Q_{36}Q_{38}}{Q_{37}^2 + Q_{38}^2}, \quad B_{18} = \frac{Q_{36}Q_{37} - Q_{35}Q_{38}}{Q_{37}^2 + Q_{38}^2}$$

$$P_1 = e^{A_1 + A_2 y}\left(\cos B_1 \cos B_2 y - \sin B_1 \sin B_2 y\right)$$

$$P_2 = e^{A_1 + A_2 y}\left(\sin B_1 \cos B_2 y + \cos B_1 \sin B_2 y\right)$$

$$P_3 = e^{A_1 + A_2 y}\left(\cos B_2 \cos B_1 y - \sin B_2 \sin B_1 y\right)$$

$$P_4 = e^{A_1 + A_2 y}\left(\sin B_2 \cos B_1 y + \cos B_2 \sin B_1 y\right)$$

$$P_5 = So\ Sc\ e^{A_2}\left[\left(A_1^2 - B_1^2\right)\cos B_2 - 2A_1B_1\sin B_2\right]$$

$$P_6 = So\ Sc\ e^{A_2}\left[\left(A_1^2 - B_1^2\right)\sin B_2 + 2A_1B_1\cos B_2\right]$$

$$P_7 = \left(e^{A_1}\cos B_1 - e^{A_2}\cos B_2\right)\left(A_1^2 - B_1^2 + Sc\lambda A_1\right) - \left(e^{A_1}\sin B_1 - e^{A_2}\sin B_2\right)$$
$$\left(2A_1B_1 + Sc\,\lambda\,B_1 - \omega Sc\right)$$

$$P_8 = \left(e^{A_1}\sin B_1 - e^{A_2}\sin B_2\right)\left(A_1^2 - B_1^2 + Sc\lambda A_1\right) - \left(e^{A_1}\cos B_1 - e^{A_2}\cos B_2\right)$$
$$\left(2A_1B_1 + Sc\,\lambda B_1 - \omega\ Sc\right)$$

$$P_9 = SoSc\ e^{A_1}\left[2A_2B_2\sin B_1 - \left(A_2^2 - B_2^2\right)\cos B_1\right]$$

$$P_{10} = So\,Sc\,e^{A_1}\left[-2A_2B_2\cos B_1 - \left(A_2^2 - B_2^2\right)\sin B_1\right]$$

$$P_{11} = \left(e^{A_1}\cos B_1 - e^{A_2}\cos B_2\right)\left(A_2^2 - B_2^2 + Sc\lambda A_2\right) - \left(e^{A_1}\sin B_1 - e^{A_2}\sin B_2\right)$$
$$\left(2A_2B_2 + Sc\lambda B_2 - \omega Sc\right)$$

$$P_{12} = \left(e^{A_1}\cos B_1 - e^{A_2}\cos B_2\right)\left(2A_2B_2 + Sc\lambda B_2 - \omega Sc\right) + \left(e^{A_1}\sin B_1 - e^{A_2}\sin B_2\right)$$
$$\left(A_2^2 - B_2^2 + Sc\lambda A_2\right)$$

$$E_1 = \left(e^{A_1}\cos B_1 - e^{A_2}\cos B_2\right), \quad E_2 = \left(e^{A_1}\sin B_1 - e^{A_2}\sin B_2\right)$$

$$Z_1 = \frac{E_1}{E_1^2 + E_2^2}, \; Z_2 = \frac{E_2}{E_1^2 + E_2^2}$$

$$Q_1 = A_5\left(e^{A_4}\cos B_4 - e^{A_1}\cos B_1\right) - B_5\left(e^{A_2}\sin B_4 - e^{A_1}\sin B_1\right)$$

$$Q_2 = A_5\left(e^{A_4}\sin B_4 - e^{A_1}\sin B_1\right) + B_5\left(e^{A_4}\cos B_4 - e^{A_1}\cos B_1\right)$$

$$Q_3 = A_6\left(e^{A_4}\cos B_4 - e^{A_1}\cos B_2\right) - B_6\left(e^{A_4}\sin B_4 - e^{A_1}\sin B_2\right)$$

$$Q_4 = A_6\left(e^{A_4}\sin B_4 - e^{A_1}\sin B_2\right) + B_6\left(e^{A_4}\cos B_4 - e^{A_1}\cos B_2\right)$$

$$Q_5 = -e^{A_4}\cos B_4, \quad Q_6 = -e^{A_4}\sin B_4$$

$$Q_7 = (e^{A_3}\cos B_3 - e^{A_4}\cos B_4), \; Q_8 = e^{A_3}\sin B_3 - e^{A_4}\sin B_4$$

$$Q_9 = A_5\left(e^{A_1}\cos B_1 - e^{A_3}\cos B_3\right) - B_5\left(e^{A_1}\sin B_1 - e^{A_3}\sin B_3\right)$$

$$Q_{10} = A_5\left(e^{A_1}\sin B_1 - e^{A_3}\sin B_3\right) + B_5\left(e^{A_1}\cos B_1 - e^{A_3}\cos B_3\right)$$

$$Q_{11} = A_6\left(e^{A_2}\cos B_2 - e^{A_3}\cos B_3\right) - B_6\left(e^{A_2}\sin B_2 - e^{A_3}\sin B_3\right)$$

$$Q_{12} = A_6\left(e^{A_2}\sin B_2 - e^{A_3}\sin B_3\right) + B_6\left(e^{A_2}\cos B_2 - e^{A_3}\cos B_3\right)$$

$$Q_{13} = e^{A_3}\cos B_3, \qquad Q_{14} = e^{A_3}\sin B_3$$

$$Q_{15} = e^{A_1}Gr\lambda^2\cos B_1 + A_6Gc\lambda^2\left(e^{A_1}\cos B_1 - e^{A_2}\cos B_2\right) - B_6Gc\lambda^2\left(e^{A_1}\sin B_1 - e^{A_2}\sin B_2\right)$$

$$Q_{16} = e^{A_1}Gr\lambda^2\sin B_1 + B_6Gc\lambda^2\left(e^{A_1}\cos B_1 - e^{A_2}\cos B_2\right) + A_6Gc\lambda^2\left(e^{A_1}\sin B_1 - e^{A_2}\sin B_2\right)$$

$$Q_{17} = \left(e^{A_1}\cos B_1 - e^{A_2}\cos B_2\right)\left\{A_2^2 - B_2^2 + \lambda A_2 - \left(\frac{\lambda^2}{K} + M^2\right)\right\}$$
$$-\left(e^{A_1}\sin B_1 - e^{A_2}\sin B_2\right)\left(2A_2B_2 + \lambda B_2 - \omega\right)$$

$$Q_{18} = \left(e^{A_1}\sin B_1 - e^{A_2}\sin B_2\right)\left\{A_2^2 - B_2^2 + \lambda A_2 - \left(\frac{\lambda^2}{K} + M^2\right)\right\}$$
$$+\left(e^{A_1}\cos B_1 - e^{A_2}\cos B_2\right)\left(2A_2B_2 + \lambda B_2 - \omega\right)$$

$$Q_{19} = e^{A_2}Gr\lambda^2\cos B_2 - A_5Gc\lambda^2\left(e^{A_1}\cos B_1 - e^{A_2}\cos B_2\right) + B_5Gc\lambda^2\left(e^{A_1}\sin B_1 - e^{A_2}\sin B_2\right)$$

$$Q_{20} = e^{A_2}Gr + \lambda^2\sin B_2 - B_5Gc\lambda^2\left(e^{A_1}\cos B_1 - e^{A_2}\cos B_2\right) - A_5Gc\lambda^2\left(e^{A_1}\sin B_1 - e^{A_2}\sin B_2\right)$$

$$Q_{21} = \left(e^{A_1}\cos B_1 - e^{A_2}\cos B_2\right)\left\{A_1^2 - B_1^2 - \lambda A_1\left(\frac{\lambda^2}{K} + M^2\right)\right\}$$
$$-\left(e^{A_1}\sin B_1 - e^{A_2}\sin B_2\right)\left(2A_1B_1 - \lambda B_1 - \omega\right)$$

$$Q_{22}=\left(e^{A_1}\cos B_1-e^{A_2}\cos B_2\right)\left(2A_1B_1+\lambda B_1-\omega\right)+\left(e^{A_1}\sin B_1-e^{A_2}\sin B_2\right)$$
$$.\left[A_1^2-B_1^2+\lambda A_1-\left(\frac{\lambda^2}{K}+M^2\right)\right]$$
$$Q_{23}=Gc\,\lambda^2A_7,\qquad Q_{24}=Gc\,\lambda^2B_7$$
$$Q_{25}=A_3^2-B_3^2+\lambda A_3-\left(\frac{\lambda^2}{K}+M^2\right),\quad Q_{26}=\left(2A_3B_3+\lambda B_3-\omega\right)$$
$$Q_{27}=Gc\lambda^2A_8,\qquad Q_{28}=Gc\lambda^2B_8$$
$$Q_{29}=A_4^2-B_4^2+\lambda A_4-\left(\frac{\lambda^2}{K}+M^2\right),\quad Q_{30}=\left(2A_4B_4+\lambda B_4-\omega\right)$$
$$Q_{31}=-A_{16}+e^{A_{10}}A_{15}\cos B_{10}-e^{A_{10}}B_{15}\sin B_{10}$$
$$Q_{32}=-B_{16}+e^{A_{10}}A_{15}\sin B_{10}+e^{A_{10}}B_{15}\cos B_{10}$$
$$Q_{33}=\left(1-h_1A_9\right)\left(e^{A_{10}}\cos B_{10}-e^{A_9}\cos B_9\right)+\left(h_1B_9\right)\left(e^{A_{10}}\sin B_{10}-e^{A_9}\sin B_9\right)$$
$$Q_{34}=\left(1-h_1A_9\right)\left(e^{A_{10}}\sin B_{10}-e^{A_9}\sin B_9\right)-\left(h_1B_9\right)\left(e^{A_{10}}\cos B_{10}-e^{A_9}\cos B_9\right)$$
$$Q_{35}=A_{16}-e^{A_9}A_{15}\cos B_9-e^{A_9}B_{15}\sin B_9$$
$$Q_{36}=B_{16}-e^{A_9}A_{15}\sin B_9-e^{A_9}B_{15}\cos B_9$$
$$Q_{37}=\left(1-h_1A_{10}\right)\left(e^{A_{10}}\cos B_{10}-e^{A_9}\cos B_9\right)+\left(h_1B_{10}\right)\left(e^{A_{10}}\sin B_{10}-e^{A_9}\sin B_9\right)$$
$$Q_{38}=\left(1-h_1A_{10}\right)\left(e^{A_{10}}\sin B_{10}-e^{A_9}\sin B_9\right)-\left(h_1B_{10}\right)\left(e^{A_{10}}\cos B_{10}-e^{A_9}\cos B_9\right)$$

IV. SKIN FRICTION AND NUSSELT NUMBER

With the help of velocity and temperature profiles, the important parameters skin friction (C_f) and Nusselt number (Nu) at the plate y = 0 and y = 1 in terms of their amplitude and phase are given as:

Skin Friction

$$\tau_0=\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}\ \text{and}\ \tau_1=\mu\left(\frac{\partial u}{\partial y}\right)_{y=1}$$

In non-dimensional form after dropping the asterisks over them

$$(C_f)_{y=0}=\frac{\tau_0 d}{\mu U}=\left(\frac{\partial u_0}{\partial y}\right)_{y=0}+\in|\,J\,|\cos\left(t+\alpha_1\right)\ \ldots(32)$$

$$(C_f)_{y=1}=\frac{\tau_1 d}{\mu U}=\left(\frac{\partial u_0}{\partial y}\right)_{y=1}+\in|\,H\,|\cos\left(t+\alpha_2\right)\qquad\ldots(33)$$

where the sinusoidal skin-friction at plate y = 0

$$C_f^1=\left(\frac{\partial u_0}{\partial y}\right)_{y=0}=D_1R_{15}+D_2R_{16}-R_2R_{17}+R_1R_{18}+Sc\lambda R_{20}\qquad\ldots(34)$$

where

$$J\ =\ J_r+i\,J_i=\left(\frac{\partial u_1}{\partial y}\right)_{y=0},\tan\alpha_1=\frac{J_i}{J_r}$$

$$J_r=A_9A_{17}-B_9B_{17}+A_{10}A_{18}-B_{10}B_{18}-A_2A_{11}+B_2B_{11}+A_1A_{12}-B_1B_{12}$$

$$-A_3A_{13}+B_3B_{13}-A_4A_{14}+B_4B_{14}$$

$$J_i=A_9B_{17}+B_9A_{17}+A_{10}B_{18}+B_{10}A_{18}-A_2B_{11}-B_2A_{11}+A_1B_{12}$$
$$+B_1A_{12}-A_3B_{13}-B_3A_{13}-A_4B_{14}-B_4A_{14}$$

$$\left(\frac{\partial u_0}{\partial y}\right)_{y=0}=D_1R_{15}+D_2R_{16}-R_2R_{17}+R_1R_{18}+Sc\lambda R_{20}$$

and

$$H = H_r+i\,H_i=\left(\frac{\partial u_1}{\partial y}\right)_{y=1},\ \tan\alpha_2=\frac{H_i}{H_r}$$

$$H_r=A_9e^{A_9}\left[A_{17}\cos B_9-B_{17}\sin B_9\right]+e^{A_9}\left[-B_9A_{17}\sin B_9-B_9B_{17}\cos B_9\right]$$
$$+A_{10}e^{A_{10}}\left[A_{18}\cos B_{10}-B_{18}\sin B_{10}\right]+e^{A_{10}}\left[-B_{10}A_{18}\sin B_{10}-B_{10}B_{18}\cos B_{10}\right]$$
$$+A_2e^{A_2}\left[B_{11}\sin B_2-A_{11}\cos B_2\right]+e^{A_2}\left[B_2B_{11}\cos B_2+B_2A_{11}\sin B_2\right]$$
$$+A_1e^{A_1}\left[A_{12}\cos B_1-B_{12}\sin B_1\right]+e^{A_1}\left[-B_1A_{12}\sin B_1-B_1B_{12}\cos B_1\right]$$
$$+A_3e^{A_3}\left[B_{13}\sin B_3-A_{13}\cos B_3\right]+e^{A_3}\left[B_3B_{13}\cos B_3+B_3A_{13}\sin B_3\right]$$
$$+A_4e^{A_4}\left[B_{14}\sin B_4-A_{14}\cos B_4\right]+e^{A_4}\left[B_4B_{14}\cos B_4+B_4A_{14}\sin B_4\right]$$

$$H_i=A_9e^{A_9}\left[B_{17}\cos B_9+A_{17}\sin B_9\right]+e^{A_9}\left[-B_9B_{17}\sin B_9+B_9A_{17}\cos B_9\right]$$
$$+A_{10}e^{A_{10}}\left[B_{18}\cos B_{10}+A_{18}\sin B_{10}\right]+e^{A_{10}}\left[-B_{10}B_{18}\sin B_{10}+B_{10}A_{18}\cos B_{10}\right]$$
$$-A_2e^{A_2}\left[B_{11}\cos B_2+A_{11}\sin B_2\right]-e^{A_2}\left[-B_2B_{11}\sin B_2+B_2A_{11}\cos B_2\right]$$
$$+A_1e^{A_1}\left[B_{12}\cos B_1+A_{12}\sin B_1\right]+e^{A_1}\left[-B_1B_{12}\sin B_1+B_1A_{12}\cos B_1\right]$$
$$-A_3e^{A_3}\left[B_{17}\cos B_3+A_{13}\sin B_3\right]-e^{A_3}\left[-B_3B_{13}\sin B_3+B_3A_{13}\cos B_3\right]$$
$$-A_4e^{A_4}\left[B_{14}\cos B_4+A_{14}\sin B_4\right]-e^{A_4}\left[-B_4B_{14}\sin B_4+B_4A_{14}\cos B_4\right]$$

$$\left(\frac{\partial u_0}{\partial y}\right)_{y=1}=D_1R_{15}e^{R_{15}}+D_2R_{16}e^{R_{16}}-R_2R_{17}e^{R_2}+R_1R_{18}e^{R_1}+Sc\lambda R_{20}e^{-Sc\lambda}$$

Nusselt Number

$$q_0=\frac{-L_1}{(T_0-T_d)}\left(\frac{\partial T}{\partial y}\right)_{y=0}\ \text{and}\ q_1=\frac{-L_1}{(T_0-T_d)}\left(\frac{\partial T}{\partial y}\right)_{y=1}$$

In non-dimensional form after dropping the asterisk over them

$$(Nu)_{y=0}=h_1\left[\left(\frac{\partial\theta_0}{\partial y}\right)_{y=0}+\in|F|\cos(t+\beta_1)\right] \quad \ldots(35)$$

$$(Nu)_{y=1}=h_1\left[\left(\frac{\partial\theta_0}{\partial y}\right)_{y=1}+\in|R|\cos(t+\beta_2)\right] \quad \ldots(36)$$

where the sinusoidal rate of heat transfer at plate y = 0

$$Nu^1=\left(\frac{\partial\theta_0}{\partial y}\right)_{y=0}=\frac{1}{e^{R_1}-e^{R_2}}\left(R_2e^{R_1}-R_1e^{R_2}\right) \quad \ldots(37)$$

where

$$F=F_r+i\,F_i=\left(\frac{\partial\theta_1}{\partial y}\right)_{y=0},\ \tan\beta_1=\frac{F_i}{F_r}$$

$$F_r=e^{A_1}\left[Z_1(-B_2\sin B_1)+Z_2(B_2\cos B_1)\right]+A_2e^{A_1}\left[Z_1(\cos B_1)+Z_2(\sin B_1)\right]$$

$$-e^{A_2}\left[Z_1\left(-B_1\sin B_2\right)+Z_2B_1\cos B_2\right]-A_1e^{A_2}\left[Z_1\left(\cos B_2\right)+Z_2\left(\sin B_2\right)\right]$$

$$F_i=e^{A_1}\left[Z_1\left(B_2\cos B_1\right)-Z_2\left(-B_2\sin B_1\right)\right]+A_2e^{A_1}\left[Z_1\left(\sin B_1\right)-Z_2\left(\cos B_1\right)\right]$$

$$-e^{A_2}\left[Z_1\left(B_1\cos B_2\right)-Z_2\left(-B_1\sin B_2\right)\right]-A_1e^{A_2}\left[Z_1\left(\sin B_2\right)-Z_2\left(\cos B_2\right)\right]$$

$$\left(\frac{\partial\theta_0}{\partial y}\right)_{y=0}=\frac{1}{e^{R_1}-e^{R_2}}\left(R_2e^{R_1}-R_1e^{R_2}\right)$$

and

$$R\ =\ R_r+i\,R_i=\left(\frac{\partial\theta_1}{\partial y}\right)_{y=1}\ ,\ \tan\beta_2=\frac{R_i}{R_r}$$

$$R_r=e^{A_1+A_2}\left[z_1\left(-B_2\cos B_1\sin B_2-B_2\sin B_1\cos B_2\right)+Z_2\left(-B_2\sin B_1\sin B_2\right.\right.$$

$$\left.+B_2\cos B_1\cos B_2\right]+A_2e^{A_1+A_2y}\left[z_1\left(\cos B_1\cos B_2-\sin B_1\sin B_2\right)\right.$$

$$\left.+Z_2\left(\sin B_1\cos B_2+\cos B_1\sin B_2\right)\right]-e^{A_1+A_2}\left[Z_1\left(-B_1\cos B_2\sin B_1\right.\right.$$

$$\left.\left.-B_1\sin B_2\cos B_1\right)+Z_2\left(-B_1\sin B_2\sin B_1+B_1\cos B_2\cos B_1\right)\right]$$

$$-A_1e^{A_1+A_2}\left[Z_1\left(\cos B_2\cos B_1-\sin B_2\sin B_1\right)+Z_2\left(\sin B_2\cos B_1+\cos B_2\sin B_1\right)\right]$$

$$R_i=e^{A_1+A_2}\left[z_1\left(-B_2\sin B_1\sin B_2-B_2\cos B_1\cos B_2\right)-Z_2\left(-B_2\cos B_1\sin B_2\,d\right.\right.$$

$$\left.-B_2\sin B_1\sin B_2\right]+A_2e^{A_1+A_2y}\left[z_1\left(\sin B_1\cos B_2+\cos B_1\sin B_2\right)\right.$$

$$\left.-Z_2\left(\cos B_1\cos B_2-\sin B_1\sin B_2\right)\right]-e^{A_1+A_2}\left[Z_1\left(-B_1\sin B_2\sin B_1\right.\right.$$

$$\left.\left.+B_1\cos B_2\cos B_1\right)-Z_2\left(-B_1\cos B_2\sin B_1-B_1\sin B_2\cos B_1\right)\right]$$

$$-A_1e^{A_1+A_2}\left[Z_1\left(\sin B_2\cos B_1+\cos B_2\sin B_1\right)-Z_2\left(\cos B_2\cos B_1\sin B_2\sin B_1\right)\right]$$

$$\left(\frac{\partial\theta_0}{\partial y}\right)_{y=1}=\frac{1}{e^{R_1}-e^{R_2}}\left(R_2e^{R_1+R_2}-R_1e^{R_2+R_1}\right)$$

(Table 1. Amplitude | J | and phase α_1 of skin-friction at plate y = 0 for fixed)

S.No.	Gr	Gc	Sc	Pr	λ	h_1	ω	| J |	Tan α_1
1	5.0	2.0	0.6	0.71	5.0	0.05	5.0	3.67263	-0.44213
2	7.0	2.0	0.6	0.71	5.0	0.05	5.0	4.14786	-0.57285
3	7.0	3.0	0.6	0.71	5.0	0.05	5.0	3.84867	-0.47512
4	5.0	2.0	0.94	0.71	5.0	0.05	5.0	3.74312	-0.46314
5	5.0	2.0	0.94	7.0	5.0	0.05	5.0	0.98213	0.43198
6	5.0	2.0	0.6	0.71	5.0	0.0	5.0	4.01254	-0.51254
7	7.0	2.0	0.6	0.71	5.0	0.05	20.0	5.31243	-1.00342
8	5.0	2.0	0.6	0.71	8.0	0.0	5.0	4.82165	-0.75431

Values of K = 1.0, So = 1.0, S = 0.2 and M = 0.5

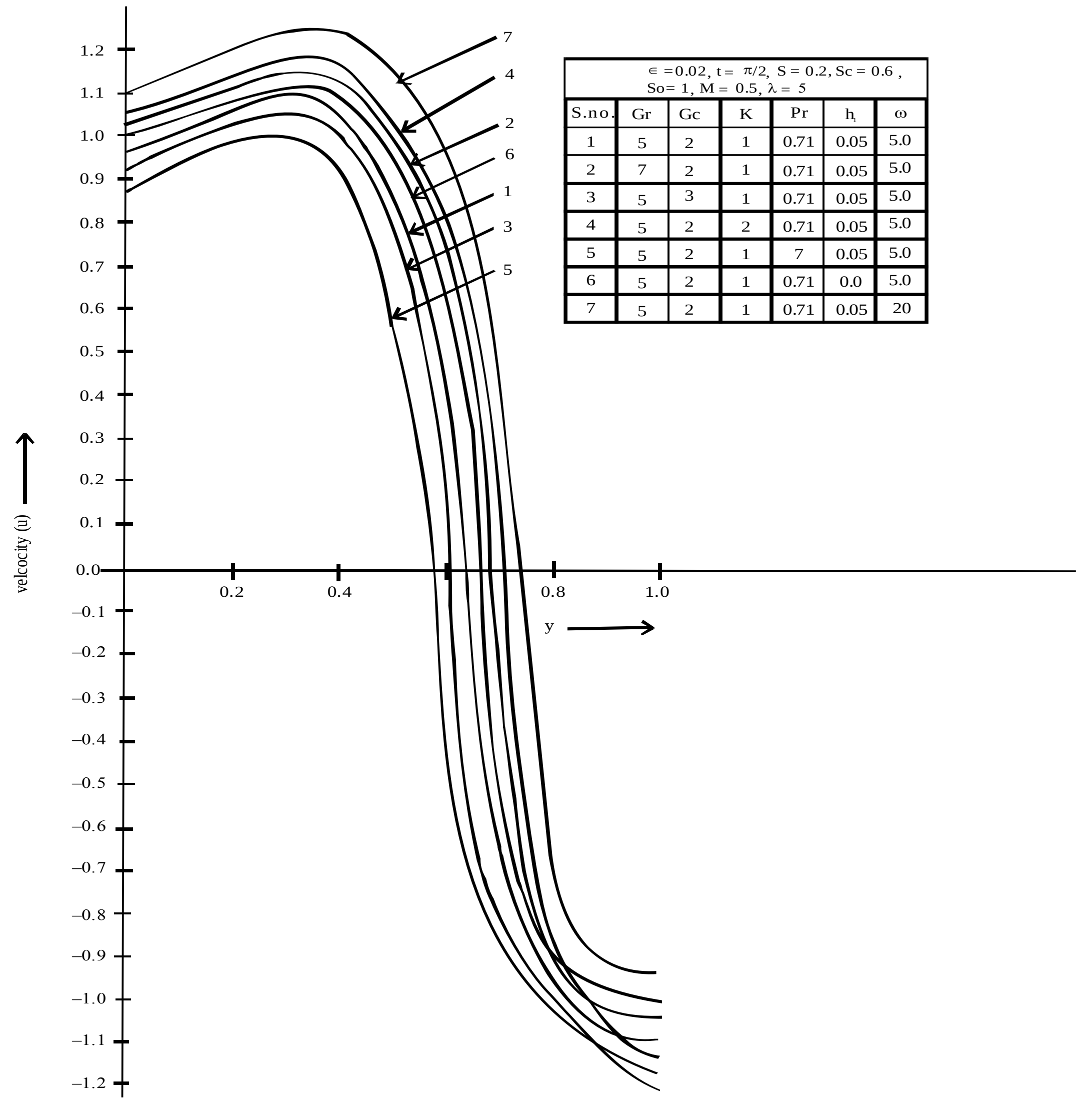
(Table 2. Amplitude | F | and phase tan β_1 of heat transfer at plate y = 0)

S.No.	Pr	λ	S	ω	| F |	Tan β_1
1	0.71	5.0	0.2	5.0	1.08639	1.00513
2	7.0	5.0	0.2	5.0	6.56256	0.53165
3	0.71	8.0	0.2	5.0	0.74542	2.48025
4	0.71	5.0	0.6	5.0	0.83412	2.15968
5	0.71	5.0	0.2	20.0	2.37215	1.20461

V. RESULT AND DISCUSSIONS

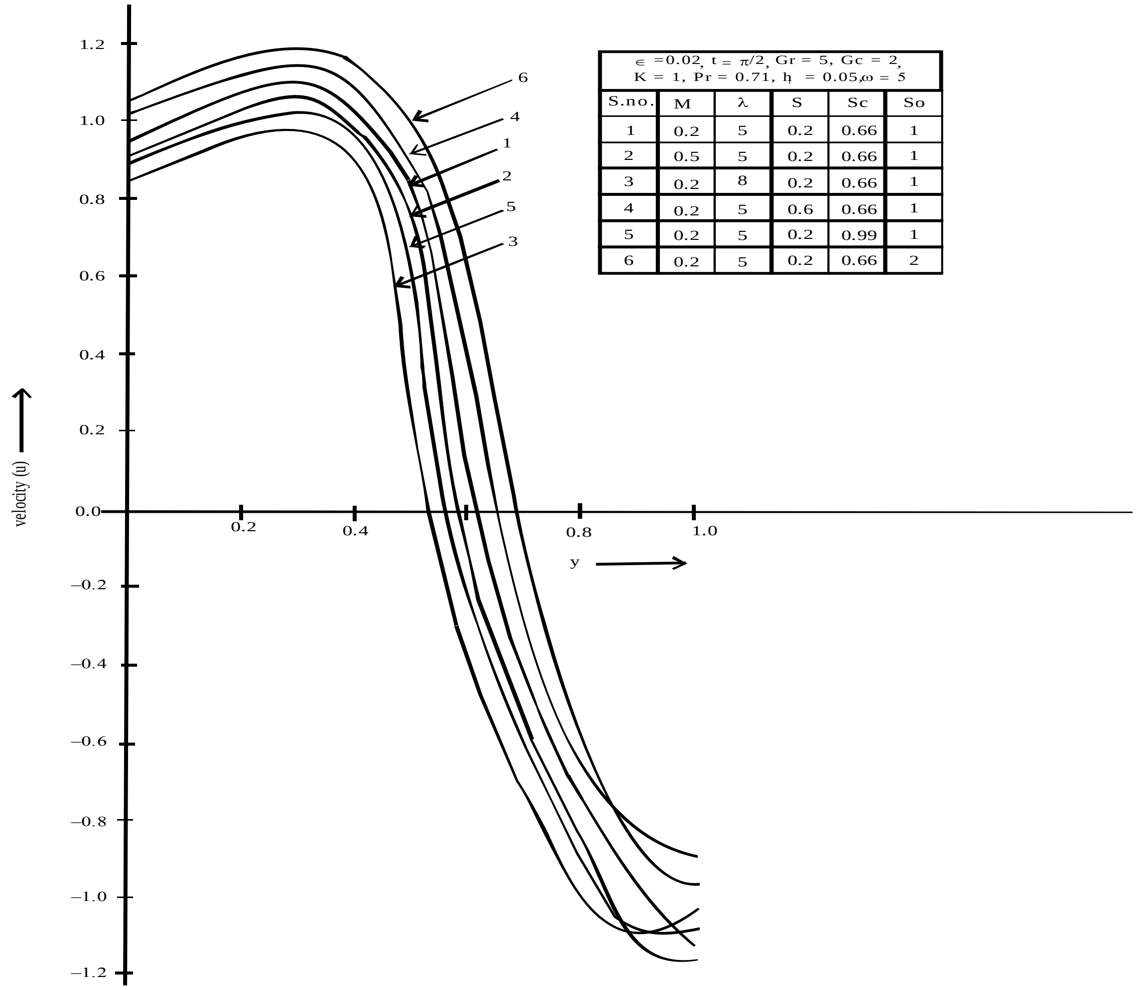
In order to understand the physical importance of the flow between the two plates, calculations have been carried out for velocity, temperature, concentration, skin friction and the rate of heat transfer. Effects for different values of thermal Grashof number (Gr), mass Grashof number (Gc), permeability parameter (K), Prandtl number (Pr), velocity slip parameter (h_1), frequency (ω), Hartmann number (M), suction parameter (□), heat source parameter (S), Schmidt number (Sc) and soret number (So) are shown graphically. The values of Prandtl number are chosen as 0.71 and 7, which represent air and water respectively at 20^{o}C. In terms of amplitude and phase, the skin friction and the rate of heat transfer are reported in table 1 and 2 at plate y = 0. Moreover sinusoidal skin friction and Nusselt number are also calculated and shown graphically at the plate y = 0. We fix $\in = 0.02$ $t = \frac{\pi}{2}$ throughout our calculations.

In figure 2 and 3 , velocity profiles are plotted against y. From the figures, we observe that on increasing Gr, K, □ , S and So, the velocity of the fluid increases. On the other hand velocity drops on increasing Gc, Pr, h_1, M, □ and Sc. Here, we notice that velocity remains positive near the plate y = 0 but after some distance it becomes negative, this is due to the fact that our plates are moving in opposite directions, specifically plate at y = 1 is moving downwards. Also, physically increase in the permeability parameter (K) implies that medium becomes more porous that is more fluid can flow through, hence increasing the velocity of the fluid.



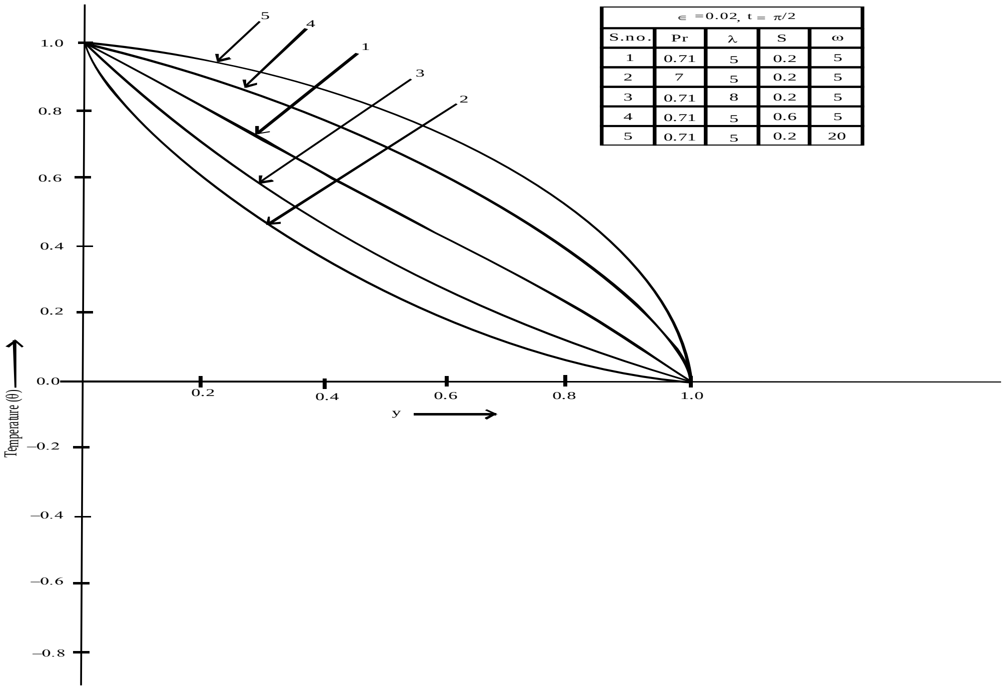
S.no.	Gr	Gc	K	Pr	h_1	ω
1	5	2	1	0.71	0.05	5.0
2	7	2	1	0.71	0.05	5.0
3	5	3	1	0.71	0.05	5.0
4	5	2	2	0.71	0.05	5.0
5	5	2	1	7	0.05	5.0
6	5	2	1	0.71	0.0	5.0
7	5	2	1	0.71	0.05	20

(Figure 2: Velocity profiles plotted against y for different values of Gr, Gc,, K, Pr, h_1 and □ .)



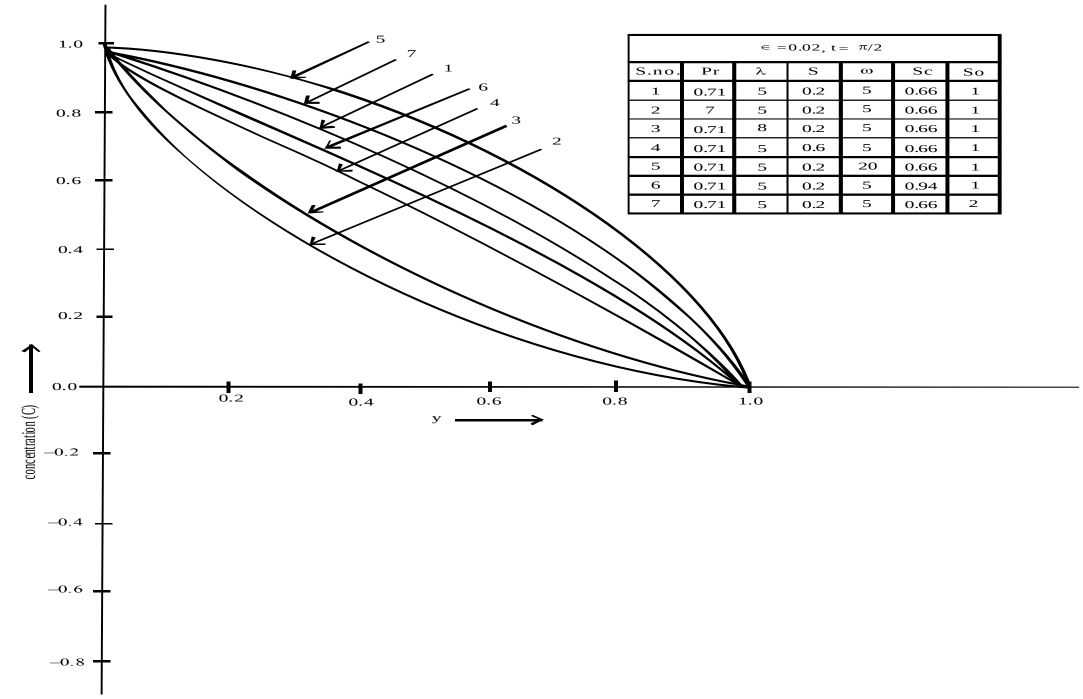
S.no.	M	λ	S	Sc	So
1	0.2	5	0.2	0.66	1
2	0.5	5	0.2	0.66	1
3	0.2	8	0.2	0.66	1
4	0.2	5	0.6	0.66	1
5	0.2	5	0.2	0.99	1
6	0.2	5	0.2	0.66	2

(Figure 3: Velocity profiles plotted against y for different values of M, □ , S, Sc and So.)



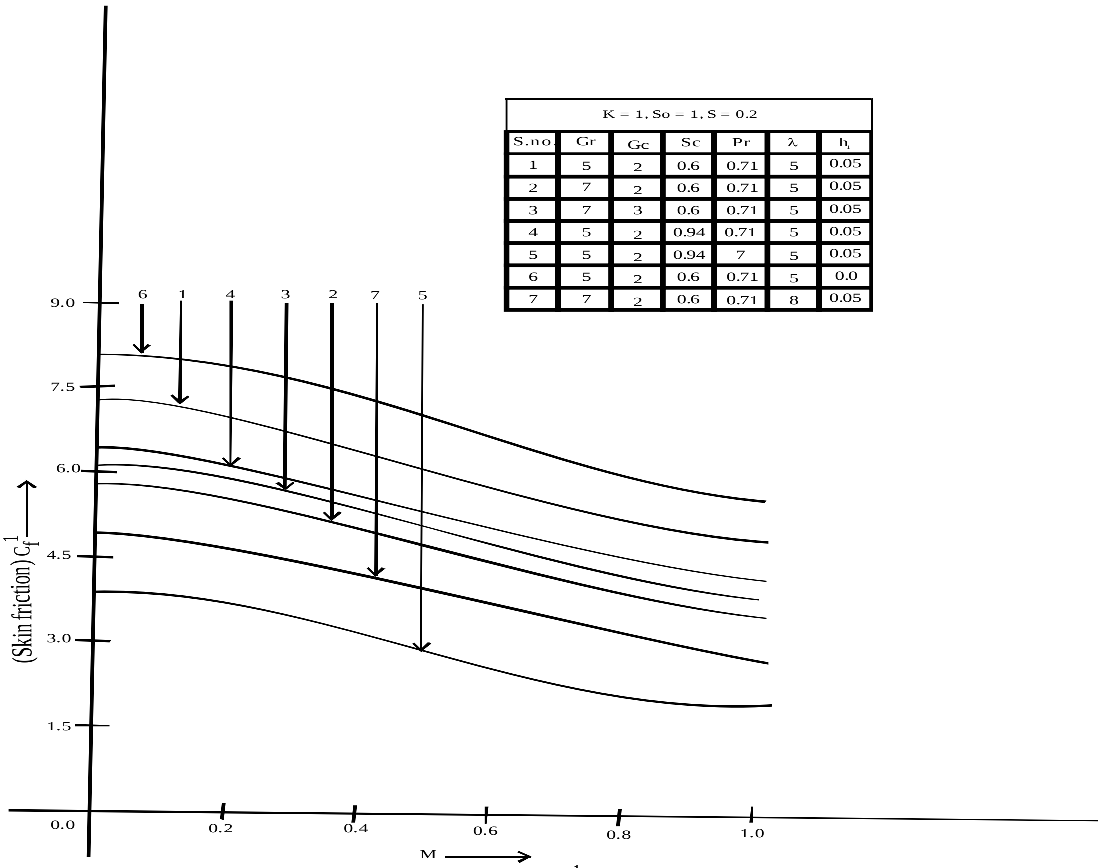
S.no.	Pr	λ	S	ω
1	0.71	5	0.2	5
2	7	5	0.2	5
3	0.71	8	0.2	5
4	0.71	5	0.6	5
5	0.71	5	0.2	20

(Figure 4: Temperature profiles plotted against y for different values of Pr, □ , S and □ .)



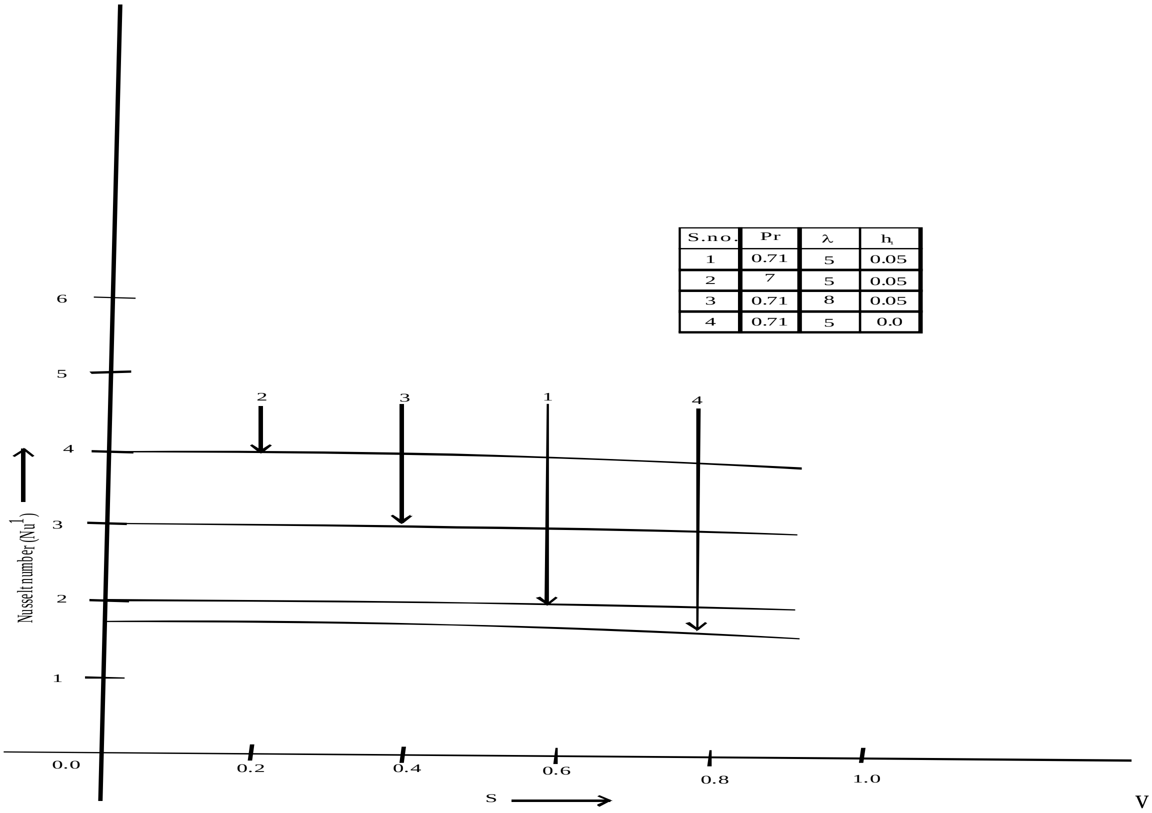
∈ =0.02, t= π/2						
S.no.	Pr	λ	S	ω	Sc	So
1	0.71	5	0.2	5	0.66	1
2	7	5	0.2	5	0.66	1
3	0.71	8	0.2	5	0.66	1
4	0.71	5	0.6	5	0.66	1
5	0.71	5	0.2	20	0.66	1
6	0.71	5	0.2	5	0.94	1
7	0.71	5	0.2	5	0.66	2

(Figure 5: Concentration profiles plotted against y for different values of Pr, □ , S, □ , Sc and So.)



K = 1, So = 1, S = 0.2						
S.no	Gr	Gc	Sc	Pr	λ	h
1	5	2	0.6	0.71	5	0.05
2	7	2	0.6	0.71	5	0.05
3	7	3	0.6	0.71	5	0.05
4	5	2	0.94	0.71	5	0.05
5	5	2	0.94	7	5	0.05
6	5	2	0.6	0.71	5	0.0
7	7	2	0.6	0.71	8	0.05

(Figure 6: Sinusoidal skin friction C_f^1 plotted against M at plate y = 0



(Figure 7: Sinusoidal rate of heat transfer Nu^1 plotted against S at plate y = 0)

Temperature profiles are plotted against y, in figure 4, we observe that on increasing the values of the source parameter (S) and frequency (□), temperature rises, where as on increasing Pr and □ , temperature drops. It is noteworthy that as we increase the heat source S i.e. we add heat and hence the temperature rises. In figure 5, concentration profiles are plotted against y. From the figure we observe that concentration profiles are less for higher values of Pr, □ , S and Sc. On the other hand for increase in □ and So we get higher concentration profiles.

Table 1, shows the amplitude |J| and phase angle tan $□_1$ of the skin friction at the plate y = 0, fixing K = 1, So = 1, S = 0.2 and M = 0.5, from the table we observe that when values of □ , Gr, Sc and □ are increased, the amplitude |J| increases, but increase in the values of Gc, Pr and h_1 drops the amplitude. The values of tan $□_1$ shows that there is always a phase lag. Also tan $□_1$ is higher for water as compared for air. The sinusoidal skin friction at the plate y = 0 is shown in figure 6, fixing K = 1, So = 1 and S = 0.2. From the figure we observe that increasing the values of h_1, □, Gr, Sc and Pr, decreases C_f^1, where as C_f^1 rises with increase in the value of Gc. Physically, increase in the value of velocity slip parameter (h_1) will reduce the friction near the plate hence decreasing C_f^1 since more the slip less will be the friction at the plate. Moreover, skin friction is higher for air (Pr = 0.71) as compared for water (Pr = 7).

Amplitude |F| and phase angle tan $□_1$, of the rate of heat transfer are shown in Table 2, at the plate y = 0. We observe that when Pr and □ are increased, it increases the amplitude |F| but increase is □ and S decreases it. From the values of tan $□_1$, we observe that it is less for water as compared for air. This table shows that there always remains a phase lag. Further, the sinusoidal rate of heat transfer at plate y = 0 is shown in figure 7. From the figure we observe increase in □ and h_1, increases Nu^1. Also Nusselt number is higher for water (Pr = 7) as compared for air (Pr = 0.71).

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