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The fundamental formulas for vertices of convex hull

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***Abstract:* -** This paper represents four formulas for solution of convex hull problem. It aims to analyze how many points are vertices out of total input points, how many vertices lie on a horizontal or vertical lines, position of vertices and number of vertices on lower and higher lines(horizontal or vertical).

Keywords: - Jarvis's March method, horizontal line (HL), vertical line(VL), vertices , Convex Hull(CH).

I. INTRODUCTION

Convex hull is a part of computational geometry. Convex hull of a set S of points is the smallest convex polygon P for which each point in S is either on the boundary of P or in its interior. We denote the convex hull of S by CH(S). Convexity has a number of properties that makes convex polygons easier to work with than arbitrary polygons. For example, every diagonal of a convex polygon is a chord, every vertex of convex polygon is convex (that means its interior angle is less than or equal to 180 degree). There are some methods generated for solving convex hull problem. Among these methods Graham Scan method[1], Jarvis's March method[1], Divide and Conquer method[2], Incremental method[3] and Prune-Search methods[4] are remarkable. Number of horizontal line indicates the number of different values of y among the input points. Number of vertical line indicates the number of different values of x among the input points. The end points where two segments meet are called it vertices. The vertices of a polygon are classified as convex or reflex. A vertex is convex if the interior angle at the vertex--through the polygon interior—measures less than or equal to 180 degrees. A vertex is reflex otherwise (its interior angle measures greater than180 degrees).

II. DESCRIPTION

Statement of formulas

i) Every vertices of the convex hull must be the starting or ending points (among input points) of any horizontal or vertical line.
ii) For top and bottom horizontal line or leftmost and rightmost vertical line the starting and ending points are must be vertices of desired convex hull.
iii) The highest number of vertices in one line (horizontal or vertical) is less than or equal to two.
iv) If the number of lines(horizontal or vertical) is K at which all the points lie then the total number of vertices is less than or equal to 2K.

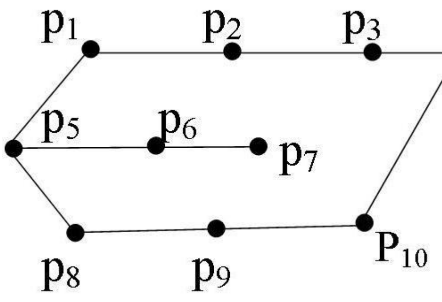
III. PROOF OF FORMULAS

1st Formula

Every vertices of the convex hull must be starting or end points of any HL or VL.

Proof

Given a set of input points $P=\{P_1,P_2,P_3,-----P_{10}\}$. Let all the points that lies in 3 HL or 3VL: The objective of this formula is to prove the vertices are starting or end points of any HL or VL. These point are shown in following figure.



According to definition of convex hull all the input points will be enclosed by smallest polygon and the edge by connecting any two points in inside the polygon and the interior angle of the vertex which will be less than or equal to 180^0.

From the above figure it is seen that by connecting p_1 and p_4 the point p_2 and p_3 can be included into the polygon. By connecting p_5 and p_7 the point p_6 can be included into the polygon. By connecting p_8 and p_{10} the point p_9 can be included into the polygon. So, the desired vertices exist among P_7 which is not a vertex because the edge P_4 P_{10} is then outside the polygon. P_6 can not be vertex because then the point P_5 and P_7 are outside the polygon. So the desired convex hull among these points is P_1 P_5 P_8 P_{10} P_4. All these vertex points are either starting or end points among these 3 HL.

Example

Let there are 31 points are in 7 HL. The point are (5,17), (10,17), (14,17), (17,17), (20, 17), (25, 17), (-5, 13), (3, 13), (13,13), (19, 13), (27, 13), (10, 10), (2, 7) (6, 7), (11, 7), (17,7), (22, 7), (-2, 5), (15, 5), (25, 5), (1, 4), (4, 4), (8, 4), (6, 7), (11, 7), (17, 7) (22, 7), (-2, 5), (15, 5), (25, 5), (1,4), (4, 4), (8, 4), (13, 4), (9, 4), (21, 4), (2, 2), (5, 2), (9, 2), (16,2), (19, 2). By applying Jarvis's march method the desired convex hull is (2,2), (19,2), (25,5), (27, 13), (25, 17), (5, 17), (-5, 13), (-2, 5). Which is shown in Figure2. From the Figure it seen that all these vertices are either starting point or end point of any HL. So the first formula is proved (under the above discussion).

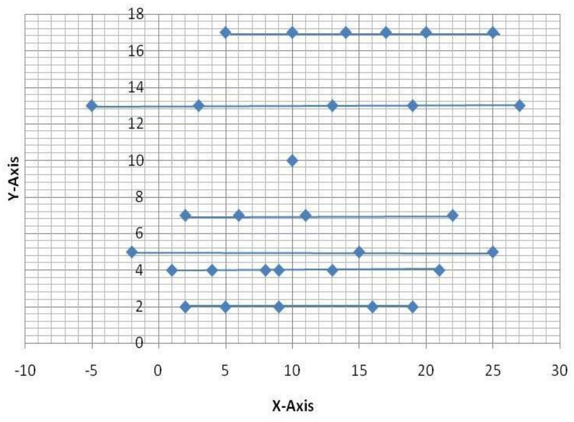


Figure1: Input points

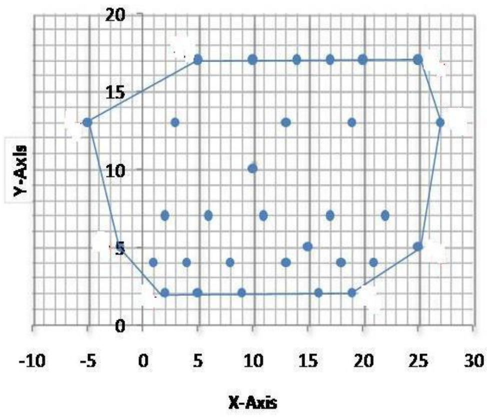


Figure2: Convex hull

2nd Formula

For top and bottom HL or leftmost(LM) and rightmost(RM) VL the starting and the end points must be vertices of the desired CH.

Proof

From the first formula, it is proved that vertices will exist among starting and end points of HL or VL. Let there are two points in top HL. Let these two points are vertices. To prove it at first it is considered that LM point is not a vertex. Since it is an input point to include into CH it needs another point at left side of LM point or upside of LM point, since there is no point at the left side of LM point and there is no point up of the LM point. So if the LM point is included into the CH then LM point will be a vertex. Again it is considered that RM point is not a vertex. Since RM point is an input point so to include the RM point into CH it needs another point at right side of RM point at the same line or up of the RM point. But it is the top HL so there is no point up of the RM point and there is no another point at the right side of RM point. So RM point also will be a vertex. Similarly for Bottom HL it can be shown that starting and end points of Bottom HL are also vertices. If there is one point in both top HL and bottom HL then that point is vertex. For both top HL and bottom HL then that point is vertex. For both LM VL and RM VL it is also shown that starting and end points are vertices.

Example

From the first example, the vertex points are (2.2), (19,2), (25,5), (27,13), (25, 17), (5,17) (-5, 13) and (-2, 5). The leftmost point of the top HL is (5, 17) and the right most point (25, 17) which are vertices of the desired CH.

The leftmost point is the bottom HL is (2, 2) and the rightmost point is (19,2) which are the vertices of the desired CH. So the second formula is proved.

3rd Formula

The number of vertices in one HL or VL is less than or equal to two.

Proof

This formula gives the concept of highest no of vertices in one HL or VL. Let there are 3 HL or VL and each HL or VL has n no. of points that means it is given 3n no of input points; they are:

$P_{11},\ P_{12},\ P_{13},$ -------- P_{1n}
$P_{21},\ P_{22},\ P_{23},$ -------- P_{2n}
$P_{31},\ P_{32},\ P_{33},$ -------- P_{3n}

According to first law, the vertices of the desired convex hull will be included among $P_{11},\ P_{1n},\ P_{21},\ P_{2n},$ $P_{31},\ P_{3n}$ are vertices of the desired CH. Now in middle line, the starting and end point are P_{21} and P_{2n}. If there is any vertex in this line then that point is either P_{21} and P_{2n}. Because if P_{22} or P_{23} or ---- or P_{2n-1} is vertex then the condition of the convexity will be broken. Moreover, by connecting starting and end points we can include all the points in that line into CH.

Example

From Figure2 it seen that the no. of vertices in first HL=2, in second HL the no. of vertices =2, In 3rd HL the no. of vertices = 0. In 4th HL the no. of vertices=0, In 5th HL the no. of vertices= 2, In 6th HL the no. of vertices = 0, In 7th HL the no. of vertices =2. Each line has highest no. of vertices 2. So from the discussion, the third formula is proved. (counting order of HL from up to down).

4th formula

If the number of the HL or VL is k then the number of vertices less than or equal to 2k.

Proof

This formula gives the concept about the highest no of vertices into among input points. It is related with the no. of line (VL or HL) at which all the input points exist. From 3rd formula it is proved that in one line there exist highest 2 vertices. So if the no of line (HL or VL) is K then highest no. of vertices are 2k. Among these the starting and end points in top and bottom HL or leftmost and rightmost VL are vetices by 2nd formula. But the starting and end point of other HLs or VLs may not be vertices because other HLs or VLs exist among top and bottom HLs or leftmost and rightmost VLs. If all starting and end points are vertices then total no. of vertices will be 2k otherwise less than 2k.

Example

In Figure2, there are 7 HLs. 1st line has 2, second has 2, 3rd has 0, 4th has 0, 5th has 2, 6th has 0 and 7th has 2 no. of vertices. So total no. of vertices = 2+2+0+0+2+0+2=8 which is less than 14. So the 4th formula is also proved.

Applications

By applying these formulas the complexity of any convex hull algorithm can be decreased as remarkable rate. Since it is proved that the vertices of the CH are must be starting and end points of any HL or VL. So by analyzing only the starting and end points of all HLs or VLs it can determined the desired Convex Hull. That means if the total no of line is equal to K then analyzing only 2k no. of points can be determined the desired convex hull although there exists Kn no. of points (each line has n points). If there exists h no. of vertices among this input point then the new complexity of Jarvis's march method will be 2kh after applying these formulas. Whereas present complexity of Jarvis's march method is Khn, that is so larger than 2kh. Since starting and end points of top and bottom HL or leftmost and rightmost VL are vertices so the CH can be determined more easily which is shown in "A new technique for solving convex hull problem"[5]. Another advantage is it can check easily a point is inside or outside of the CH and merging of two convex hull.

Results and Discussion

Now these four formulas are proved for different convex hull. In the table number of horizontal line is expressed as HL.

Number of points	Number of lines=k	Highest Number of vertices =2k	Input points	Total number of vertices	Vertices	1st formula	2nd formula	3rd formula	4th formula
8	HL=2	4	(2,1),(5,1),(9,1),(12,1), (-2,5), (2,5), (8,5), (10,5)	4	(2,1),(12,1), (-2,5) ,(10,5)	True	True	True	True
10	HL=3	6	(-1,7), (4,7), (6,7), (10,7), (1,2),(5,2),(7,2), (-5,1), (1,1), (10,1)	4	(-5,1), (10,1), (-1,7), (10,7)	True	True	True	True
15	HL=4	8	(-6,3),(-3,3),(0,3),(5,3), (2,5), (4,5), (10,5), (12,5), (15,5), (5,10) (7,10),(10,10),(15,10), (1,12), (5,12)	6	(-6,3),(5,3), (15,5),(15,10), (1,12),(5,12)	True	True	True	True
20	HL=4	8	(-6,3),(-3,3),(0,3),(5,3), (10,3), (-10,5), (4,5), (10,5), (12,5), (15,5) (-10,10),(5,10),(7,10), (10,10),(15,10) (-2,12),(0,12),(1,12),(5,12), (7,12)	8	(-6,3), (10,3), (-10,5), (15,5), (-10,10), (15,10), (-2,12), (7,12)	True	True	True	True
15	HL=5	10	(1,2),(4,2),(10,2), (-5,6), (5,6), (12,6), (1,10), (3,10), (5,10), (-3,15)(3,15),(4,15), (6,18),(10,18), (13,18)	7	(1,2),(10,2), (-5,6),(12,6), (-3,15),(6,18), (13,18)	True	True	True	True

From the above table it is seen that for all the inputs, four formulas are true. So it can be said that for any input four formulas are true.

IV. CONCLUSION

These four formulas can be applied for any convex hull problem. So it can be said these formulas are successful formulas for solution of convex hull problem. These formulas describe the fundamental characteristics of vertices of convex hull.

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