

Fuzzy gc-super Irresolute Mappings

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Abstract: - In this paper we study the concept of fuzzy gc-super irresolute mappings and introduced some of their basic properties in fuzzy topology.

Keywords: - fuzzy super closure ,fuzzy super interior, fuzzy super closed set, fuzzy super open set ,fuzzy super continuity ,fuzzy g -super closed sets and fuzzy g –super open sets, fuzzy g -super continuous mappings.

I. INTRODUCTION

After the introduction of fuzzy sets by Zadeh [15] in 1965 and fuzzy topology by Chang [4] in 1967. Several researches were conducted on the generalizations of the notions of fuzzy sets and fuzzy topology. The concept of fuzzy sets was introduced by Atanassov [1] as a generalization of fuzzy sets. In the last 20 years various concepts of fuzzy mathematics have been extended for fuzzy sets. In 1997 Coker [5] introduced the concept of fuzzy topological spaces. Recently many fuzzy topological concepts such as fuzzy compactness [8], fuzzy connectedness [14], fuzzy multi functions [9] fuzzy g -super closed set [11] and fuzzy g -super continuity [12] have been generalized for fuzzy topological spaces. Topological space. In the present paper we introduce and study the concept of fuzzy gc-super irresolute mappings in fuzzy topological space.

II. PRELIMINARIES

Definition 2.1[8,9,12]: A fuzzy set A of a fuzzy topological space (X, \mathfrak{T}) is called a

- fuzzy generalized super closed (fuzzy g -super closed) if $\text{cl}(A) \leq O$
Whenever $A \leq O$ and O is fuzzy super open.
- Fuzzy generalized super open if its complement is fuzzy generalized super closed.

Remark 2.1 [8,9,12]: Every fuzzy super closed set is fuzzy g -super closed set but its converse may not be true.

Definition 2.2[9]: Let (X, \mathfrak{T}) and (Y, Φ) be two fuzzy topological spaces and let $f: X \rightarrow Y$ be a function. Then

- f is said to be fuzzy super continuous if the pre image of each fuzzy open set in Y is an fuzzy super open set in X . [8]
- f is said to be fuzzy g -super continuous if the inverse image of every fuzzy super closed set of Y is fuzzy g -super closed set in X . [13]

Definition 2.3[8,9,13]: An fuzzy topological space X is called fuzzy g -super connected if there is no proper fuzzy set of X which is both fuzzy g -super open and fuzzy g -super closed.

Definition 2.4[8,9,13]: An fuzzy set B of a fuzzy topological space (X, \mathfrak{T}) is said to be fuzzy GO- super compact relative to X , if for every collection $\{A_i; i \in \Lambda\}$ of fuzzy g –super open sets of X such that $B \leq \cup\{A_i; i \in \Lambda\}$. There exists a finite subset Λ_0 of Λ such that $B \leq \cup\{A_i; i \in \Lambda_0\}$.

Definition 2.5[8,9,13] : A crisp subset Y of an fuzzy topological space (X, \mathfrak{T}) is said to be fuzzy GO- super compact if Y is fuzzy GO- super compact as a fuzzy subspace of X .

Definition 2.6.[8,9,13]: Let (X, \mathfrak{S}) be an fuzzy topological space. The generalized closure of a fuzzy set A of X denoted by $\text{gcl}(A)$ is the intersection of all fuzzy g -super closed sets of X which contains A .

III. FUZZY GC-SUPER IRRESOLUTE MAPPINGS

Definition 3.1: A mapping f from an fuzzy topological space (X, \mathfrak{S}) to another fuzzy topological space (Y, σ) is said to be fuzzy g -super irresolute if the inverse image of every fuzzy g -super closed set of Y is fuzzy g -super closed in X .

Theorem 3.1: A mapping $f : (X, \mathfrak{S}) \rightarrow (Y, \sigma)$ is fuzzy g -super irresolute if and only if the inverse image of every fuzzy g -super open set in Y is fuzzy g -super open in X .

Proof: It is obvious because $f^{-1}(U^c) = (f^{-1}(U))^c$, for every fuzzy set U of Y .

Remark 3.1: Since every fuzzy closed set is fuzzy g -super closed it is clear that every fuzzy g -super irresolute mapping is fuzzy g -super continuous but the converse may not be true.

Remark 3.2: Example (3.1) and example (3.2) asserts that the concepts of fuzzy g -super irresolute and fuzzy super continuous mappings are independent.

Theorem 3.2: If a mapping $f : (X, \mathfrak{S}) \rightarrow (Y, \sigma)$ is fuzzy g -super irresolute then

(a) $f(\text{gcl}(A)) \leq \text{gcl}(f(A))$

(b) $\text{gcl}(f^{-1}(B)) \leq f^{-1}(\text{gcl}(B))$.

Proof: Obvious.

Theorem 3.3: Let $f : (X, \mathfrak{S}) \rightarrow (Y, \sigma)$ is bijective fuzzy super open and fuzzy g -super continuous then f is fuzzy g -super irresolute.

Proof: Let A be a fuzzy g -super closed set in Y and let $f^{-1}(A) \leq G$ where G is fuzzy open set in X . Then $A \leq f(G)$. Since $f(G)$ is fuzzy super open and A is fuzzy g -super closed in Y , $\text{cl}(A) \leq f(G)$ and $f^{-1}(\text{cl}(A)) \leq G$. Since f is fuzzy g -super continuous and $\text{cl}(A)$ is fuzzy super closed in Y , $\text{cl}(f^{-1}(\text{cl}(A))) \leq G$. And so $\text{cl}(f^{-1}(A)) \leq G$. Therefore $f^{-1}(A)$ is fuzzy g -super closed in X . Hence f is fuzzy g -super irresolute.

Theorem 3.4: Let $f : (X, \mathfrak{S}) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \eta)$ be two fuzzy g -super irresolute mappings, then $\text{gof} : (X, \mathfrak{S}) \rightarrow (Z, \eta)$ is fuzzy g -super irresolute.

Proof : Obvious.

Theorem 3.5: Let $f : (X, \mathfrak{S}) \rightarrow (Y, \sigma)$ is fuzzy g -super irresolute and $g : (Y, \sigma) \rightarrow (Z, \eta)$ is fuzzy g -super continuous then the $\text{gof} : (X, \mathfrak{S}) \rightarrow (Z, \eta)$ is fuzzy g -super continuous.

Proof: Obvious.

Theorem 3.6: Let $f : (X, \mathfrak{S}) \rightarrow (Y, \sigma)$ is fuzzy g -super irresolute mappings, then $\text{gof} : (X, \mathfrak{S}) \rightarrow (Z, \eta)$ is fuzzy g -super irresolute and if B is fuzzy GO- super compact relative to X , then the image $f(B)$ is fuzzy GO- super compact relative to Y .

Proof : Let $\{A_i : i \in \Lambda\}$ be any collection of fuzzy g -super open set of Y such that $f(B) \leq \cup \{A_i : i \in \Lambda\}$. Then $B \leq \cup \{f^{-1}(A_i) : i \in \Lambda\}$. By using the assumption, there exists a finite subset Λ_0 of Λ such that $B \leq \cup \{f^{-1}(A_i) : i \in \Lambda_0\}$. Therefore, $f(B) \leq \cup \{A_i : i \in \Lambda_0\}$. Which shows that $f(B)$ is fuzzy GO- super compact relative to Y .

Corollary 3. 1: A fuzzy g -super irresolute image of a fuzzy GO- super compact space is fuzzy GO- super compact.

Theorem 3.8: Let $(X \times Y, \mathfrak{S} \times \sigma)$ be the fuzzy product space of non-empty fuzzy topological spaces (X, \mathfrak{S}) and (Y, σ) . Then the projection mapping $p : X \times Y \rightarrow X$ is fuzzy g -super irresolute.

Proof: Let F be any fuzzy g -super closed set of X . Then $p^{-1}(F) (= p^{-1}(F))$ is fuzzy g -super closed and hence p is fuzzy g -super irresolute.

Theorem 3.9: If the product space $(X \times Y, \mathfrak{S} \times \sigma)$ of two non empty fuzzy topological spaces (X, \mathfrak{S}) and (Y, σ) is fuzzy GO- super compact, then each factor space is fuzzy GO- super compact.

Proof: Obvious.

Theorem: 3.10: Let $f : (X, \mathfrak{S}) \rightarrow (Y, \sigma)$ is an fuzzy g -super irresolute surjection and (X, \mathfrak{S}) is fuzzy GO- super connected, then (Y, σ) is fuzzy GO- super connected.

Proof: Suppose Y is not fuzzy GO- super connected then there exists a proper fuzzy set G of Y which is both fuzzy g -super open and Fuzzy g -super closed, therefore $f^{-1}(G)$ is a proper fuzzy set of X , which is both fuzzy g -super open and fuzzy g -super closed, because f is fuzzy g -super continuous surjection. Therefore X is not fuzzy GO- super connected, which is a contradiction. Hence Y is fuzzy GO- super connected.

Theorem 3.11: If the product space $(X \times Y, \mathfrak{S} \times \sigma)$ of two non-empty fuzzy topological spaces (X, \mathfrak{S}) and (Y, σ) is fuzzy GO- super connected, then each factor fuzzy space is fuzzy GO- super connected.

Proof: Obvious.

REFERENCES

- [1] K. Atanassova, Fuzzy Sets, In VII ITKR's Session, (V.Sgurev,Ed.) Sofia, Bulgaria, (1983)
- [2] K. Atanassov and S. Stoeva., Fuzzy Sets, In Polish Symposium on Interval and Fuzzy Mathematics , Poznan, (1983), 23-26
- [3] K. Atanassov, Fuzzy Sets. Fuzzy Sets and Systems, 20(1986), 87-96.
- [4] C.L. Chang, Fuzzy Topological Spaces, J.Math.Anal.Appl. 24(1968) 182-190.
- [5] D. Coker, An Introduction to Fuzzy Topological Spaces, Fuzzy Sets and Systems .88(1997), 81-89.
- [6] D.Coker and A. Es. Hyder., On Fuzzy Compactness in Fuzzy Topological Spaces, The Journal of Fuzzy Mathematics, 3-4(1995),899-909.
- [7] H. Gurcay, D. Coker and Es., A.Haydar, On Fuzzy Continuity in Fuzzy Topological Spaces. The Journal of Fuzzy Mathematics Vol.5, no.2, 1997, 365-378.
- [8] Mishra M.K. ,et all on “ Fuzzy super closed set” International Journal International Journal of Mathematics and applied Statistics.
- [9] Mishra M.K. ,et all on “ Fuzzy super continuity” International Review in Fuzzy Mathematics ISSN : 0973-4392July –December2012.
- [10] Mishra M.K., Shukla M. “Fuzzy Regular Generalized Super Closed Set” Accepted for publication in International Journal of Scientific and Research Publication ISSN2250-3153. July December 2012.
- [11] N. Levine., Generalized Closed Sets In Topology, Rend. Cerc. Mat. Palermo.19(2), 1970, 571-599.
- [12] O. Ozbakir and D. Coker., Fuzzy Multifunctions in Fuzzy Topological Spaces Notes on IFS 5(1999) No. 3.
- [13] S.S. Thakur and Malviya R., Generalized Closed Sets In Fuzzy Topology, Math. Notae 38(1995), 137-140.
- [14] N.Turnali and D. Coker, Fuzzy Connectedness in Fuzzy Topological Spaces. Fuzzy Sets And Systems 116(2000) (3), 369-375.
- [15] L.A.Zadeh, Fuzzy Sets, Information and Control, 18(1965), 338-353.