

## Design and Control of a hybrid 2-DOF Balancing Table System

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**ABSTRACT:** This paper presents the design and control of a hybrid 2-degree-of-freedom (2-DOF) balancing table system. It consists of a mobile table which is attached to a two degree of freedoms serial system. This system is expected to handle payloads up to 5kg in weight and can maintain the orientation in the horizontal plane while is being mounted on a moving floor. Experiment results on a prototype show that the controller can give a good response in maintaining the orientation of the balancing table around its equilibrium pose with respects to the horizontal plane.

**KEYWORDS:** Balancing table, position control.

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### I. INTRODUCTION

Studying stabilization systems that have a robotic-based mechanism operating on a moving base is one of the most challenging tasks in robotics. These systems can be found in application fields such as offshore access systems or from the more popular fly-cam UAVs.

Several high degree of freedoms robotic based stabilization systems can be listed such as the Offshore Access System [1], Ampelmann [2], Personal Transfer System [3] or the Maxcess Transfer System [4]. Most of recent studies focus on the dynamic modeling of such systems [5-7].

Studies on less degree of freedoms stabilization systems can be mentioned such as in [8], where a 2-DOF spherical parallel mechanism which is used as a motion sensor device to measure two DOFs of orientations of the mobile platform. The system consists of two parallel legs that support a moving table rotating about a pivot universal joint, each parallel leg has three revolute joints. However, the work focused mainly on the design and mathematical modeling of the system.

The control problem of such stabilization systems, especially the fully 6-dof systems such as the Ampelmann [2], is a difficult problem and has rarely been discussed in literature.

In this paper, we present a hybrid 2-DOF balancing table system which is used to keep a top platform balancing at its equilibrium around the two horizontal axes of the world coordinate system regardless of the motions generated by the base. A cable-driven routing system is used to move the second rigid link of the system.

The paper is organized as follows. Section 2 presents the hybrid 2-dof balancing table system. Its mathematical models including kinematics and dynamics are presented in Section 3. The control system and experiment results are discussed in Section 4.

### II. TWO DEGREE OF FREEDOMS BALANCING TABLE

The design of the 2-dof balancing table is shown in Fig 2. The system consists of a base, two links, two revolute joints and a platform. The joint 1 and link 1 are directly driven by the motor 1. The joint 2 and link 2 are driven by the motor 2 via the pulley 1, 2, 3 and the cable. The top platform where placed the payloads of the table is the balancing control object. The MRU-PD is mounted on the platform to measure the angles of the platform. The motor 1, 2 are DC servo motors which are attached to the planetary reducers.

By moving the driving motor of joint 2 down to the base, the mass of link 1 is reduced. It leads to reduce driving torque of joint 2 and increase stability of the system. Furthermore, by isolating the driving groups from the table, it is able to isolate the noise that generates from the motors.

Beside the above advantages, there are problems of the cable and pulley driving system. Wiring the cable via the pulleys and roller is complicated. The tension of the cable must be always greater than a specific

value that guarantees the motion control accuracy of the joint. One pulley is used to tense the cable (by moving the pulley position vertically).

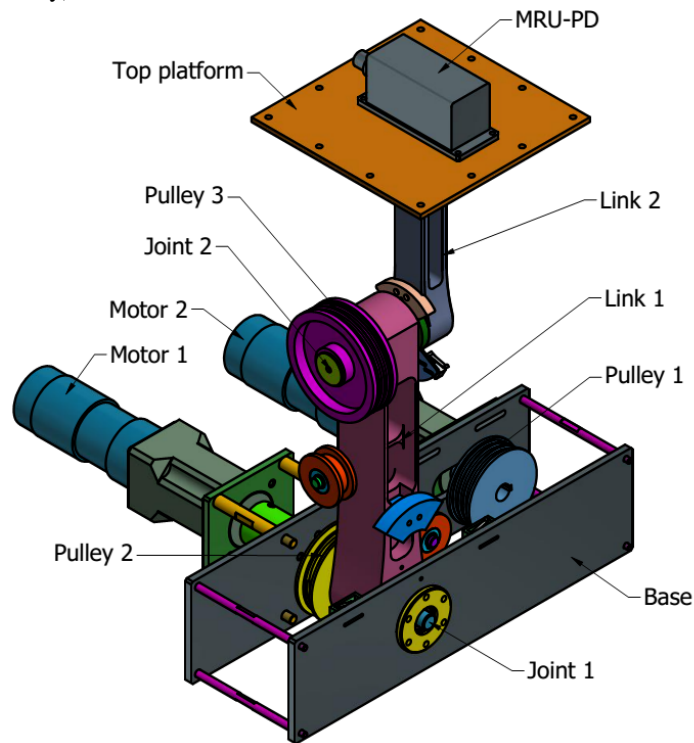


Fig. 1. Design of the 2-dof balancing table

### III. MATHEMATIC MODELS OF 2-DOF BALANCING TABLE

Figure 2 shows the structure of 2-dof balancing table. Its kinematic parameters are given in Table 1, following Craig rules [9] with  $r_2 = 0.05m$ ,  $d_2 = 0.26m$  and  $r_3 = 0.24m$ .

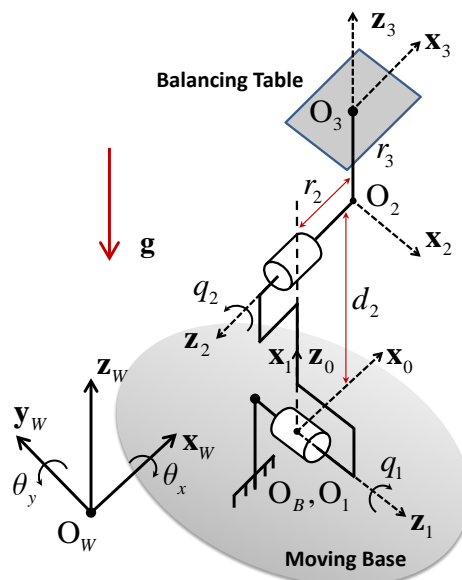


Fig. 2. Geometric structure of 2-dof balancing table

**Table 1. Kinematic parameters of 2-dof balancing table**

j	a(j)	$\sigma_j$	$\alpha_j$	$d_j$	$\theta_j$	$r_j$
1	0	0	$\pi/2$	0	$q_1$	0
2	1	0	$-\pi/2$	$d_2$	$q_2$	$-r_2$
3	2	2	$-\pi/2$	0	$\pi/2$	$r_3$

The initial joint values for the configuration showing in Fig.2 are:

$$\begin{cases} q_1(0) = \frac{\pi}{2} \\ q_2(0) = -\frac{\pi}{2} \end{cases} \tag{1}$$

Since our objective is to control the table orientation with respects to the horizontal plane, it is necessary to compute two rotation matrices:

- The rotation matrix that defines the rotation between the table frame and the base frame  ${}^0\mathbf{A}_3$  :

$${}^0\mathbf{A}_3 = {}^0\mathbf{A}_1 {}^1\mathbf{A}_2 {}^2\mathbf{A}_3 = \begin{bmatrix} S_1 & -C_1C_2 & -C_1S_2 \\ 0 & -S_2 & C_2 \\ -C_1 & -C_2S_1 & -S_1S_2 \end{bmatrix} \tag{2}$$

Here  ${}^{i-1}\mathbf{A}_i$  ( $i=1,2,3$ ) is the rotation matrix between any two consecutive coordinate frames of the system and  $S_1 = \sin(q_1), C_1 = \cos(q_1), S_2 = \sin(q_2), C_2 = \cos(q_2)$ .

- The rotation matrix that defines the rotation between the base frame and the world coordinate frame  ${}^W\mathbf{A}_0$  :

$${}^W\mathbf{A}_0 = \mathbf{rot}(\mathbf{x}, \theta_x) \mathbf{rot}(\mathbf{y}, \theta_y) = \begin{bmatrix} C_y & 0 & S_y \\ S_xS_y & C_x & -C_yS_x \\ -C_xS_y & S_x & C_xC_y \end{bmatrix} \tag{3}$$

where the matrix  $\mathbf{rot}(\mathbf{u}, \theta)$  represents the rotation about a vector  $\mathbf{u}$  with an angle  $\theta$  and  $S_x = \sin(\theta_x), C_x = \cos(\theta_x), S_y = \sin(\theta_y), C_y = \cos(\theta_y)$ .

In order for the top platform is maintained at its equilibrium (parallel to the horizontal plane), the following conditions must be met:

$${}^W\mathbf{A}_3 = {}^W\mathbf{A}_0 {}^0\mathbf{A}_3 = \mathbf{1}_{3 \times 3} \tag{4}$$

or:

$${}^0\mathbf{A}_3 = ({}^W\mathbf{A}_0)^{-1} \Leftrightarrow \begin{bmatrix} S_1 & -C_1C_2 & -C_1S_2 \\ 0 & -S_2 & C_2 \\ -C_1 & -C_2S_1 & -S_1S_2 \end{bmatrix} = \begin{bmatrix} C_y & S_xS_y & -C_xS_y \\ 0 & C_x & S_x \\ S_y & -C_yS_x & C_xC_y \end{bmatrix} \tag{5}$$

It is not difficult to show that one solution which satisfies (5) is the following:

$$\begin{cases} q_1 = \theta_y + \frac{\pi}{2} \\ q_2 = \theta_x - \frac{\pi}{2} \end{cases} \tag{6}$$

The solution in (6) can be used as the desired pose for the control problem of the balancing table system.

In this work, we consider the impact of the system dynamics on the control system including the effects from the base motion and the dynamics of the two moving links of the balancing table. In order to do so, it is

necessary to derive the inverse dynamic model (IDM) of the robotic system. This model can be written in the following form:

$$\Gamma = \mathbf{A}\ddot{\mathbf{q}} + \mathbf{H}(\mathbf{q}, \dot{\mathbf{q}}, \dot{\mathbf{X}}_B, \ddot{\mathbf{X}}_B) \tag{7}$$

Here,  $\Gamma = [\Gamma_1 \ \Gamma_2]^T$  is the actuator torque vector of the system,  $\mathbf{q} = [q_1 \ q_2]^T$ ,  $\dot{\mathbf{q}} = [\dot{q}_1 \ \dot{q}_2]^T$ ,  $\ddot{\mathbf{q}} = [\ddot{q}_1 \ \ddot{q}_2]^T$  are the joint position vector, joint velocity vector, and joint acceleration vector, respectively. And  $\dot{\mathbf{X}}_B = [{}^W \mathbf{V}_0^T, {}^W \boldsymbol{\omega}_0^T]^T$ ,  $\ddot{\mathbf{X}}_B = [{}^W \mathbf{a}_0^T, {}^W \boldsymbol{\alpha}_0^T]^T$  define the space velocity and acceleration vectors of the base with respect to the world coordinate frame  $\langle O_W \rangle$ , with  ${}^W \mathbf{V}_0$ ,  ${}^W \boldsymbol{\omega}_0$ ,  ${}^W \mathbf{a}_0$  and  ${}^W \boldsymbol{\alpha}_0$  being the velocity, angular velocity, acceleration and angular acceleration vectors of the base expressed in the frame  $\langle O_W \rangle$ .

Equation (7) is derived by using the Newton-Euler method presented in [10]. The notation of the dynamic parameters used in this paper is taken from [9]. Table 2 shows the dynamic parameters of the two moving link of the balancing table (the link 2 consists also the top platform). Each moving link has ten dynamic parameters that form the inertial matrix  $\mathbf{I}_j$ , the principle moment  $MS_j$  and the mass  $M_j$  of the link ( $j=1, 2$ ):

$$\mathbf{I}_j = \begin{bmatrix} XX_j & XY_j & XZ_j \\ XY_j & YY_j & YZ_j \\ XZ_j & YZ_j & ZZ_j \end{bmatrix},$$

$$MS_j = [MX_j \ MY_j \ MZ_j]^T$$

$$\rightarrow Y_j = [XX_j \ XY_j \ XZ_j \ YY_j \ YZ_j \ ZZ_j \ MX_j \ MY_j \ MZ_j \ M_j]^T \tag{8}$$

Note that the matrix  $\mathbf{I}_j$  is computed with respects to the origin of the coordinate frame  $\langle O_j \rangle$ .

**Table 2. Dynamic parameters of 2-dof balancing table**

Link	$XX_j$ ( $kg\ m^2$ )	$XY_j$ ( $kg\ m^2$ )	$XZ_j$ ( $kg\ m^2$ )	$YY_j$ ( $kg\ m^2$ )	$YZ_j$ ( $kg\ m^2$ )	$ZZ_j$ ( $kg\ m^2$ )	$MX_j$ ( $kg\ m$ )	$MY_j$ ( $kg\ m$ )	$MZ_j$ ( $kg\ m$ )	$M_j$ ( $kg$ )
1	0.0021	-2.96e-4	4.004e-4	0.0255	1.159e-5	0.0245	0.1313	0.0038	-0.0051	1.686
2	0.0287	-2.29e-5	7.064e-6	0.0056	0.0028	0.0271	3.02e-4	0.1218	-0.0376	1.607

#### IV. CONTROL SYSTEM OF 2-DOF BALANCING TABLE

The control law of 2-dof balancing table takes into account the dynamics of the base as well as the dynamics of the moving links. We adapt the computed torque control in the joint space from [9] where the control torque vector is computed as the following rules:

$$\Gamma = \hat{\mathbf{A}}(\mathbf{q})\mathbf{w}(t) + \hat{\mathbf{H}}(\mathbf{q}, \dot{\mathbf{q}}, \dot{\mathbf{X}}_B, \ddot{\mathbf{X}}_B) \tag{9}$$

with  $\hat{\mathbf{A}}$  and  $\hat{\mathbf{H}}$  are the estimates of  $\mathbf{A}$  and  $\mathbf{H}$  in (7) respectively, and  $\mathbf{w}(t)$  is the input control vector:

$$\mathbf{w}(t) = \ddot{\mathbf{q}}^d + \mathbf{K}_d(\dot{\mathbf{q}}^d - \dot{\mathbf{q}}) + \mathbf{K}_p(\mathbf{q}^d - \mathbf{q}) \tag{10}$$

$\mathbf{K}_d = \mathbf{diag}(k_{d1}, k_{d2})$ ,  $\mathbf{K}_p = \mathbf{diag}(k_{p1}, k_{p2})$  are two  $[2 \times 2]$  positive definite diagonal matrices. The desired joint position, velocity and acceleration vectors are compute as follows:

$$\mathbf{q}^d = \begin{bmatrix} \theta_y \\ \theta_x \end{bmatrix} + \begin{bmatrix} \pi/2 \\ -\pi/2 \end{bmatrix}$$

$$\dot{\mathbf{q}}^d = \begin{bmatrix} \dot{\theta}_y \\ \dot{\theta}_x \end{bmatrix}$$

$$\ddot{\mathbf{q}}^d = \begin{bmatrix} \ddot{\theta}_y \\ \ddot{\theta}_x \end{bmatrix} \tag{11}$$

The control scheme for this case is shown in Fig. 3. The “base motion” block measures the motions of the base and generates desired values for the joint positions, velocities and accelerations according to (11). This

block also estimates the space velocity and acceleration vectors of the base which are used to compute the desired control torque as the output of the inverse dynamic model of the 2-dof balancing system.

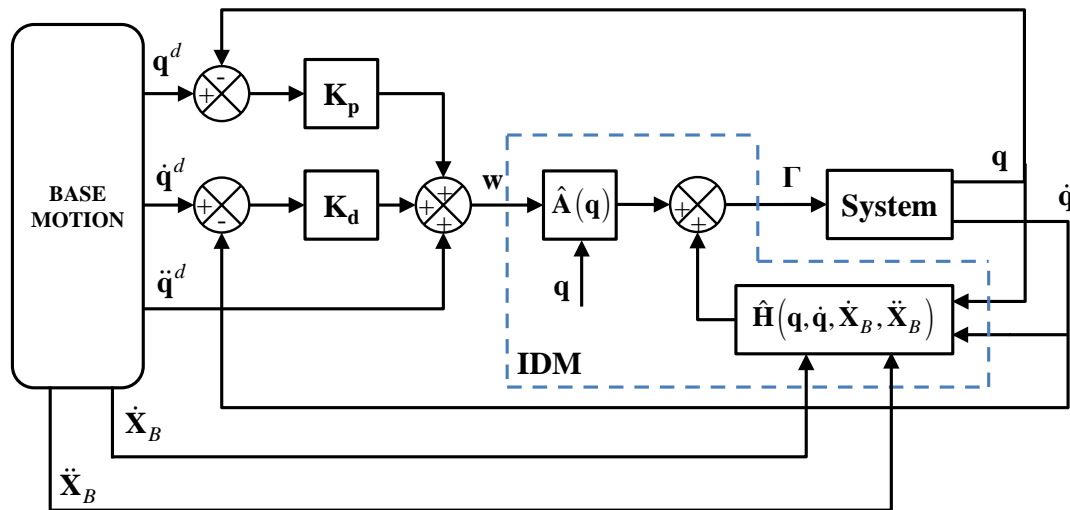


Fig. 3. Control scheme of 2-dof balancing table system

To extract the top table motion, we use the same rule that defines the base motion in (3):

$${}^w A_3 = {}^w A_0 {}^0 A_3 = \text{rot}(x, \theta_{x3}) \text{rot}(y, \theta_{y3}) = \begin{bmatrix} C_{y3} & 0 & S_{y3} \\ S_{x3} S_{y3} & C_{x3} & -C_{y3} S_{x3} \\ -C_{x3} S_{y3} & S_{x3} & C_{x3} C_{y3} \end{bmatrix} \quad (12)$$

with  $\theta_{x3}$  and  $\theta_{y3}$  are the two rotating angles of the top table. When the orientation of the base is known and the joint positions of the balancing table are known, one can compute the matrix  ${}^w A_3$ , thus the values of  $\theta_{x3}$  and  $\theta_{y3}$  can be computed as follows:

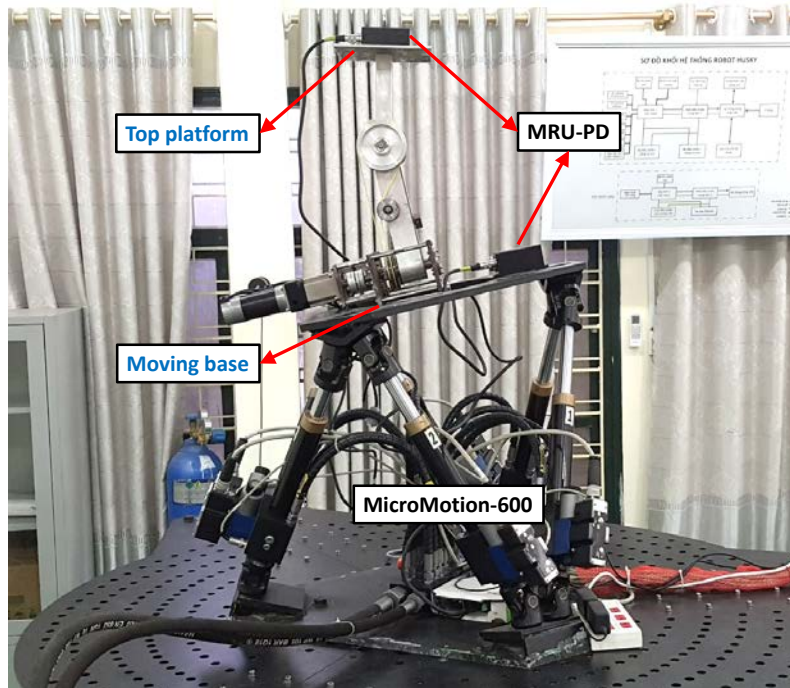


Fig. 4. Prototype of 2-dof balancing table mounting on the moving platform of a MicroMotion-600

$$\begin{cases} \theta_{x3} = \text{atan2}({}^w \mathbf{A}_3[3,2], {}^w \mathbf{A}_3[2,2]) \\ \theta_{y3} = \text{atan2}({}^w \mathbf{A}_3[1,3], {}^w \mathbf{A}_3[1,1]) \end{cases} \quad (13)$$

Fig.4 shows the prototype of balancing table system in this study case. Its base is mounted on the moving platform of a MicroMotion-600 Gough Stewart parallel robot from BoschRexroth [11]. The MicroMotion-600 system is used to generate the motion for the base in space. The position of the reference point of the balancing system's base  $O_B$  is  $0.055m$  along the vertical axis (the  $z$ -axis) with respect to the reference point of the moving platform of MicroMotion-600 system.

In this work, we generate the motion for the moving platform of the MicroMotion-600 system with two rotation angles as follows (Fig. 5):

$$\begin{cases} \theta_x = 8 \sin(2\pi \times 0.53t) \quad (\text{deg}) \\ \theta_y = 15 \sin(2\pi \times 0.27t) \quad (\text{deg}) \end{cases} \quad (14)$$

This motion is within the workspace of the MicroMotion-600 system (in terms of the angles  $\theta_x, \theta_y$ ). This means in this case the origin of the global coordinate frame  $O_w$  is placed at the reference point of the moving platform of MicroMotion-600 system. This motion will generate an angular motion and linear motion for the base of the 2-dof balancing system (6-dof motion).

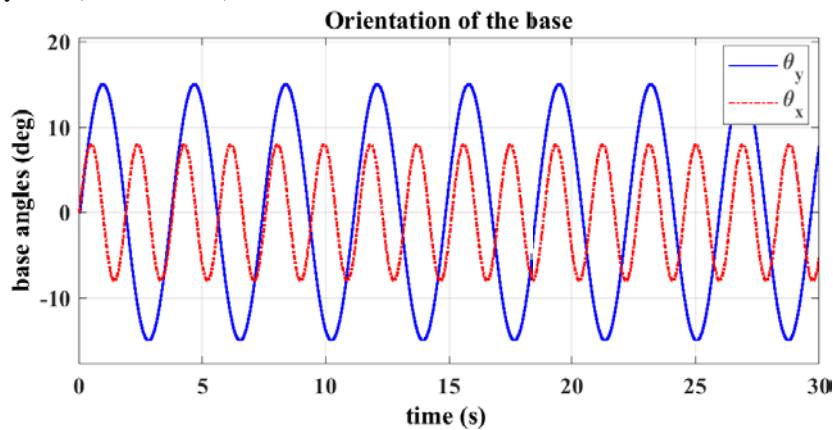


Fig. 5. Orientation of the base

In order to measure the base motion we use one MRU-PD device [12] to get the orientation as well as the angular velocity and acceleration vectors of the base. From these terms one can compute the velocity vector of the point  $O_B$  and the base angular acceleration. To measure the orientation of the top platform with respects to the horizontal plane, we use another MRU-PD device. We use SK-75 Dyneema cable [13] to drive the second link (the cable diameter is 3mm which can handle up to 500kg payload).

The control system has the sample time of 5ms. It is chosen corresponding to the maximum feedback rate of two MRU-PD devices (at 200Hz). The controller parameters are selected as follows:

$$\mathbf{K}_d = \text{diag}(9, 9), \quad \mathbf{K}_p = \text{diag}(6, 6) \quad (15)$$

The responses of the balancing table are shown in Fig. 6 – 8. It can be seen that the top platform is being kept around its equilibrium with respect to the horizontal plane. The maximum errors (at steady state) in this case are  $0.55\text{deg}$  and  $0.3\text{deg}$  in  $\theta_{y3}$  and  $\theta_{x3}$  respectively, which is a quite good performance.

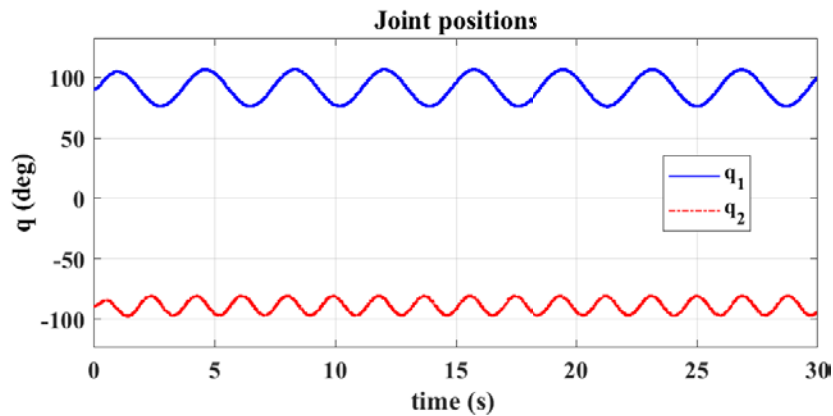


Fig. 6. Joint positions of the balancing table

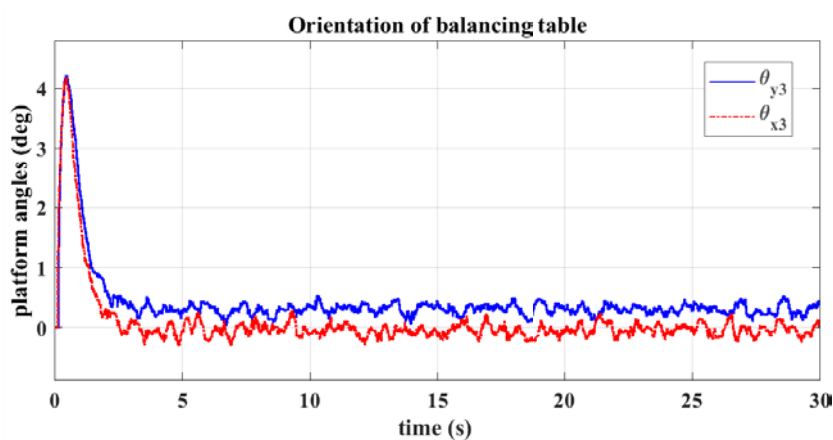


Fig. 7. Orientation of the top platform of balancing table

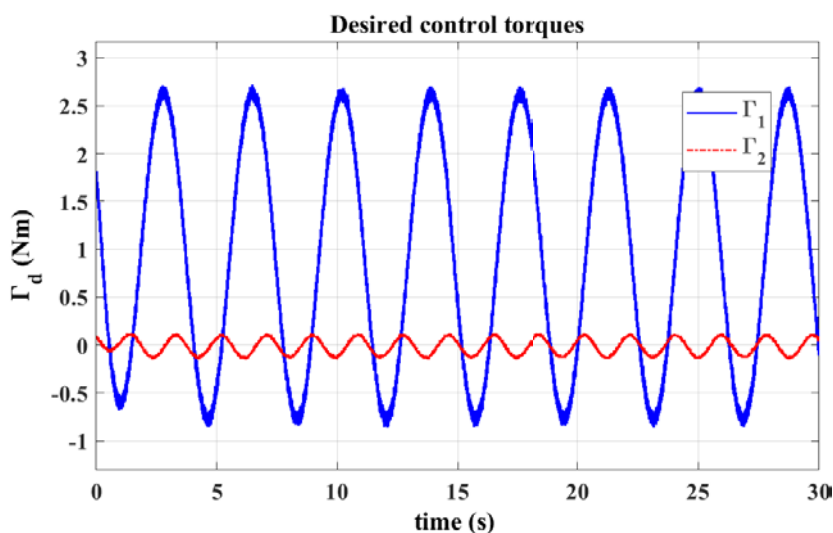


Fig. 8. Desired control torques of the balancing table

We notice that there are noises in the measurement of the two MRU-PD devices. This lead to errors in the computation of the inverse dynamic model since two inputs are the space velocity and acceleration of the base are calculated from these measurement data of the two devices. There are also the errors in mathematical model due to the machining process and assembly. Eliminating these errors in measurement and modeling is not a trivial problem.



## V. CONCLUSION

In this paper we have presented a design solution for a serial 2-dof balancing table. Although this hybrid solution by adding a cable-driven routing system makes the system becomes more complicated, it helps to reduce the control output torques needed to actuate the two moving links.

The control algorithm considering the dynamic of the system (including the motion of the base) can keep the top platform at its equilibrium with respect to the horizontal plane. Although the orientation errors in the experiment are still quite large, it has shown that the control system is working.

In our future works, we aim to implement this algorithm to balancing systems with larger degree of freedoms. More advance control methods might need to be developed for such complicated systems.

## REFERENCES

- [1]. Offshore Access System. [www.offshoresolutions.co.nz](http://www.offshoresolutions.co.nz)
- [2]. Ampelmann. [www.ampelmann.nl](http://www.ampelmann.nl)
- [3]. Personal Transfer System. [www.pts-offshore.com](http://www.pts-offshore.com)
- [4]. Maxcess Transfer System. [www.osbitpower.com](http://www.osbitpower.com)
- [5]. E.,Sadraei,M. M., Moghaddam: On a Moving Base Robotic Manipulator Dynamics. International Journal of Robotics, Vol. 4, No. 3, p. 66-74, (2015).
- [6]. M. L., Antonio: Complete dynamic modelling of a moving base 6-DoF parallel manipulator. Robotica, Vol. 28, p. 781-793 (2009).
- [7]. C. M., Wronka,M. W., Dunningan: Derivation and analysis of a dynamic model of a robotic manipulator on a moving base. Robotics and Autonomous Systems, Vol. 59, p. 758-769 (2011).
- [8]. X., Duan, Y., Yang, B., Cheng: Modeling and Analysis of a 2-DOF Spherical Parallel Manipulator. Sensors, Vol. 16, p. 1485-1499, (2016).
- [9]. W., Khalil: Dynamic Modeling of Robots Using Newton-Euler Formulation. Informatics in Control, Automation and Robotics, Vol. 89, p. 3-20 (2010).
- [10]. W., Khalil, E., Dombre: Modeling, Identification and Control of Robots. Kogan page Science Paper Edition (2002).
- [11]. MicroMotion-600 Motion System. [www.boschrexroth.com](http://www.boschrexroth.com)
- [12]. Inertial Labs Motion Reference Unit MRU-PD. <https://inertiallabs.com/products/mru/>
- [13]. SK-75 Dyneema cable. [https://www.dsm.com/dyneema/en\\_GB/home.html](https://www.dsm.com/dyneema/en_GB/home.html)

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