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Characterization of an electoral process through a multifractal distribution

José Roberto Mercado-Escalante[1], Pedro Antonio Guido-Aldana[2], Jorge Sánchez-Sesma[1]

[1]Independent Researcher, Mexico. [2]Mexican Institute of Water Technology
Corresponding Author: José Roberto Mercado Escalante

ABSTRACT : *The objective of this research is to find the qualities that an election process must show so that it can have the option of being qualified as genuine. For this purpose, irregular and regular qualities meet in the same object; the first, as a reflection of human settlements, and the second, for various political interests and opinions. Three scales are considered: micro, meso and macro. The distribution and its density are sought, which reflects the electoral results, and the Multifractal distribution is found, associated with what is called the "random electoral variable". While the Multifractal distribution density is the inverse of the singularities spectrum. But in addition an approximate distribution is found, this turns out to be the Student's t-distribution, which allows to use the Pearson's method with the first four central moments. Among the qualities obtained from the quantitative analysis, two stand out: symmetry and unimodality.*

KEYWORDS: *Multifractal distribution; election process; electoral voting system; Student's t-distribution; singularities spectrum.*

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I. INTRODUCTION

In order to develop the research, it is necessary to imagine a country where an electoral voting system is broad enough to cover the assumptions that are assumed. We propose to find the most important characteristics of the result of an election to judge it as genuine or legitimate, or in its defect, fraudulent.

Citizens must attend voting locations that will be scattered across the country and reflect the dispersion of human settlements. But also, they must deposit their respective votes for different parties differentiated by diverse colors and logos. The different colors will mark the paths of the votes without contamination of any kind, so we can imagine them as fluids through the entire system, through differentiated and parallel ducts or channels. The image of nature that evokes us is a very leafy tree.

In many cases the number of parties or options can be set at two, due to the existence of a second round in the electoral process, or because comparatively so possible triumph options are arranged, who usually defend the "status quo" and compete against those who aspire to replace it or change it, the former qualify as right-wing as their opposites as left-wing.

The values of the preferences (p_i) can be set from the surveys carried out on dates close to the election. The results can be studied from the preliminary and exit polls that are drawn up from the polls after the voting. On the other hand, in the case of several candidates, the analysis could be done in pairs.

The objective is to propose a distribution of probabilities that synthesize the electoral results and also find another one, which can be judged analogous and close, and that allows it to be estimated with the Pearson's method with the four central moments, from small samples.

Because the quality of energy supplied can adversely affect its operation, oftentimes leading to loss or degradation of equipment, product, revenue, and reputation, plant managers must weigh the advantages of implementing a monitoring program.

The second section of this paper shows three methods for monitoring systems of solar plants. The third section discusses communication and monitoring system for wind turbines, and finally the conclusion is discussed in the fourth section.

II. SYSTEM DESCRIPTION AND METHODS

2.1 Electoral voting system

The voting system can be observed as a dual object. On the one hand it is similar to a distribution system, which we describe as the series of points in the space that represents the set of voting points, and which, speaking in general and imagining it sufficiently extensive, must have a scattered and irregular appearance, which reflects the similar characteristics of human settlements; we cover these space points with a mesh of cubes of the same size, but variable, and we seek to determine their fractal dimension. On the other hand, the voting system is a macroscopic body, composed of several microscopic states: the probability of voting for one or the other of the candidates, including the possibility of null votes or blank votes, but each state is perfectly distinguishable.

As a macroscopic body, a method similar to thermodynamics is followed and we study it as Bernoulli's enhanced processes. We consider the body as a compound of several microscopic states. We define the number (p_i), as the probability of voting for a candidate, being $\sum p_i = 1$, with $0 \leq p_i \leq 1$. We define a microscopic measure by $\mu_i = p_i$. The transition from a micro level to a macro level occurs through reiteration, so we enhance the probability of voting, defining: $\nu_i(s)$, which complement each other, in the sense that they satisfy: $\sum \nu_i = 1, 0 < \nu_i < 1$; and that the micro measurements be recovered by $\nu_i(1) = p_i$. These are defined in equation (3).

On the other hand, as an irregular object it is covered by a mesh of cubes $(C_k)_k$ of side $h, 0 < h < 1$, a number that we call the resolution. The mesoscopic level is defined by the resolution h, so that the mesoscopic states are measured by the succession $\log_h p_i$, of positive numbers. Let α be the order of the singularity of the measure $\log_h p_i$, in some C_k cube of the mesh,

$$\alpha = \log_h \mu(C_k) \tag{1}$$

We define the characteristics by the total number of cubes where the measure has the singularity of the order ofα, being h small,

$$h^{\alpha+\varepsilon} \leq \mu(C_k) < h^{\alpha},\ N_h(\alpha) = \#\{k: \mu(C_k) \geq h^{\alpha}\} \tag{2}$$

Gibbs distribution is defined as the probability of finding the state with micro-configuration measured by $\mu_k, k = \{1,2, \dots, K\}$, where the number K depends on the order of the singularity $\alpha, K(\alpha)$, such as the number of macro-configurations that represent this state over the total number of available configurations of the object, or as the relative frequency of occupation of a microscopic state, [1], by:

$$\nu_k(s) = \frac{{\mu_k}^s}{\sum_i {\mu_i}^s} \tag{3}$$

The partition function is understood as the total sum of the configurations, or macro-configurations, available of a system:

$$S_h(s) = \sum_i {\mu_i}^s \tag{4}$$

where the sum extends, in general, on those cubes that intersects the support of the microscopic measurement. A power relationship is sought for the partition function, and this power is called the structure function, [2], [3], [4]:

$$S_h(s) = h^{-\tau(s)} \tag{5}$$

Otherwise, in multifractal formalism the structure function, $\tau(s)$, turns out to be the Legendre transform of the multifractal spectrum, $f(\alpha)$; where the maximum is reached for a certain α, when the concave curve $f(\alpha)$ is above and as far as possible from the line $s\alpha$ of slope s, then:

$$\tau(s) = \sup_{\alpha' \geq 0}\{f(\alpha') - s\alpha'\},\ \tau(s) = f(\alpha(s)) - s\alpha(s) \tag{6}$$

By the condition of maximum, it is obtained that the slope of the spectrum f is the reiteration power s; and, when τ is differentiable, the slope of the structure function τ is $-\alpha$:

$$\frac{d}{d\alpha}(f(\alpha) - s\alpha) = 0,\ \frac{d}{d\alpha}f = s,\ \frac{d}{ds}\tau = -\alpha \tag{7}$$

Therefore, in the distribution of the probabilities of the microscopic states, the p_i, define the probabilistic model at the microscopic level. Meanwhile, mesoscopic states are measured by the succession of positive numbers $\log_h p_i$. And the macro level is obtained by the average values of the random variables at the reiteration level s.

Let's assume that we deal with a body that has K microscopic states. After s reiterations of the spraying process, the body will be in a macroscopic state, whose entropy, according to the definition [5] is given by:

$$f(\alpha) = \sum_k \nu_k \log_h \nu_k \tag{8}$$

Then,

$$f(\alpha) = \sum_{k=1}^{K} \log_h (\nu_k)^{\nu_k} = \log_h \prod_{1}^{K} \left(\frac{p_k^s}{\sum_i p_i^s} \right)^{\nu_k} = \log_h \frac{\left(\prod_k^K p_k^{\nu_k} \right)^s}{\left(\sum_i p_i^s \right)^1} \tag{9}$$

Finally:

$$f(\alpha(s)) = -\log_h \sum_{i=1} p_i^s + s \sum_k \nu_k \log_h p_k \tag{10}$$

As $\frac{d}{d\alpha} f = s$, then, the linear part with s determines $\alpha(s)$, which is expressed by the convex combination of the meso-states $\log_h p_i$.

$$\alpha(s) = \sum_k \nu_k(s) \log_h p_k = \sum_k \left(\frac{p_k^s}{\sum_i p_i^s} \right) \log_h p_k \tag{11}$$

Meanwhile, the convex part leads us to the structure function:

$$\tau(s) = -\log_h \sum_{k=1}^{K} p_k^s \tag{12}$$

And it is observed that the singularity $\alpha(s)$ is represented by the weighted average of the values $\log_h p_i$, being the weights $\nu_i(s)$ the relative frequency of occupation of the meso-state $\log_h p_i$, or micro-state p_i.
In many cases the K value can be set to 2. In the case of two micro-states, in particular, the spectrum of the singularities is:

$$f(\alpha(s)) = -\log_h (p_1^s + p_2^s) + s \left(\frac{p_1^s}{p_1^s + p_2^s} \log_h p_1 + \frac{p_2^s}{p_1^s + p_2^s} \log_h p_2 \right) \tag{13}$$

The order of the singularities, in the s state, is:

$$\begin{aligned} \alpha(s) &= \frac{p_1^s}{p_1^s + p_2^s} \log_h p_1 + \frac{p_2^s}{p_1^s + p_2^s} \log_h p_2 \\ &= \nu_1(s) \log_h p_1 + \nu_2(s) \log_h p_2 \end{aligned} \tag{14}$$

meanwhile, the structure function:

$$\tau(s) = -\log_h (p_1^s + p_2^s) \tag{15}$$

which represents a power ratio for the partition function:

$$S_h(s) \approx h^{-\tau(s)} \tag{16}$$

The features and the $f(\alpha)$ spectrum are mutually determined because the features must be of the order of $f(\alpha)$; so the number of features follows a power law, and is given by:

$$N_h(\alpha) \approx h^{-f(\alpha)} = \frac{p_1^s+p_2^s}{\left(p_1^{p_1^s} p_2^{p_2^s}\right)^{\frac{s}{p_1^s+p_2^s}}}, 0 < h < 1 \tag{17}$$

In the graph of Figure 1 we represent the order of the singularities, in parametric form, with coffee color line $\alpha(s, 1/7,3), s$; the inverse of the spectrum of singularities $(f(s, 1/7,3), s)$, with red color line; the information dimension, with circle $(\alpha(1/7,3), 1)$ and black color line $(\alpha(1,1/7,3), (1 + 1/10(s - 5)1))$.

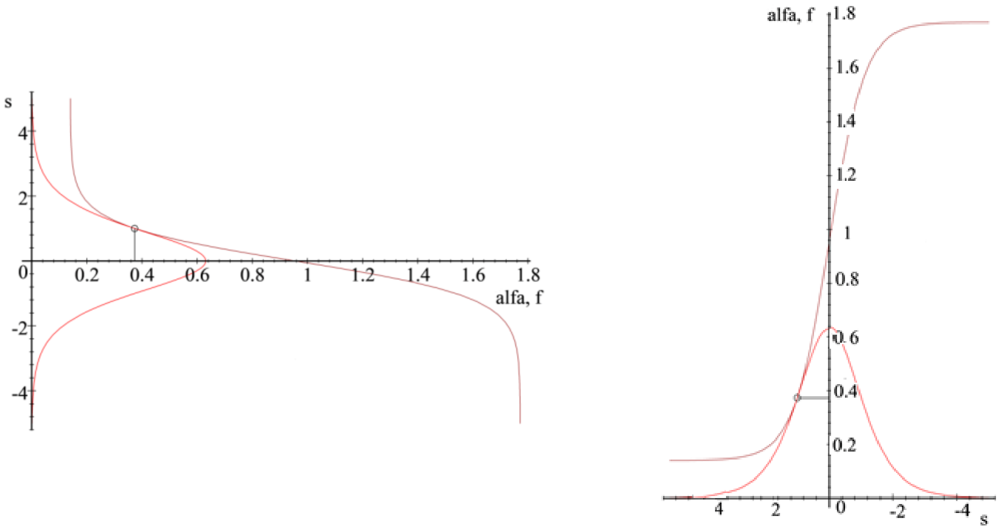

Figure 1. Order of the singularities in a parametric way.

III. RESULTS AND DISCUSSION

3.1 A Multifractal distribution

We consider the binomial multifractal in terms of the reiteration power, the probability of the Bernoulli process with $p = 1/7$, and based on resolution $h = 3$, as $f(s, p, h) = \tau(s, p, h) + s\alpha(s, p, h)$, [6], with:

$$\tau(s) = -\frac{1}{\ln 3}\ln[(1/7)^s + (6/7)^s] \tag{18a}$$

$$\alpha(s, 1/7,3) = -\frac{1}{\ln 3}\left[\frac{(1/7)^s \ln(1/7)+(6/7)^s \ln(6/7)}{(1/7)^s+6/7^s}\right] \tag{18b}$$

From this multifractal spectrum we define the Multifractal distribution $M(x)$:

$$M(x;1/7,3) = \frac{1}{1.6713}\int_{-\infty}^{x} f(s,1/7,3)ds \tag{19}$$

Because conditions are verified: 1. $f(s) \geq 0$, for almost all s; 2. $M(\infty) \to 1$; and 3. It is observed that $f(s)$ is measurable according to Lebesgue; then, in effect, $f(s)$ defines a density of a certain random magnitude, that we have call "random electoral variable". On the other hand, the probability that the random magnitude does not exceed a certain value, in the case of the Student's t-distribution, is given by the expression:

$$\Pr(\xi \leq x) = \mathrm{TDist}(x; \nu) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)\sqrt{\pi\nu}}\int_{-\infty}^{x}\left(1+\frac{1}{\nu}s^2\right)^{-\frac{\nu+1}{2}} ds \tag{20}$$

where Γ is Euler's delta function and the shape parameter ν is known as the freedom degrees. In the case when this number is equal to 4, you have:

$$\Pr(\xi \leq x) = \mathrm{TDist}(x; 4) = \frac{3}{8}\int_{-\infty}^{x}\left(1+\frac{1}{4}s^2\right)^{-\frac{5}{2}} ds \tag{21}$$

The graphs of both distributions are presented in Figure 2, with a continuous line for the Multifractal and small circles for the Student's t.

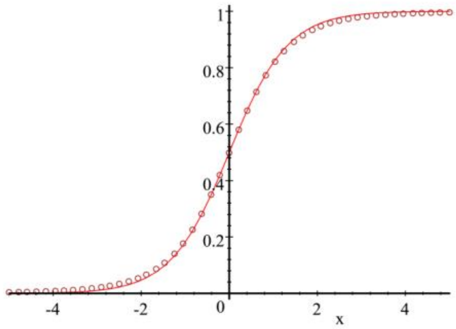

Figure 2. Distributions: Multifractal (continuous line); Student's t (circles).

The densities are also shown in the graph of Figure 3, the Multifractal with a continuous line, the Student's t with small circles and the Gaussian with crosses.

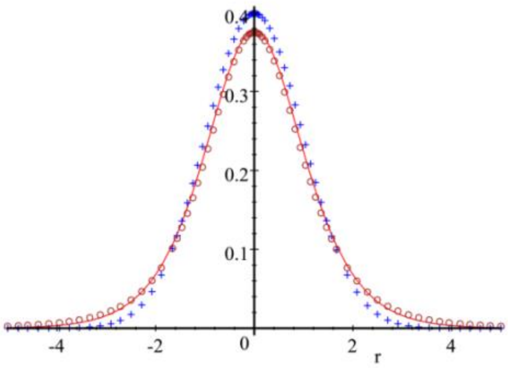

Figure 3. Distributions: Multifractal (continuous line); Student's t (circles); Gaussian (crosses).

The graph in Figure 3 shows that the three densities are bell-shaped, symmetrical, and their arithmetic means are zero. Compared to Gaussian, Multifractal and Student's t have lower height, greater variance, and the tails are larger. In particular, both the Multifractal and the Student's t-distribution are useful when the sample sizes are small.

When comparing the multifractal spectrum as a function of the power parameter with the Student's t-distribution, the graphic representation of the two shows the proximity of one to the other. This closeness is also reinforced when the two expansions are observed and compared in series. The comparison between the binomial Multifractal with probability of the Bernoulli process of $p = 1/7$, with the Student's t of 4 freedom degrees, for the distributions, is illustrated in the graph of Figure 2; and for densities, in the graph of Figure 3.

If the probability of success is increased, the tail is lifted and further away from the Gaussian, and they are shown in the graph in Figure 4. $f(s, 1/3, 3/2)$, (black); $f(s, 1/4, 3/2)$, (blue); $f(s, 1/5, 3/2)$, (brown); $f(s, 1/7, 3/2)$, (green); $f(s, 1/9, 3/2)$, (magenta); $f(s, 1/9.5, 3/2)$, (navy color).

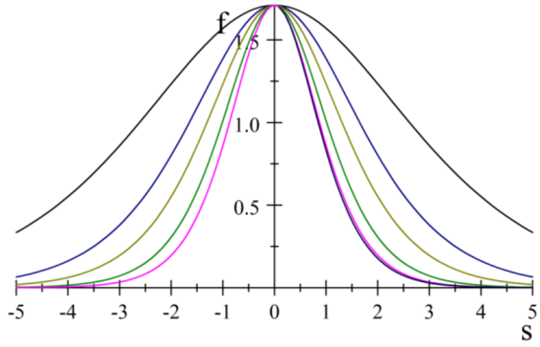

Figure 4. Distributions: Multifractal for different values of the probability p_i .

IV. CONCLUSIONS

The inverse of the spectrum of singularities is the Multifractal density distribution, which is symmetric and unimodal.

The same is close to a Gaussian and a Student's t-distribution.

If any of the densities obtained in an electoral process is not unimodal, but bimodal for example, it can be argued that there should be ballot filling and therefore the election can be qualify as fraudulent [7].

In addition, if one is not symmetric but, for example, more nourished on the left side than on the right side, it should be suspected that there could be a subtraction of votes and would also qualify as fraud.

On the contrary, if any of the densities is unimodal and symmetric, it can be considered genuine [7].

When there are more than two candidates, an elimination analysis can be done to find out who won the elections.

The opening of ballot boxes could be carried out, take a sample of type 5 to 10%, analyze it, and contribute to the transparency of the process.

This method could be applied to estimate the degree of authenticity of an account of a "Youtuber" with respect to that of a "Bot". There are many benefits to installing a monitoring system — some of which strongly interrelate with each other.

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Author's biographies

Dr. José Roberto Mercado-Escalante

Independent Researcher. Doctor of Sciences (Mathematics) by the National Autonomous University of Mexico (UNAM), Master Degree in Mathematics by the Meritorious Autonomous University of Puebla (BUAP), Mexico; graduates in Physics by the National University of Colombia. Research areas: inverse problems, fractals and fractional derivatives, mathematical aspects of hydraulics. E-mail: jrmercadoe@yahoo.com

Dr. Pedro Antonio Guido-Aldana

Researcher at the Mexican Institute of Water Technology- IMTA, Mexico. Affiliate to the Professional and Institutional Development Coordination. Doctor of Engineering and Master Degree in Hydraulics by the National Autonomous University of Mexico-UNAM. Civil Engineer. Associate Professor of the Engineering Faculty of UNAM. Research interest: water planning, hydraulics, potamology, energy and environment. E-mail: pedroguido@tlaloc.imta.mx

Dr. Jorge Sánchez-Sesma

Independent consultant and researcher. Doctor of Engineering. Researcher at IMTA and INEEL (National Institute of Electricity and Clean Energy). Research interest: Extreme climate, hydrological events, climate change, multi-millennial-scale solar activity and its influences on continental tropical climate.

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