

A Comparison between Using Finite Difference and Limit Equilibrium Methods for Landslide Analysis of Slopes Containing a Weak Layer

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ABSTRACT: Weak layers play an important role in instability of slopes. In this paper, a comparison between using Finite Difference Method and the traditional method (i.e., Limit Equilibrium Method) for stability analysis of rock slopes containing a weak layer is carried out. For this purpose, four different cases are studied: (1) a horizontal weak layer in the middle of a slope, (2) a horizontal weak layer close to the slope base, (3) an inclined weak layer with a moderate slope of 29°, and (4) a weak layer with a steep slope of 57°. The results obtained from the slope stability analysis performed by using the two methods show that Finite Difference Method calculates a lower minimum factor of safety for all cases, and predicts a smaller potential failure surface when the weak layer is horizontal or has a slope of 29°. However, for the weak layer with a 57° slope, the Limit Equilibrium Method predicts a smaller potential failure surface. The findings of this study can help engineers select the appropriate method for slope stability analysis and design of slope reinforcements based on the situation of a weak layer in slopes.

KEYWORDS: Landslide, Slope stability analysis, Rock slope, Finite difference method, Limit equilibrium method, Factor of safety

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I. INTRODUCTION

Slope instability can lead to a landslide, and landslides not only destroy the environment but can also kill people and damage buildings and properties [1]. There are different approaches to evaluate and analyze the stability of slopes and predict landslides. The most common ones are using numerical modeling and Limit Equilibrium Method (LEM) [2]. Numerical methods used to analyze slopes are usually Finite Difference Method (FDM) [3, 4], Finite Element Method (FEM) [5], Discontinuous Deformation Analysis (DDA) [6, 7], and Discrete Element Method (DEM) [8]. LEM is the most popular approach in slope stability analysis [9, 10].

There are several papers published on comparison of different methods for general slope stability analysis. For example, Fredlund et al. [11] studied the relationship between different limit equilibrium slope stability methods. FDM and LEM were compared by Cala and Flisiak [12], Cala and Flisiak [13], and Mansour and Kalantari [14]. Hammah et al. [15], Rawat and Gupta [16], and Koca T.K, Koca [17] compared FEM and LEM for slope stability analysis. Aksoy et al. [18] used FEM-based software called PLAXIS 3D to compare two different failure criteria (i.e., Hoek-Brown and Mohr-Coulomb) for deep open coal mine slope stability analysis. Deng et al. [19] and Kumar et al. [20] used the Hoek-Brown strength criterion for rock slope analysis. Finally, Chen and Lin [21] investigated the consistency of the Hoek-Brown and equivalent Mohr-Coulomb parameters in calculating slope safety factor.

Weak layers, consisting of sediment or rock that has strength significantly lower than that of adjacent units, for instance fractured zones or faults, have significant effects on the stability of slopes [22]. Thus, because of their weak mechanical properties, these layers cannot be overlooked in slope stability analysis because they are an important disaster-causing factor leading to slope instability [23, 24]. Although there are many studies on the influence of weak layers on slope stability, there is no research published on the comparison of FDM and LEM for stability analysis of rock slopes containing a weak layer. For example, effects of a weak layer on the stability of slopes were studied by Yasu [22]. Deng et al. [25] used a procedure based on LEM and Newmark's sliding block method to evaluate the displacement of slopes containing a weak layer. Ho [26] performed a numerical study of slope-stabilizing piles in slopes with a weak thin layer. Zhou et al. [27]

developed a translational failure mechanism to evaluate the stability of slopes with a weak layer. Li and Li [23] studied the effect of a weak layer on slope sliding mode and stability. Finally, Li et al. [24] carried out a three-dimensional analysis of complex open-pit mine rock slope stability affected by a fault and weak layer.

In this paper, a comparison between using FDM and LEM for stability analysis of rock slopes containing a weak layer is carried out. For this purpose, Fast Lagrangian Analysis of Continua (FLAC)/Slope program version 8.0 [28] and SLIDE version 9 [29] are used for slope stability analysis using FDM and LEM, respectively. In this study, different locations and slopes of a weak layer are considered to compare the results obtained from the two methods for slope stability analysis.

II. FINITE DIFFERENCE METHOD

The Finite Difference Method (FDM) has been broadly used for slope stability analysis by geotechnical, civil, and mining engineers and is one of the oldest numerical methods to calculate factors of safety [30]. FDM is used to solve sets of differential equations. In FDM, every derivative in the set of governing equations is replaced with an algebraic expression written in terms of the field variables, which are undefined within elements, at discrete points in space [28].

Compared with LEM, FDM has the following advantages in terms of slope stability analysis [30]:

- Unlike LEM, there is no need to define a range of trial surfaces and possible failure modes to find minimum factor of safety for a slope.
- No function is needed to be assumed for interslice force.
- Different failure surfaces can occur at the same time.
- Kinematically feasible mechanisms are consistent in the solution.

FLAC is a numerical modeling code using FDM for advanced geotechnical analysis of soil, rock, and structural support [28]. The general calculation sequence used in FLAC is shown in Fig. 1. This sequence first considers the equations of motion to derive new velocities and displacements from stresses and forces. Then, strain rates are derived from velocities, and new stresses are derived from strain rates. This program is used in this study for stability analysis of rock slopes containing a weak layer.

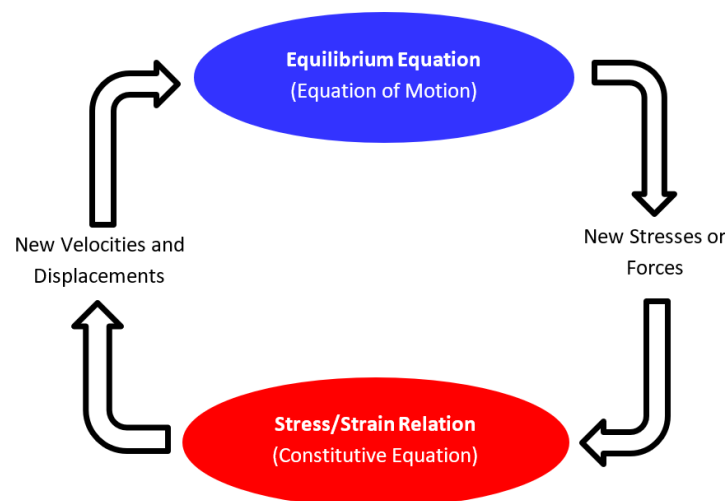


Fig. 1. Calculation Cycle in FLAC (Adapted from Itasca Consulting Group Inc. [28])

III. LIMIT EQUILIBRIUM METHOD

In Limit Equilibrium Method (LEM), which is the most widespread method in slope stability analysis [9, 10], failure surfaces are preassumed and are divided into several slices based on force and/or moment equilibrium. It is assumed that factor of safety is constant along the potential failure surface. There are several methods that assume different interslice and equilibrium conditions to calculate force and/or moment equilibrium for the slices [17]. For example, the Fellenius method neglects the interslice forces and calculates factor of safety using the moment equilibrium for circular failure surfaces. Another method, called Bishop, satisfies moment equilibrium with additionally considering the interslice normal forces. However, this method does not consider interslice shear forces and can only be used for circular failure surfaces. In Janbu's method, only horizontal

force equilibrium of wedge is accounted for to achieve a factor of safety for the slope failure. Spencer and Morgenstern-Price methods consider both force and moment equilibrium so that can be used to evaluate both for circular and noncircular potential failure surfaces. These methods that satisfy all equilibrium (force and moment) conditions are more accurate than the ones satisfying only one equilibrium condition [17]. Therefore, in this study, the Morgenstern-Price method is used.

The Morgenstern-Price method incorporates the use of interslice shear-normal forces [31]. Two types of factors of safety are used in this method. One is with respect to moment equilibrium (F_m), and the other is with respect to force equilibrium (F_f) [17]:

$$F_m = \frac{\sum(c' \beta R + (N - u\beta) R \tan \phi')}{\sum Wx - \sum Nf - \sum Dd} \quad (1)$$

and

$$F_f = \frac{\sum(c' \beta \cos \alpha + (N - u\beta) \tan \phi' \cos \alpha)}{\sum N \sin \alpha - \sum D \cos \omega} \quad (2)$$

where:

c' = effective cohesion;

ϕ' = friction angle;

u = pore water pressure;

w = slice weight;

D = concentrated point load;

α = slice base inclination with the horizontal;

b, f, d, x = geometric parameters;

N = slice base normal force.

As shown in Fig.2, the value of the normal force at the base of each slice (N) is dependent on the shear forces acting on the slices and can be calculated as follows [17]:

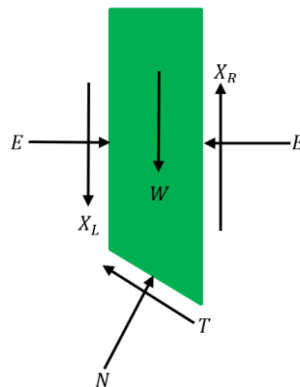


Fig. 2. Interslice Forces (Adapted from [16])

$$N = \frac{W + (X_R - X_L) - \left(\frac{c' \beta \sin \alpha + u\beta \sin \alpha \tan \phi'}{F} \right)}{\cos \alpha + \frac{\sin \alpha \tan \phi'}{F}} \quad (3)$$

where:

X_R = shear force on right side of slice;

X_L = shear force on left side of slice;

E = normal forces of interslice;

T = shear force at base of slice;

W = weight of slice.

Morgenstern and Price [31] proposed the relationship between the shear and normal forces on a slice as shown in Eq (4) below:

$$X = E \cdot f(x) \cdot \lambda \quad (4)$$

Where:

X = interslice shear force;

$f(x)$ = interslice function;

λ = percentage of function used.

To find the global factor of safety of slopes, both the conditions of moment and force are calculated until convergence is reached between the two factors of safety [Eq. (1) and (2)]. In this study, the SLIDE software [29] was used to model slopes utilizing the Morgenstern-Price method as this is one of the most accurate limit equilibrium methods.

IV. METHODOLOGY AND MODELING

To carry out a comparison between using FDM and LEM for stability analysis of rock slopes containing a weak layer, the slope shown in Fig. 3 was studied.

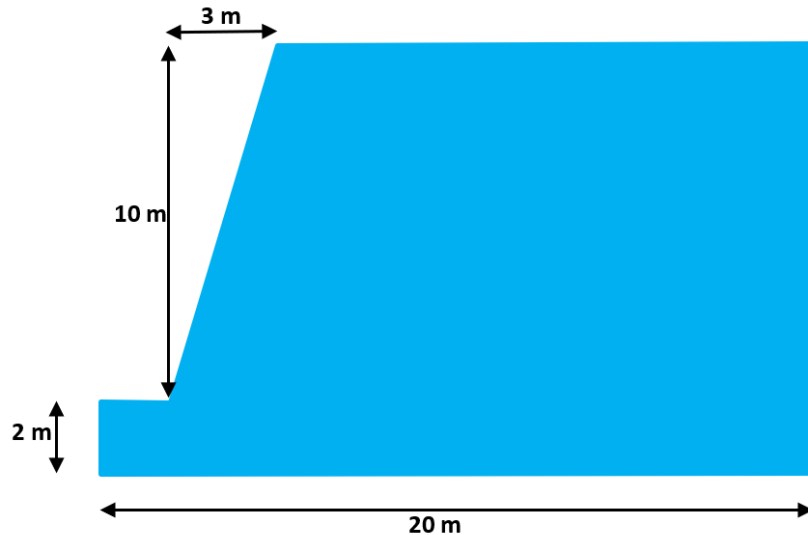


Fig. 3. Dimensions of the Analyzed Slope

To investigate the effect of a weak layer on results, different cases are considered as shown in Fig. 4. As can be seen, four cases are studied: (1) the slope contains a horizontal 0.5-m-thick weak layer located at 5 m above the toe; (2) there is a horizontal 0.5-m-thick weak layer 1 m above the toe; (3) the slope contains an inclined weak layer with a thickness of 0.5 m and an angle of 29° with respect to horizontal; and (4) there is a 0.5-m-thick weak layer with an inclination of 57°.

The properties of the rock and weak layer considered in this study are presented in Table 1.

Table 1. Properties of Rock and Weak Layer

Material	Unit Weight (kN/m ³)	Cohesion (kPa)	Friction Angle (degrees)
Rock (Siltstone)	27	44	39.2
Weak Layer	25	8	15.9

The slope stability analysis was performed using FLAC/Slope program version 8.0 [28] and SLIDE version 9 [29].

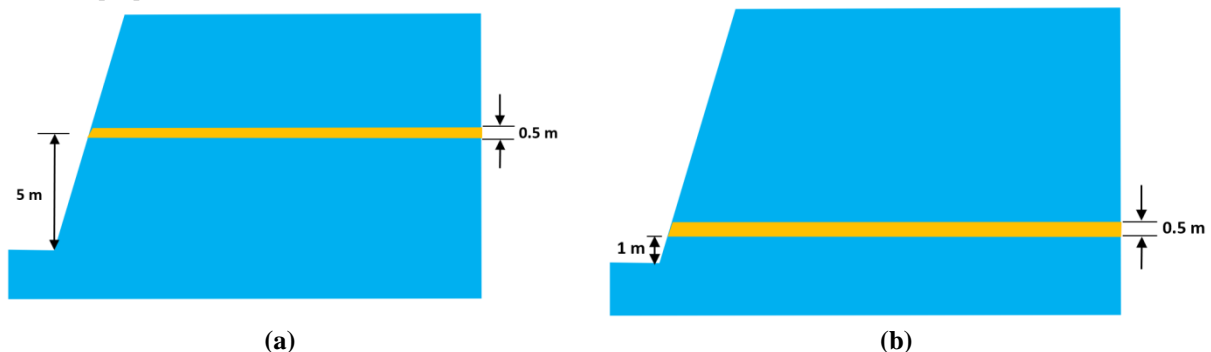




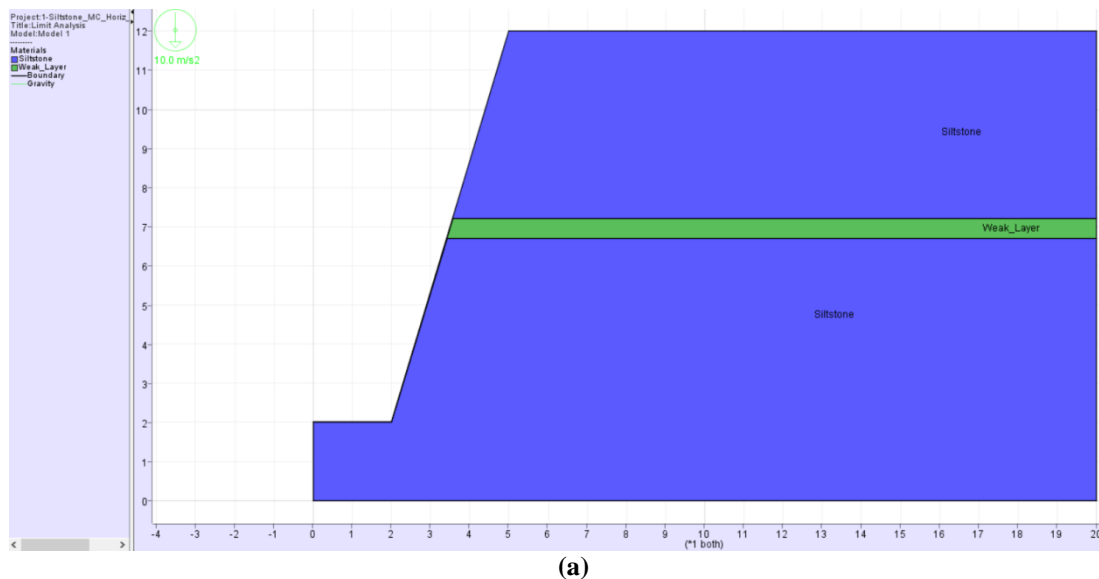
Fig. 4. Cases Considered for Investigation: (a) A Horizontal Weak Layer 5 m above Toe; (b) A Horizontal Weak Layer 1 m above Toe; (c) an Inclined Weak Layer with an Angle of 29° with Respect to Horizontal; (d) a Weak Layer with an Inclination of 57°

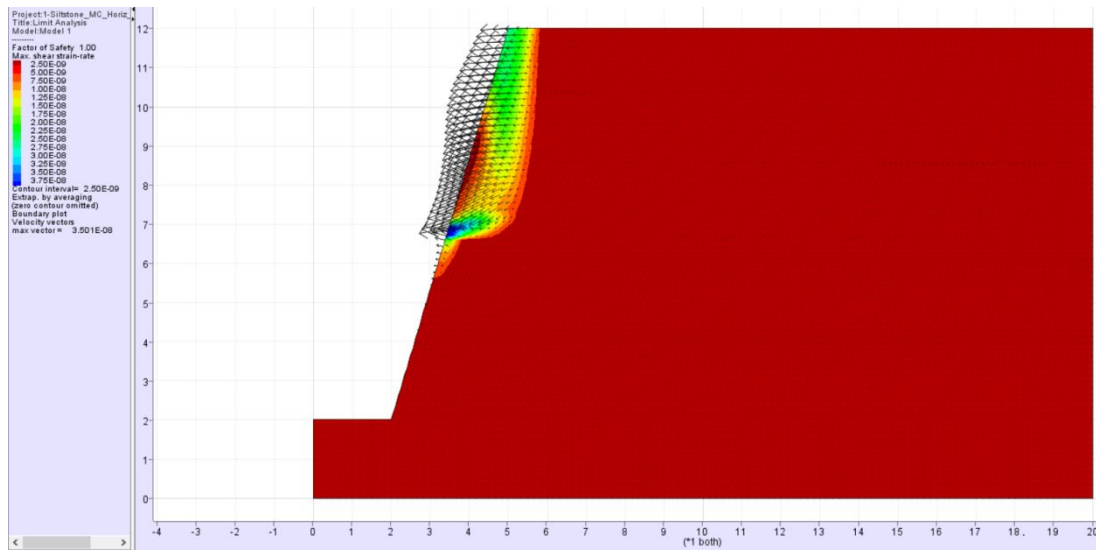
V. RESULTS AND DISCUSSION

In this section, results for each case are presented and discussed.

5.1 Case 1: Horizontal Weak Layer in the Middle of Slope

Fig. 5 shows the FLAC model configuration and modeling results obtained from finite difference modeling for case 1, in which there is a horizontal weak layer 5 m above the slope toe. The results obtained from LEM analysis for this case are shown in Fig. 6.





(b)

Fig. 5. FLAC Modeling for Case 1: (a) Model Configuration; (b) Modeling Results

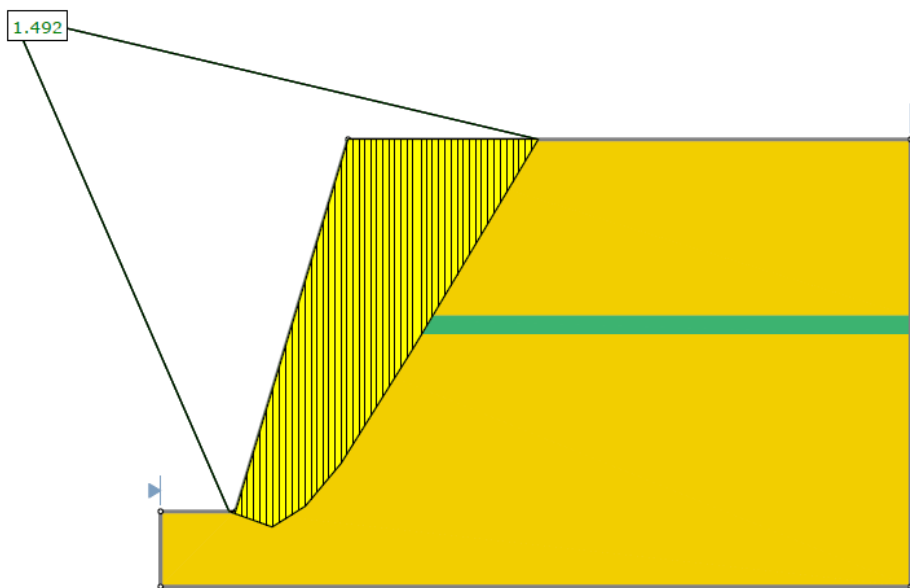


Fig. 6. LEM Analysis Results for Case 1

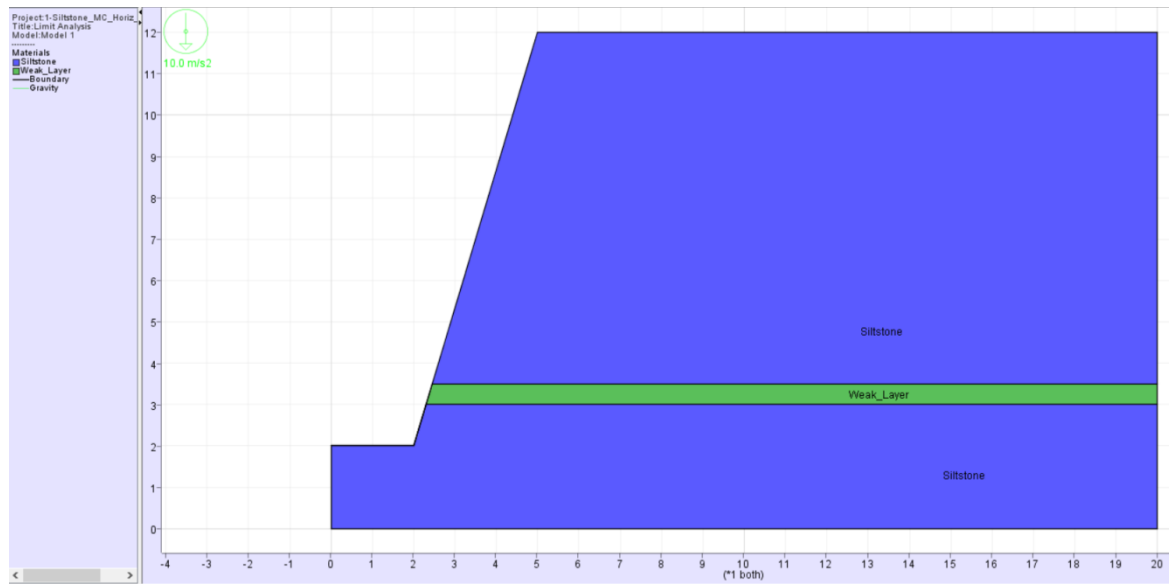
The minimum factors of safety obtained from the analyses using FDM and LEM are 1.0 and 1.492 for case 1, respectively. Also, the areas of the potential failure surface obtained from the modeling using FDM and LEM are 5.51 m^2 and 35.55 m^2 , respectively. It means, for this case, that FDM can capture a lower minimum factor of safety for a smaller area. This can help engineers design slope reinforcements for potential local slope instabilities when the slope contains a thin horizontal weak layer in the middle.

5.2 Case 2: Horizontal Weak Layer Close to Slope Base

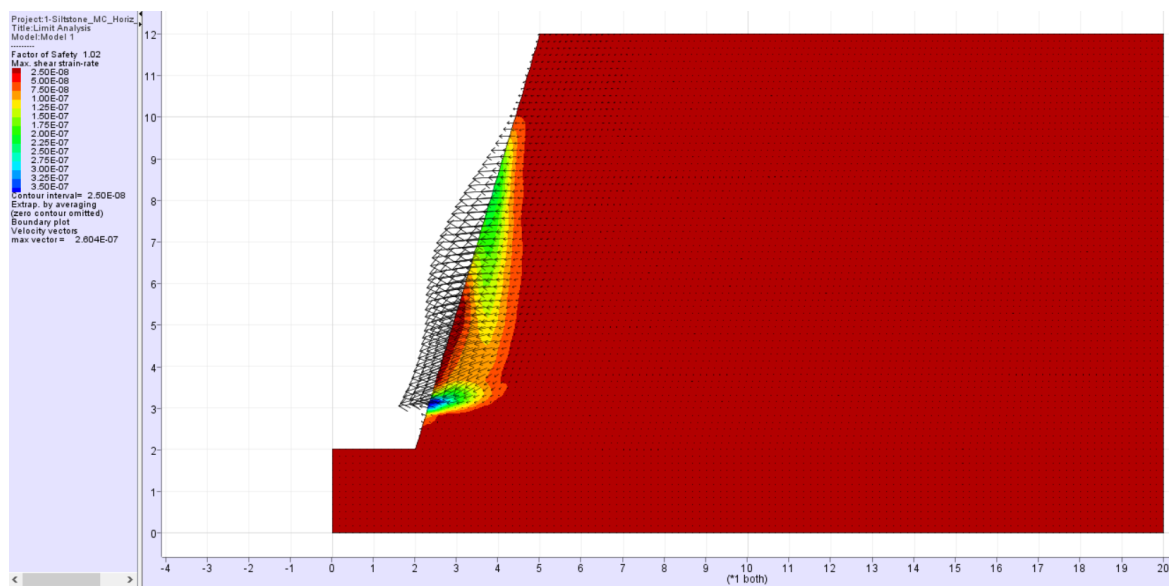
The model configuration and results obtained from the FLAC modeling for case 2 are presented in Fig. 7. Also, Fig.8 shows the LEM modeling results for the same case. In this case, the slope contains a horizontal 0.5-m-thick weak layer located at 1 m from the slope toe.

As can be seen, the minimum factors of safety in this case are 1.02 and 1.437 for potential failure surfaces with areas of 6.73 m^2 and 25.65 m^2 for FDM and LEM modeling, respectively. Like case 1, LEM gives

a higher minimum factor of safety for a larger potential failure surface. Thus, for the sake of caution, engineers should use FDM for slope stability analysis when the slope contains a thin horizontal weak layer.



(a)



(b)

Figure 7. FLAC Modeling for Case 2: (a) Model Configuration; (b) Modeling Results

5.3 Case 3: Inclined Weak Layer with a Moderate Slope

Fig.9 shows the FLAC modeling configuration and results when there is an inclined weak layer with an angle of 29° with respect to horizontal. Compared to the two previous cases with a horizontal weak layer, a minimum factor of safety of less than 1.0 (i.e., unstable slope) is expected for this case in which the slope contains an inclined weak layer. The FLAC modeling shows a minimum factor of safety of 0.48 for a potential failure surface with an area of 35.55 m². However, as shown in Fig. 10, the results obtained from the LEM modeling show a minimum factor of safety of 0.641 for a potential failure surface with an area of 67.43 m². Although both methods show that the slope is not stable, LEM gives a higher minimum factor of safety for a larger potential failure surface. Thus, like cases 1 and 2, it would be conservative to use LEM when there is an inclined weak layer with a moderate slope.

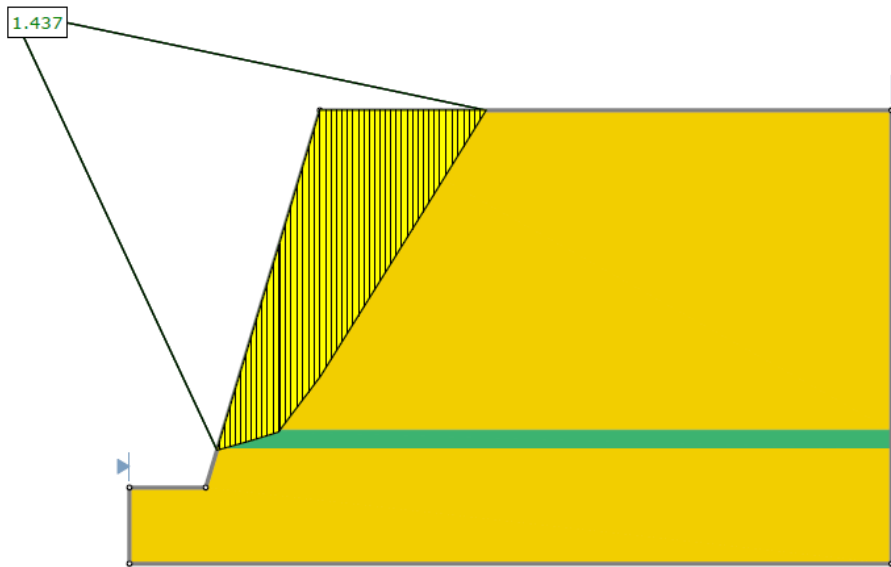
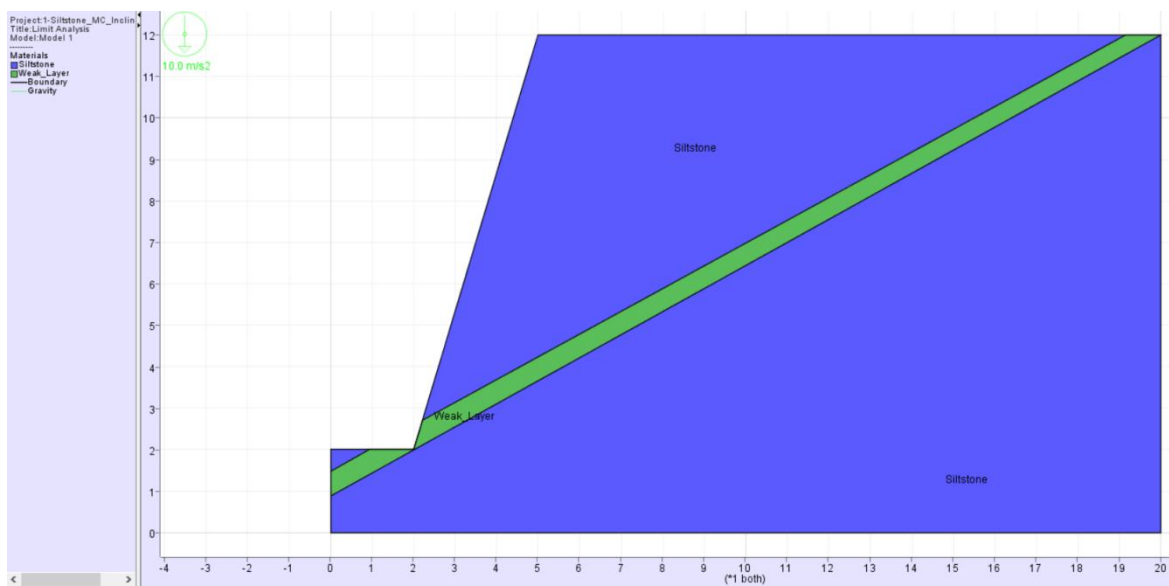
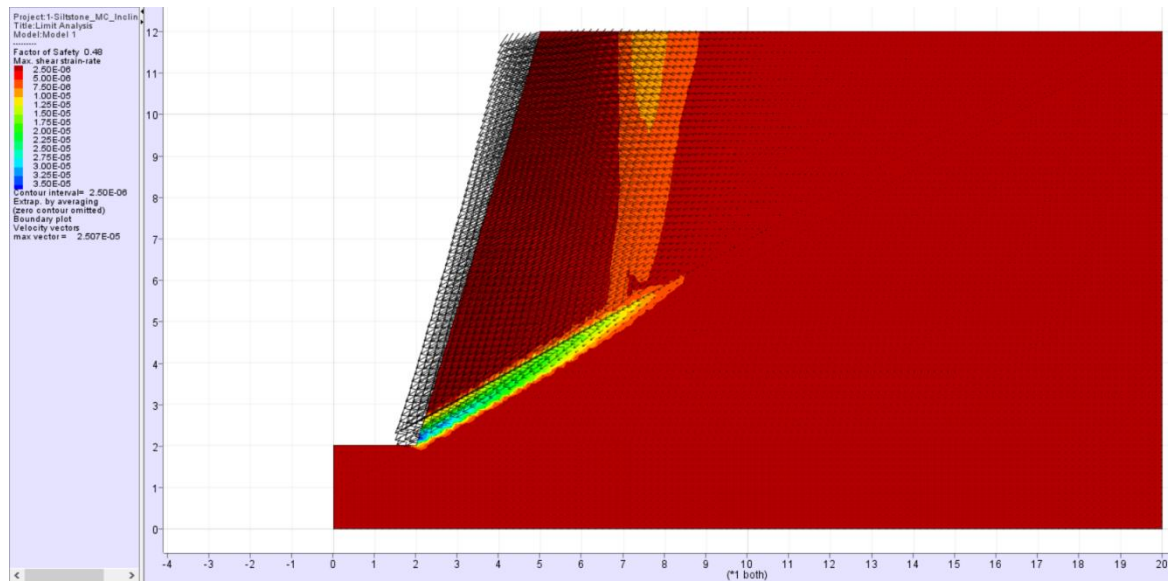


Figure 8. LEM Analysis Results for Case 2



(a)



(b)

Figure 9. FLAC Modeling for Case 3: (a) Model Configuration; (b) Modeling Results

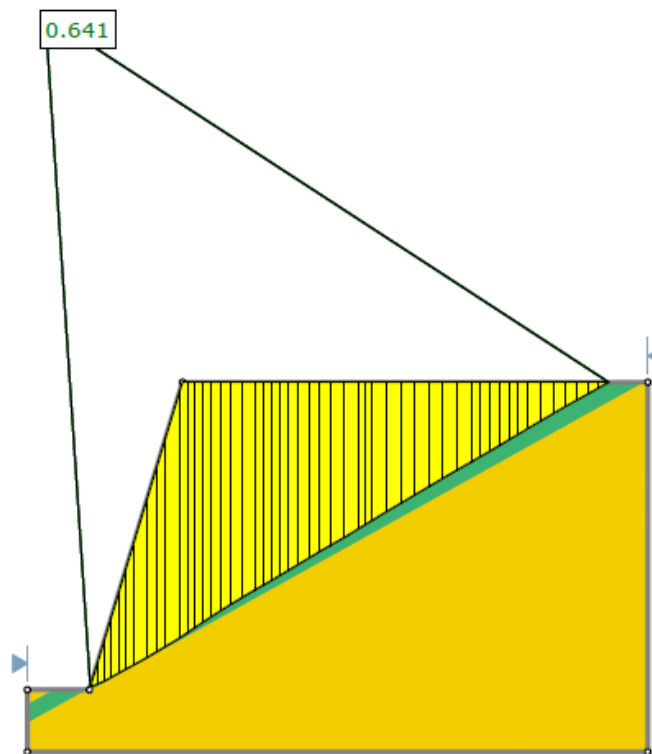
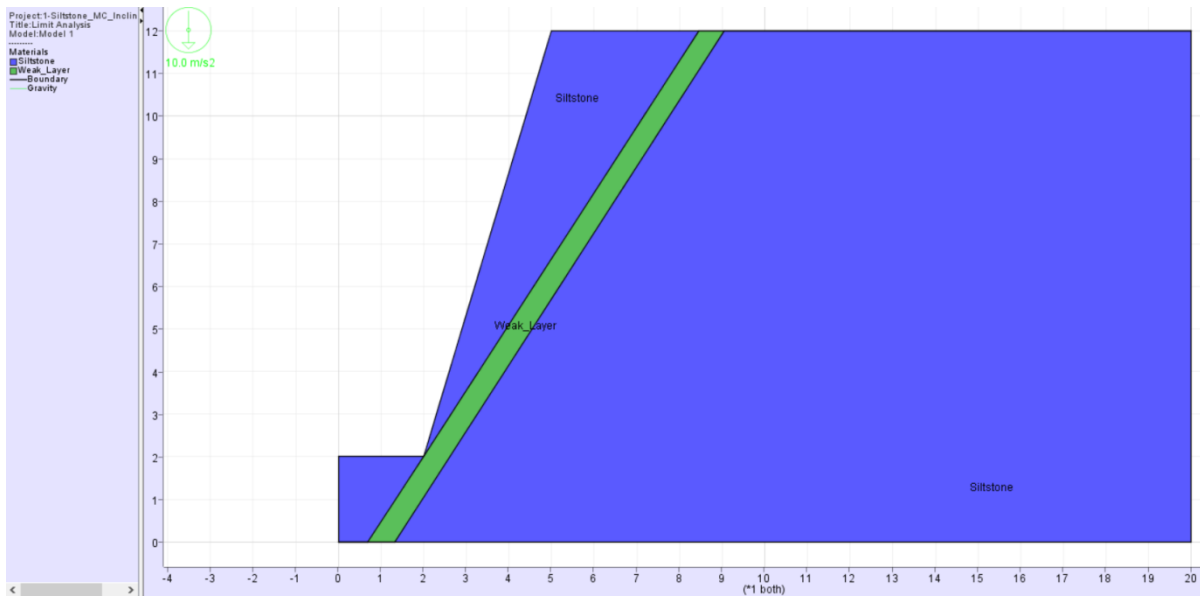


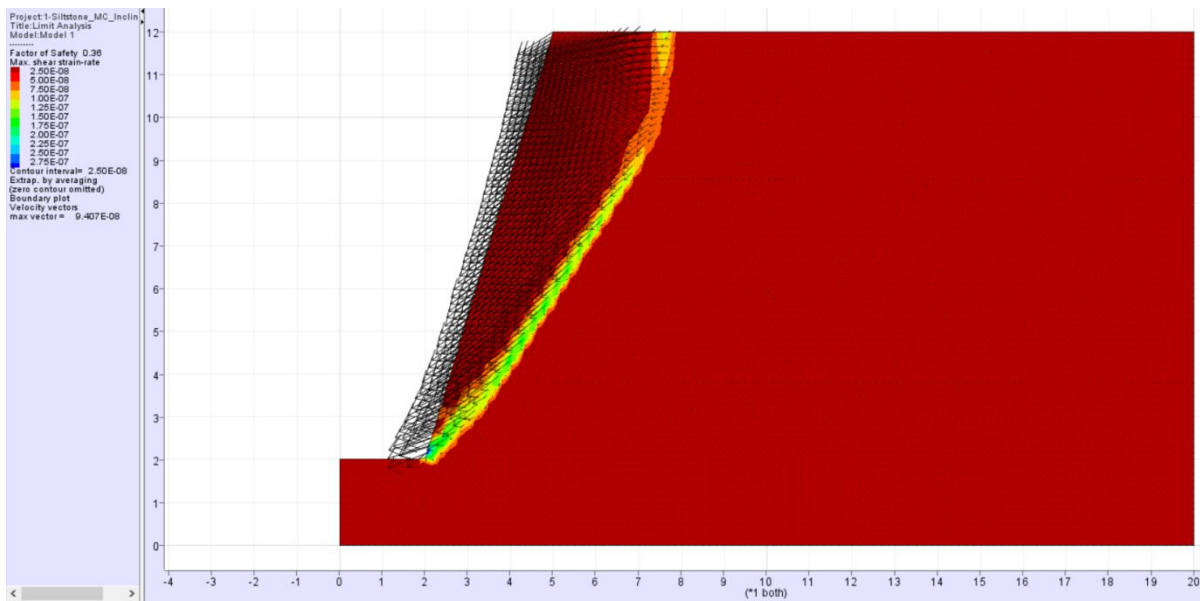
Figure 10. LEM Analysis Results for Case 3

5.4 Case 4: Inclined Weak Layer with a Steep Slope

In this case, the slope contains a 0.5-m-thick weak layer with an inclination of almost twice the one in case 3. Therefore, it is expected to get a minimum factor of safety that is lower than what was obtained for the previous case. The results obtained from the FLAC modeling and LEM modeling are presented in Fig. 11 and Fig. 12, respectively.



(a)



(b)

Figure 11. FLAC Modeling for Case 4: (a) Model Configuration; (b) Modeling Results

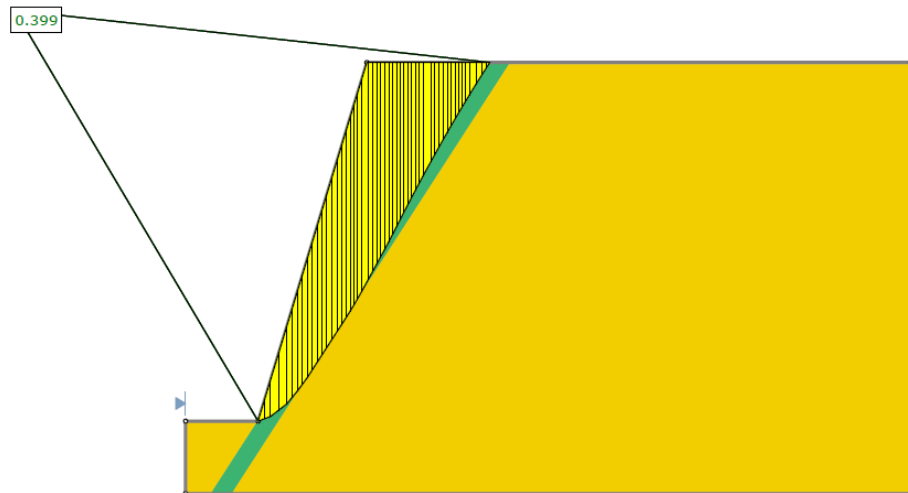


Figure 12. LEM Analysis Results for Case 4

The minimum factors of safety calculated for this case using FDM and LEM are 0.36 and 0.399, respectively, as shown in the figures. FDM and LEM predict potential failure surfaces with areas of 22.00 m² and 20.05 m², respectively. Both minimum factors of safety are lower than those obtained for case 3. For this case, using FDM is more cautious for calculating minimum factor of safety because the value calculated by using FDM is lower than that calculated by using LEM. Furthermore, it would be conservative to use LEM to predict potential failure surfaces when there is a thin weak layer with a steep slope.

VI. CONCLUSIONS

A comparison between using finite difference and limit equilibrium methods for landslide analysis of slopes containing a weak layer was performed for four different cases including a 0.5-m-thick weak layer. In two cases, the weak layer was horizontal but was located at different distances from the slope toe. In the other two cases, different inclination angles were considered for the weak layer. A comparison between the results obtained from modeling using FDM and LEM shows that the predicted potential failure surface is larger for the two cases in which the weak layer is horizontal and the case in which the weak layer has a moderate slope when LEM is used. However, for the case in which the weak layer has a steep slope, the predicted potential failure surface is larger when FDM is used for the slope stability analysis. However, FDM calculates a lower minimum factor of safety for all cases. Therefore, for the sake of caution, it is recommended that geotechnical, civil, and mining engineers use FDM for slope stability analysis and design of slope reinforcements based on the potential failure surface predicted by LEM modeling when the slope contains a horizontal thin weak layer or a weak layer with a moderate slope. Also, it would be more cautious to use FDM to predict the potential failure surface when there is a thin weak layer with a steep slope. Finally, it is recommended that FDM be used to calculate the minimum factor of safety instead of LEM because FDM calculates lower values for all cases.

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