

## A Dynamic Model of Induction Electromotor with Wound Rotor

Biya Motto Frederic<sup>1k</sup>, Tchuidjan Roger<sup>2</sup>, Ndzana Benoît<sup>2</sup>, Atangana Jacques<sup>3</sup>.

1-Faculty of Science, University of Yaounde I, PO.Box 812 Yaoundé, Cameroon;

2-National Advanced School of Engineering of Yaounde;

3-Higher Teacher Training College Yaoundé, University of Yaoundé I, PO Box 47 Yaoundé, Cameroon.

Corresponding Author: Biya Motto Frederic

**ABSTRACT:** In this paper, we present the dynamic equations of electromagnetic processes in induction electromotor with supply of stator winding from three-phase source of sinusoidal e.m.f and direct current e.m.f source. For induction electromotor with a rectifier in rotor circuit, we present voltages equations in rectified current loop.

**KEYWORDS:** Dynamic model, Induction electromotor, wound rotor

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### I. INTRODUCTION

The utility of induction motors are more than 50% of the total electric energy generated worldwide. A small improvement in efficiency would significantly save the total electric energy. Hence, it is important to optimize the efficiency of motor drive systems if significant energy savings are to be obtained. The induction motor (IM), especially the squirrel-cage type, is widely used in electrical drives and is responsible for most of the energy consumed by electric motors [3].

If equal resistances are included in each secondary phase of three phase induction motor, the speed decreases as the secondary resistances increases. A study is made in which the impedances to be connected into the secondary circuits of the motor are not resistors but passive impedances. [2]

A novel concept for obtaining various Torque-speed characteristics from a wound rotor induction motor by operating such a motor close to its resonance have been introduced. Being essentially a one speed device, the induction motor produces maximum torque when the rotor resistance is approximately equal to the slip times the rotor reactance.

$X_r$  is normally much greater than  $R_r$ , and the machine is hardly ever operated at the maximum torque conditions continuously. In order to get the resonant condition, a capacitive reactance has been introduced in the rotor circuit for cancelling the inductive reactance of the rotor circuit. Speed control of an induction motor is possible by having a resonant rotor circuit, which is adjusted according to the slip frequency. The main drawback of this method is that a wound rotor machine is more costly than a squirrel-cage machine and reactive components capable of conducting large currents and withstanding high voltages are relatively expensive. Also, some form of a control system will be needed to carry out a reactive component switching strategy [3]

In order to overcome the problem faced in [3], a novel system has been presented for the control of the phase difference between voltage and current in inductive circuits, using a switched capacitor. The system provides good results, even if the parameters of the circuit are unknown. The power factor control is one of the main directions in power electronics research. Lately, the work in this field has been facilitated by the development of the buck, boost or buck-boost converters using soft switching techniques (multiresonant or quasi resonant versions). Novel control strategies based on concepts such as: delta modulation or

fuzzy logic has been developed [4]

The switched capacitor concept [4] has been adopted with the use of non-resistive secondary control of an induction motor to improve the efficiency, power factor and torque. It utilizes the concept of switched capacitor which makes use of four thyristors which forms a H-bridge circuit and a single capacitor for each rotor phase. The complementary switch pairs are switched using a PWM strategy [5]. The main drawback of this method is the usage of more number of switches and three capacitors in the rotor circuit which are costly.

Many techniques are available for control of three phase machines to extract the best performance. It proposes a novel method for improving performance of a Single Phase Induction Motor using indirect current control of VSI with dynamic capacitor. Normally

a fixed capacitor is included in the auxiliary winding in order to make a phase angle difference of  $90^\circ$  between main and auxiliary windings. Instead of using a fixed capacitor, in the proposed approach, a dynamic capacitor is included with the auxiliary winding to make the two winding currents always in  $90^\circ$  phase quadrature irrespective of the load conditions [6].

The efficiency optimization of wound rotor induction motor using soft computing techniques has been presented in [7].

This paper presents new dynamic equations of electromagnetic processes in induction electromotor with supply of stator winding from three-phase source of sinusoidal e.m.f. and direct current e.m.f source.

## II. DYNAMICS EQUATIONS OF ELECTROMAGNETIC PROCESSES WITH STATOR WINDINGS SUPPLY FROM THREE-PHASE EMF SOURCE

We present the dynamics equations of electromagnetic processes in induction electromotor with control of rotor circuit parameters. We assume that the stator windings supply of electromotor is done from sinusoidal e.m.f. source, with amplitude and frequency having nominal values:

$$U_S = \begin{bmatrix} u_A \\ u_B \\ u_C \end{bmatrix} = U_B \begin{bmatrix} \cos(\omega_1 t) \\ \cos(\omega_1 t - \rho) \\ \cos(\omega_1 t + \rho) \end{bmatrix} \text{ with } \rho = \frac{2\pi}{3};$$

$U_B$  – phase stator voltage amplitude;

$\omega_1 = \omega_0$  – angular frequency

The stator voltage complex amplitude:

$$U_{m1} = U_B = \sqrt{2}U_N.$$

The dynamics of electromagnetic processes in inductor electromotor are described by differential equations:

$$\begin{cases} \dot{u}_1 = R_1 \cdot i_1 + j\omega_1 \cdot L_{01} \cdot i_1 + L_{01} \cdot p \cdot i_1 - j\omega_1 \cdot L_0 \cdot i_2 - L_0 p i_2 \\ \dot{u}_2 = R_2 \cdot i_2 + j\omega_2 \cdot L_{02} \cdot i_2 + L_{02} \cdot p \cdot i_2 - j\omega_2 \cdot L_0 \cdot i_1 - L_0 p i_1 \end{cases} \quad (1)$$

We introduce a new complex variable:

$i_{02} = i_{02u} + j \cdot i_{02v}$ , called the rotor magnetization current vector, and we change the variables, considering  $i_1 = (i_{02} + i_2) \cdot L_0 / L_{01}$ .

We have new voltages equations in stator and rotor windings:

$$\dot{u}_1 = R_1(i_{02} + i_2) \cdot L_0 / L_{01} + j \cdot \omega_1 \cdot L_0 \cdot i_{02} + L_0 \cdot p \cdot i_{02} \quad (2)$$

$$\dot{u}_2 = R_2 \cdot i_2 + j\omega_2 \cdot L'_K \cdot i_2 + L'_K \cdot p \cdot i_2 = (j\omega_2 \cdot L_0 \cdot i_{02} + L_0 p i_{02}) \cdot L_0 / L_{01} \quad (3)$$

With  $L'_K = (L_{01} \cdot L_{02} - L_0^2) / L_{01}$

We simplify the equation (2) assuming that the stator winding receives supply from three-phase sinusoidal e.m.f source. The active resistance  $R_1$  is considerably less than inductive resistance  $\omega_1 \cdot L_0$ .

$$U_B = j \cdot \omega_1 \cdot L_0 \cdot i_{02} + L_0 \cdot p i_{02}.$$

We consider  $U_B$  – constant expression, and  $i_{02} = \dot{I}_{02}$  a constant expression,  $p i_{02} \approx 0$ .

$$\dot{I}_{02} = I_{02u} + j \cdot I_{02v} = j \cdot I_{02v} = -j \cdot U_B / (\omega_1 L_0) \quad (4)$$

With  $I_{02u} = 0$ ;  $I_{02v} = -U_B / (\omega_1 L_0)$ .

The equation (3) will look as follows:

$$R_2 \cdot i_2 + j\omega_2 \cdot L'_K \cdot i_2 + L'_K \cdot p i_2 = \omega_2^* \cdot U_B \cdot L_0 / L_{01} - \dot{u}_2 \quad (5)$$

With  $\omega_2^* = \omega_2 / \omega_1$  – the slip. We have  $L_0 / L_{01} \approx 1$  and  $L'_K = L_K = L_{01} - L_0^2 / L_{02} = L_1 + L_2$ .

Equation (5) can be replaced by expression:

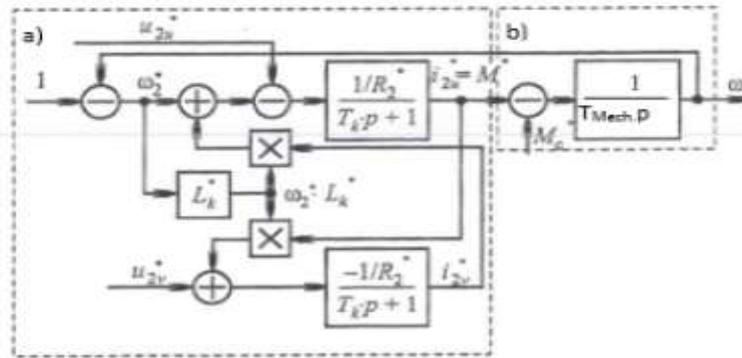
$$R_2 \cdot i_2 + j\omega_2 \cdot L_K \cdot i_2 + L_K \cdot p i_2 = \omega_2^* \cdot U_B - \dot{u}_2 \quad (6)$$

We represent equation (6) in the aspect of equations with real and imaginary parts:

$$\begin{cases} -\omega_2^* \cdot L'_K \cdot i_{2v}^* + R_2^* \cdot (T_K \cdot P + 1) \cdot i_{2u}^* = \omega_2^* - u_{2u}^* \\ \omega_2^* \cdot L'_K \cdot i_{2u}^* + R_2^* \cdot (T_K \cdot P + 1) \cdot i_{2v}^* = -u_{2v}^* \end{cases} \quad (7)$$

With  $T_K = L_K / R_2$ ;  $\dot{u}_2 = u_{2u} + j u_{2v}$ ;  $i_2 = i_{2u} + j i_{2v}$ .

These equations correspond to the structural circuit represented on figure 1a.



**Figure 1:** Structural circuit of electric drive power channel with stator supply from three-phase e.m.f source and regulation of rotor voltage

The asynchronous electromagnetic torque in pair of poles is defined as follows:

$$M = (m/2) \cdot L_0 \cdot (I_{02v} \cdot i_{2u} - I_{02u} \cdot i_{2v}) = -(m/2) \cdot U_B \cdot i_{2u} / \omega_1$$

In per units  $M^* = -i_{2u}^*$ .

If we add to (7) the movement equation:

$$T_{Mech} \cdot P \omega^* = M^* - M_r^* \quad (8)$$

Then we have the system of equations, that characterizes the processes dynamics in the electric drive with control on rotor circuit. The movement equation corresponds to figure 1b.

The equations (7) and (8) correspond to the structural circuit represented on figure 1.

### III. DYNAMICS EQUATIONS OF ELECTROMAGNETIC PROCESSES WITH STATOR WINDING SUPPLY FROM CONSTANT E.M.F. SOURCE

We assume that the electromotor stator windings supply is done from constant e.m.f. source.

Voltages vector on stator phase windings is represented:

$$U_{S0} = \begin{bmatrix} u_A \\ u_B \\ u_C \end{bmatrix} = U_{S0} \cdot \begin{bmatrix} 0 \\ -1/2 \\ 1/2 \end{bmatrix}$$

The angular frequency of phase source e.m.f voltages  $\omega_1 = 0$ . In u and v coordinate system, the stator phase winding voltages.

$$U_{10} = \frac{2}{3} \cdot \nabla(\omega_1, t)^T \cdot D_S \cdot U_{S0} = \begin{bmatrix} 0 \\ -1/\sqrt{3} \end{bmatrix}$$

With  $D_S^T = \begin{bmatrix} 1 & \cos(\rho_S) & \cos(2\rho_S) \\ 0 & \sin(\rho_S) & \sin(2\rho_S) \end{bmatrix} = \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix}$

- Phase matrix of three-phase electromotor;  $\rho_S = 2\pi/3$ ,  $\omega_1 = 0$ .

In complex form  $\dot{U}_{10} = -jU_{S0}/\sqrt{3}$

The electromotor stator winding current in coordinate system u and v is:

$$i_1 = \dot{I}_{10} + i_{1n}$$

With  $\dot{I}_{10}$  – constant current component created by voltage  $U_{S0}$ ;

$i_{1n}$  - dynamic current component of stator winding that is transformed in stator current loop from rotor circuit.

Stator winding supply from constant e.m.f. source,  $\omega_1 = 0$ ;  $\omega_2 = \omega$  equation (1) will look like:

$$\dot{U}_{10} = R_1 \cdot \dot{I}_{10} + R_1 \cdot i_{1n} + L_{01} P \cdot \dot{I}_{10} - L_0 P i_2; \quad (9)$$

$$\dot{u}_2 + R_2 \cdot i_2 + j \cdot \omega_2 \cdot L_k \cdot p i_2 + L_{02} P \cdot i_2 = j \omega_2 \cdot L_0 \cdot \dot{I}_{10} + j \omega_2 \cdot L_0 \cdot i_{1n} + L_0 P i_{1n}$$

Or  $\dot{U}_{10} = R_1 \cdot \dot{I}_{10}$ ;  $0 = R_1 \cdot i_{1n} + L_{01} P i_{1n} - L_0 P i_2$ . (10)

Thus  $\dot{I}_{10} = \frac{\dot{U}_{10}}{R_1} = -j \cdot U_{S0} / (\sqrt{3} \cdot R_1)$ .

The voltage  $U_{S0}$  is chosen so that  $\dot{I}_{10} = \dot{I}_{02}$  with  $\dot{I}_{02}$  defined by equation (4). Therefore

$$U_{S0} = U_B \cdot \sqrt{3} \cdot R_1 / (\omega_1 \cdot L_0)$$

If in equation(10) we neglect  $R_1$ , then  $P i_{1n} = L_0 / L_{01} P \cdot i_2$

And  $\dot{u}_2 + R_2 \cdot i_2 + j \omega_2 \cdot L_k \cdot p i_2 = j \omega_2 \cdot L_0 \cdot \dot{I}_{10}$  (11)

If we consider  $\dot{I}_{10} = \dot{I}_{02} = -j \cdot U_B / (\omega_1 \cdot L_0)$ , then

$$R_2 \cdot i_2 + j \cdot \omega_2 \cdot L_k \cdot i_2 + L_k \cdot p i_2 = \omega_2^* \cdot U_B - \dot{u}_2 \quad (12)$$

By comparing equations (6) and (12), we realize that they have the same aspect that they have the same aspect.

Finally, the electromagnetic dynamic processes in induction electromotor with wound rotor and supply of stator windings from three-phase e.m.f. source are described by similar equations compared to the constant e.m.f. source.

#### IV. INDUCTION ELECTROMOTOR DYNAMIC MODEL ON ROTOR RECTIFIED CURRENT WITH SUPPLY FROM STATOR SIDE

Very often on phase rotor windings endings of induction electromotor we attach a three-phase rectifier bridge, while to the constant current bridge, we attach the load. The energy expression consumed by the load is proportional to the rotor slip and it is called the "slip energy".

In the vector form, equation (6) takes the following aspect:

$$\omega_2 \cdot L_k \cdot E \cdot I_2 + R_2 \cdot I_2 + L_k \cdot P \cdot I_2 = \omega_2^* \cdot U_1 - U_2 \quad (13)$$

With  $U_2 = \begin{bmatrix} u_{2u} \\ u_{2v} \end{bmatrix}$ ;  $U_1 = \begin{bmatrix} U_B \\ 0 \end{bmatrix}$ ;  $I_2 = \begin{bmatrix} i_{2u} \\ i_{2v} \end{bmatrix}$ ;  $E = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

In real coordinate system,

$$U_I = D_S^T \cdot \nabla(\omega_2 t) \cdot U_1; U_{II} = D_R^T \cdot \nabla(\omega_2 \cdot t) \cdot U_2, \\ I_{II} = D_R^T \cdot \nabla(\omega_2 \cdot t) \cdot I_2$$

With  $D_S = D_R$  – phase matrices of stator and rotor windings.

If we make the variables change, then  $R_2 \cdot I_{II} + L_k \cdot P I_{II} = \omega_2^* \cdot U_I - U_{II} \quad (14)$

That equation (14) will determine the induction electromotor dynamic model with stator windings supply from three-phase network. It corresponds to equivalent circuit of rotor network shown on figure 2.

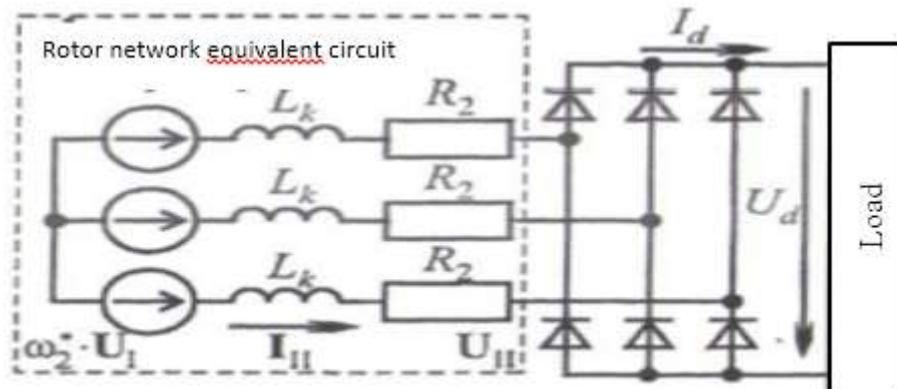


Figure 2: Rotor network equivalent circuit with a load on rectified rotor current

The equivalent circuit of rotor with load in rectified rotor current network will have a rectified bridge and it is nonlinear. For the control system design, it is necessary to linearize it. We assume that in the rectified bridge, at each moment current is supplied on two diodes.

The equivalent circuit of rectified current network will have the aspect shown on figure 3.

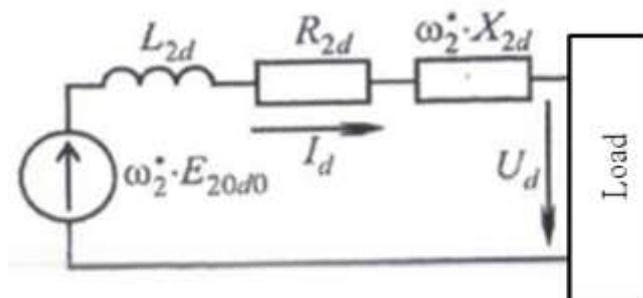


Figure 3: Equivalent circuit network

The three-phase e.m.f. system  $\omega_2^* \cdot E_{2od0} = \omega_2^* K_C \cdot U_N$ ,

With  $E_{2od0}$  – rotor rectified e.m.f. for slip  $\omega_2^* = 1$ ,  $K_C = 3 \cdot \sqrt{6}/\pi$  – coefficient of circuit for three-phase rectifier bridge;

$U_N$  – Nominal active value of stator circuit phase voltage.

The equivalent inductance carried out in rectified current from rotor side  $L_{2d} = 2L_K$  and  $L_K = L_1 + L_2 R_{2d} = 2 \cdot R_2$ , with  $R_2$  – Rotor winding active resistance.

We also consider commutation resistance:  $\omega_2^* \cdot X_{2d} = 3/\pi \cdot \omega_2 \cdot L_K$  with  $X_{2d} = 3/\pi \cdot \omega_1 \cdot L_K$ .

The differential equation that describes the electromagnetic dynamic processes in constant current circuits is written from the Kirchhoff second law:

$$L_{2d} \cdot PI_d + R_{2d} \cdot I_d + \omega_2^* \cdot X_{2d} \cdot I_d + U_d = \omega_2^* E_{2od0} \quad (15)$$

In per units:

$$L_{2d}^* \cdot PI_d^* + R_{2d}^* \cdot I_d^* + \omega_2^* \cdot X_{2d}^* \cdot I_d^* + U_d^* = \omega_2^* \cdot K_C \quad (16)$$

## V. TRANSFORMATION OF DIRECT CURRENT CIRCUIT PARAMETERS TO ALTERNATIVE CURRENT CIRCUIT PARAMETERS

The rotor rectified current  $I_d$  is linked with active first harmonic value of rotor current  $I_2$ :

$$I_2 = K_C \cdot I_d / 3 = 0,78 \cdot I_d \quad (17)$$

If we introduce that expression in equation (16) we have the equation in alternative current circuit parameters.

$$\omega_2^* = L_{2d2}^* \cdot PI_2^* + R_{2d2}^* \cdot I_2^* + \omega_2^* \cdot X_{2d2}^* \cdot I_2^* + U_{d2}^* \quad (18)$$

With  $L_{2d2}^* = K_{2d} \cdot L_{2d}^* = K_{2d2} \cdot L_2^*$  - inductance of direct current circuit, transformed to alternative current circuit;  $R_{2d2}^* = K_{2d} \cdot R_{2d}^*$  - active resistance of direct current circuit, transformed to alternative current circuit;

$$U_{d2}^* = U_d^* / K_C$$

The coefficient of parameters calculations from direct current circuit to alternative current circuit:

$$K_{2d} = 3/K_C^2 = \pi^2/18 \approx 0,548 \quad (19)$$

The coefficient of parameters calculations from alternative current circuit to direct current circuit and reciprocally in alternative current circuit  $K_{2d2} = 2 \cdot K_{2d} = 6/K_C^2 = \pi^2/9 \approx 1,096$ .

Finally the expression (16) of direct current circuit is written in direct current circuit parameters, while equation (18) of direct current circuit is written in alternative current circuit parameters.

For calculation of  $I_d$  current to fact current that circulate along rectified current circuit, we should multiply  $I_d$  with the coefficient of transformation  $K_{S/R} = W_S/W_R$ , with  $W_S, W_R$  are the number of turns of stator and rotor windings respectively.

## VI. LINK BETWEEN ELECTROMAGNETIC TORQUE AND ROTOR RECTIFIED CURRENT

The link between electromagnetic torque and rotor rectified current is found from the energetic point of view: the power slip is  $P_S = M \cdot \omega_2^* \cdot \omega_1$

With  $M$  – electromagnetic torque.

Considering that the commutation resistance  $X_{2d}$  does not influence the transformation of electrical energy into thermal energy from the equivalent circuit, we can write:

$$P_S = \omega_2^* \cdot E_{2od0} \cdot I_d - \omega_2^* \cdot X_{2d} \cdot I_d^2$$

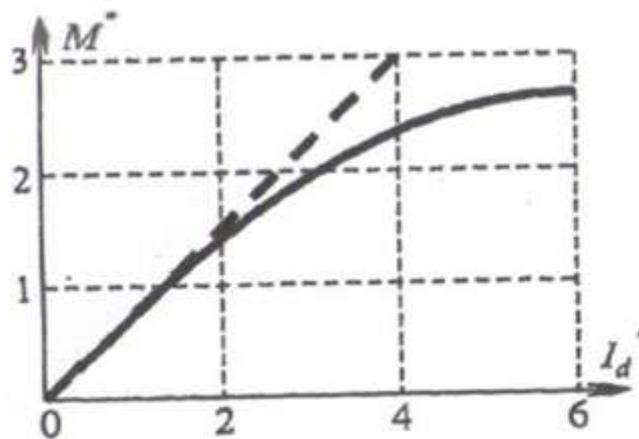
$$\text{Thus, } M = (E_{2od0} \cdot I_d - X_{2d} \cdot I_d^2) / \omega_1 \quad (20)$$

Using basic expressions, in per units:

$$M^* = (K_C \cdot I_d^* - X_{2d}^* \cdot I_d^{*2}) / 3 \approx K_C \cdot I_d^* / 3 \quad (21)$$

If we replace (17) in (21), then we have the equation linking electromagnetic torque with rotor first harmonic current:  $M^* = I_2^* - X_{2d2}^* \cdot I_d^{*2} \approx I_2^*$  (22)

The plot of dependence for the torque  $M^*$  on rectified rotor current  $I_d^*$  with middle statistic values of induction electromotor parameters is shown in figure 4.



**Figure 4: Plot of dependence of electro machine torque on rectified rotor current**

## VII. CONCLUSIONS

For the electromotor stator windings supply from sinusoidal e.m.f. source, with nominal values of amplitude and frequency, the dynamic processes can be characterized by differential equations system of second order. The control of electromagnetic processes in that case can be done by rotor circuit parameters.

Very often the rotor circuit parameters control is done by direct parameters change in rotor rectified current circuit. The analysis and design of dynamic processes are done by using the equivalent circuit of induction electromotor transformed to rotor rectified current circuit whose expression is practically proportional to the electromagnetic torque.

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