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Scheffe Optimization of Compressive Strength of Palm Nut Fibre Concrete.

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ABSTRACT: *In this paper, a regression model is developed to optimize the compressive strength of palm nut fiber reinforced concrete (PFRC) using Scheffe's regression theory. Using Scheffe's Simplex method, the compressive strength of PFRC was determined for the different componential ratios. Control experiments were also carried out to test the adequacy of the model. After the test has been conducted, the adequacy of the model was tested using student's t-test and analysis of variance (ANOVA) at 95% confidence level. The result of the adequacy of the model test shows a good correlation between the model and control results. The optimum compressive strength was found to be 31.53Nmm^2 corresponding to mix ratio of 0.525:1.0:1.45:1.75:0.6 and minimum strength was found to be 17.25Nmm^2 corresponding to mix ratio of 0.6:1.0:1.8:2.5:1.2. For water, cement, fine and coarse aggregate and palm nut fibre respectively. With the aid of computer software SPSS and MATLAB the experimental analysis was modelled, the mix ratios corresponding to a desired strength value can be predicted with reasonable accuracy and without waste of time.*

KEYWORDS*: Scheffe;Model; Palm nut fibre; Concrete Compressive Strength; MATLAB*

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I. INTRODUCTION

Concrete plays a vital role as a construction material in the world. But the use of concrete as a structural material is limited to certain extent by deficiencies like brittleness, poor tensile strength and poor resistance to impact strength, fatigue, low ductility and low durability [1].Attempts have been made in the past to optimize the concrete mixture design using either the empirical/experimental methods or analytical /statistical methods. Empirical/experimental methods involve an extensive series of tests, sometimes conducted on a trial-and-error basis, and the optimization results are often applicable only to a narrow range of local materials [2].In order to reduce the number of trial mixtures required to obtain an optimal mixture, efforts have been made towards developing analytical/statistical methods rationalizing the initial mixture proportioning into a more logical and systematic process. Analytical/ statistical methods help in searching for an optimum concrete mixture based on detailed knowledge of specific weight functions of the mixture components and on certain basic formulas, which result from previous experience without conducting expensive and time-consuming experimental works [3].

Scheffe's method, named after the American statistician Henry Scheffe, is a method for adjusting significance levels in a linear regression analysis to account for multiple comparisons [4]. It is particularly useful in analysis of variance (a special case of regression analysis), and in constructing simultaneous confidence bands for regressions involving basis functions. It is a single-step multiple comparison procedure which applies to the set of estimates of all possible contrasts among the factor level means. It is a theory where a polynomial expression of any degrees, is used to characterize a simplex lattice mixture components. The theory lends path to a unifying equation model capable of taking varying componential ratios to fix approximately equal mixture properties. The optimization is the selectability, from some criterial (mainly economic) view point, the optimal ratio from the component ratios list that can be automatedly generated [5].

1.1 Fibre Concrete:

Concrete plays a vital role as a construction material in the world. But the use of concrete as a structural material is limited to certain extent by deficiencies like brittleness, poor tensile strength and poor resistance to impact strength, fatigue, low ductility and low durability [6-7]. In the present scenario, waste materials from various industries and admixtures are added to the mix. The concept of using fibers to improve the characteristics of construction materials is very old. Early applications include addition of straw to mud bricks, horse hair to reinforce plaster and asbestos to reinforce pottery. Use of continuous reinforcement in concrete (reinforced concrete) increases strength and ductility, but requires careful placement and labour skill. Alternatively, introduction of fibers in discrete form in plain or reinforced concrete may provide a better solution [7-8]. Addition of fibers to concrete makes it a homogeneous and isotropic material. When concrete cracks, the randomly oriented fibers start functioning, arrest crack formation and propagation, and thus improve strength and ductility. The failure modes of FRC are either bond failure between fiber and matrix or material failure. The plain concrete fails suddenly when the deflection corresponding to the ultimate flexural strength is exceeded, on the other hand fiber-reinforced concrete continue to sustain considerable loads even at deflections considerably in excess of the fracture deflection of the plain concrete [9]. Their main purpose is to increase the energy absorption capacity and toughness of the material, but also increase tensile and flexural strength of concrete. There is considerable improvement in the post-cracking behavior of concretes containing fibers. Although in the fiber-reinforced concrete the ultimate tensile strengths do not increase appreciably, the tensile strains at rupture do. Compared to plain concrete, fiber reinforced concrete is much tougher and more resistant to impact [10].

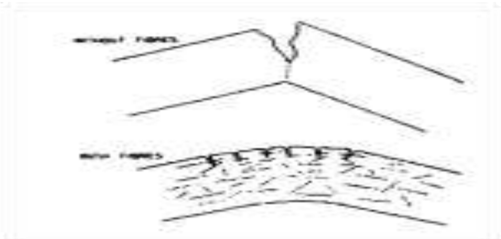

Fig 1: Examination of fractured specimens of fiber-reinforced concrete shows that failure takes place primarily due to fiber pull-out or debonding

1.2 Aims and Objectives of Study

The work aims amongst others to:

i. Investigate the use of palm nut fibre as a fifth component in concrete.
ii. Determine the optimal combination of materials in palm nut fibre concrete in terms of compressive strength characteristics.
iii. Investigate the use of Scheffe's optimization theory in a five-component concrete mix.
iv. Develop models for the optimization of mechanical properties of five component concrete mix.

II. METHODOLOGY

2.1 Mathematical Modelling and Formulation

Henry Scheffe developed a theory for experiments with mixture of which the property studied depends on the proportions of the components present and not on the quantity of the mixture [11]. Scheffe showed that if q represents the number of constituent components of the mixture, the space of the variables known also as the factor space is a $(q - 1)$ dimensional simplex lattice. The composition may be expressed as molar, weight, or volume fraction or percentage. A simplex lattice is a structural representation of lines or planes joining the assumed coordinates (points) of the constituent materials of the mixture [12-15]. According to Scheffe [16], in exploring the whole factor space of a mix design with a uniformly spaced distribution of points over the factor space, we have what we shall call a [q, m] simplex lattice.

2.1.1 Scheffe's Optimization Theory

This is a theory where a polynomial expression of any degrees, is used to characterize a simplex lattice mixture components. In the theory only a single phase mixture is covered. The theory lends path to a unifying equation model capable of taking varying componential ratios to fix approximately equal mixture properties. The optimization is the selectability, from some criterial (mainly economic) view point, the optimal ratio from the component ratios list that can be automatedly generated. His theory is one of the adaptations to this work in the formulation of response function for compressive strength of palm nut fibre concrete [17, 18].

2.1.2 Scheffe's Factor Space

When a product is formed by mixing together two or more ingredients, the product is called a mixture, and the ingredients are called mixture components. In a general mixture problem, the measured response is assumed to depend only on the proportions of the ingredients in the mixture, not the amount of the mixture. The response in a mixture experiment usually is described by a polynomial function. This function represents how the components affect the response. To better study the shape of the response surface, the natural choice for a design would be the one whose points are spread evenly over the whole simplex [19]. An ordered arrangement consisting of a uniformly spaced distribution of points on a simplex is known as a lattice. Simplex is a structural representation (shape) of lines or planes joining assumed positions or points of the constituent materials (atoms) of a mixture [15], and they are equidistant from each other. Mathematically, a simplex lattice is a space of constituent variables of X_1, X_2, X_3... and Xi which obey these laws:

$Xi < 0$

$X \neq$ negative

$$0 \leq x_i \leq 1 \quad (1)$$

$$\sum_{i=1} x_i = 1 \quad (2)$$

A {q, m} simplex lattice design for q components consists of points defined by the following coordinate settings: the proportions assumed by each component take the m+1 equally spaced values from 0 to 1,

$$x_i = 0, \frac{1}{m}, \frac{2}{m}, \ldots\ldots, 1 i = 1, 2, \ldots\ldots\ldots, q \quad (3)$$

2.2 Number of Coefficients

P = 5, M = 2

$$N = \frac{(p+m-1)!}{m!(p-1)!} \quad (4)$$

$$N = \frac{(5+2-1)!}{2!\,(5-1)!} = N = \frac{6!}{2!\,4!} = 15$$

2.3 Five Component Factor Space

The mixture at the vertices of the simplex are pure component blends; a 100% mixture of the single factor assigned to each vertex. All blends along the vertices of the simplex are binary component blends. For the five component mixture, we have five vertices and ten spread in between the vertices of the simplex. All mixture interior to the perimeter of the simplex region are blends of all of the q-components. The factor space is the space within which all the experimental points will be distributed.

The first five pseudo component are located at the vertices of the tetrahedron simplex.

A1 [1:0:0: 0:0], **A2** [0:1:0:0:0], **A3** [0:0:1:0:0], **A4** [0:0:0:1:0], **A5** [0:0:0:0:1].

Ten other pseudo mix ratios located at mid points of the lines joining the vertices of the simplex are

A12 [0.5:0.5:0:0:0], **A13** [0.5:0:0.5:0:0], **A14** [0.5:0:0:0.5:0], **A15** [0.5:0:0:0:0.5], **A23** [0:0.5:05:0:0], **A24** [05:0:0:0.5:0], **A25** [0:0.5:0:0:0.5], **A34** [0:0:0.5:05:0], **A35** [0:0:0.5:0:0.5], **A45** [0:0:0:0.5:0.5].

2.4 Responses

Responses are the properties of fresh and hardened concrete. A simplex lattice is described as a structural representation of lines joining the atoms of a mixture. The atoms are constituent components of the mixture [20-25]. For a normal concrete mixture, the constituent elements are water, cement, fine and coarse aggregates and palmnutfibre. And so, it gives a simplex of a mixture of five components. Hence the simplex lattice of this five-component mixture is a three- dimensional solid equilateral tetrahedron. Mixture components are subject to the constraint that the sum of all the components must be equal to one [4].

As a rule, the response surfaces in multi-component systems are very intricate. To describe such surfaces adequately, high degree polynomials are required, and hence a great many experimental trials. A polynomial of degree n in q variable has $C^n{}_{q+n}$ coefficients. If a mixture has a total of q components and x_1 be the proportion of the i^{th} component in the mixture such that,

$$X_i \geq 0 (I = 1,2....q) \quad (5)$$

Then the sum of the component proportion is a whole unity i.e.

$$X_1 + X_2 + X_3 + X_4 + X_5 = 1 \text{ or } \Sigma X_i - 1 = 0 \quad (6)$$

$$n = b_0 + \sum biXi + \sum bijXiXj + \sum bijkXiXj\,XK + \ldots\ldots \quad + \sum bi1\,i2 \ldots inX_{i1}X_{i\ldots}X_{in} \quad (7)$$

Where l≤i≤q, l≤i≤j≤q, l≤i≤j≤k≤qand 1≤i1≤i1≤ ……..≤in≤q respectively (Simon et al., 1997).

$$Y = b_0 + b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4 + b_5X_5 + b_{11}X_1^2 + b_{12}X_1X_2 + b_{13}X_1X_3 + b_{14}X_1X_4 + b_{15}X_1X_5 + b_{22}X_2^2 + b_{23}X_2X_3 + b_{24}X_2X_4 + b_{25}X_2X_5 + b_{33}X_3^2 + b_{34}X_3X_4 + b_{35}X_3X_5 + b_{44}X_4^2 + b_{45}X_4X_5 + b_{55}X_5^2 \quad (8)$$

Where b is a constant coefficient.

The relationship obtainable from Eqn.[8] is subjected to the normalization condition of Eqn. [6] for a sum of independent variables. For a ternary mixture, the reduced second degree polynomial can be obtained as follows:

From Eqn. [2] $X_1+X_2+X_3+X_4+X_5 = 1$ (9)

i.e. $b_0X_1 + b_0X_2 + b_0X_3+ b_0X_4 + b_0X_5 = b_0$ (10)

$b_0 = b_0 (X_1 +X_2 +X_3+X_4 +X_5)$

Multiplying Eqn. [9] by X_1, X_2, X_3, X_4, and X_5 in succession gives

$X_1^2 = X_1 - X_1X_2 - X_1X_3 - X_1X_4 - X_1X_5$

$X_2^2 = X_2 - X_1X_2 - X_2X_3 - X_2X_4 - X_2X_5$ (11)

$X_3^2 = X_3 - X_1X_3 - X_2X_3 - X_3X_4 - X_3X_5$

$X_4^2 = X_4 - X_1X_4 - X_2X_4 - X_3X_4 - X_4X_5$

$X_5^2 = X_5 - X_1X_5 - X_2X_5 - X_3X_5 - X_4X_5$

Substituting Eqn. [10] and [11] into Eqn. [8], we obtain after necessary transformation that

$\hat{Y} = (b_0 + b_1 + b_{11}) X_1 + (b_0 + b_2 + b_{22}) X_2 + (b_0 + b_3 + b_{33}) X_3 + (b_0 + b_4 + b_{44}) X_4 + (b_0 + b_5 + b_{55}) X_5 + (b_{12} - b_{11} - b_{22}) X_1X_2 + (b_{13} - b_{11} - b_{33}) X_1X_3 + (b_{14} - b_{11} - b_{44}) X_1X_4 + (b_{15} - b_{11} - b_{55}) X_1X_5 + (b_{23} - b_{22} - b_{33}) X_2X_3 + (b_{24} - b_{22} - b_{44}) X_2X_4 + (b_{25} - b_{11} - b_{55}) X_2X_5 + (b_{34} - b_{33} - b_{44}) X_3X_4 + (b_{35} - b_{33} - b_{55}) X_3X_5 + (b_{45} - b_{44} - b_{55}) X_4X_5$ (12)

If we denote

$\beta i = b_0 + b_i + b_{ii}$

And $\beta ij = b_{ij} - b_{ii} - b_{jj}$,

Then we arrive at the second-degree polynomial:

$\hat{Y} = \beta_1X_1+ \beta_2X_2 + \beta_3X_3 + \beta_4X_4 + \beta_5X_5 + \beta_{12}X_1X_2 + \beta_{13}X_1X_3 + \beta_{14}X_1X_4 + \beta_{15}X_1X_5 + \beta_{23}X_2X_3 + \beta_{24}X_2X_4 + \beta_{25}X_2X_5 + \beta_{34}X_3X_4 + \beta_{35}X_3X_5 + \beta_{45}X_4X_5$ (13)

And doing so in succession for the other two points if the hexahedron, we obtain $Y_1 = \beta_1$, $Y_2 = \beta_2$, $Y_3 = \beta_3$, $Y_4 = \beta_4$, $Y_5 = \beta_5$.

The substitution of the coordinates of the fourth point yields

$Y_{12} = ½ X_1 + ½X_2 + ½X_1.X_2 = ½ \beta_1 + ½ \beta_2 + 1/4 \beta_{12}$

But as $\beta_i = Y_i$ then $Y_{12} = ½ \beta_1 - ½ \beta_2 - 1/4 \beta_{12}$

Thus $\beta_{12} = 4 Y_{12} - 2Y_1 - 2Y_2$, $\beta_{13} = 4Y_{13} - 2Y_1 - 2Y_3$, $\beta_{14} = 4Y_{14} - 2Y_1 - 2Y_4$,

$\beta_{15} = 4Y_{15} - 2Y_1 - 2Y_5$, $\beta_{23} = 4Y_{23} - 2Y_2 - 2Y_3$, $\beta_{24} = 4Y_{24} - 2Y_2 - 2Y_4$,

$\beta_{25} = 4Y_{25} - 2Y_2 - 2Y_5$, $\beta_{34} = 4Y_{34} - 2Y_3 - 2Y_4$, $\beta_{35} = 4Y_{35} - 2Y_3 - 2Y_5$, $\beta_{45} = 4Y_{45} - 2Y_4 - 2Y_5$

(14)

2.4.1 Actual Components and Pseudo Components

$Z = AX$ (15)

Z represents the actual components while X represents the pseudo components, where A is the constant; a five by five matrix.The value of matrix A will be obtained from the first five mix ratios. **The mix ratios are**

Z_1 [0.45:1.0:1.25:1.45:0.2], **Z_2**[0.5:1.0:1.35:1.6:0.4], **Z_3**[0.55:1.0:1.55:1.9:0.8], **Z_4**[0.6:1.0:1.8:2.5:1.2], **Z_5** [0.65:1.0:2.0:3.0:1.8].

The corresponding pseudo mix ratios are

X_1 [1:0:0:0:0], **X_2**[0:l:0:0:0], **X_3** [0:0:1:0:0], **X_4** [0:0:0:1:0], **X_5**[0:0:0:0:l].

Substitution ofX_i and Z_i into Equation [14] use the corresponding pseudo components to determine the corresponding actual mixture components.

X_1 = fraction of water cement ratio

X_2 = fraction of ordinary Portland cement

X_3 = fraction of fine aggregate

X_4= fraction of coarse aggregate

X_5 = fraction of palm nut fibre

$$\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{pmatrix}$$

For the first run

$$\begin{pmatrix} 0.45 \\ 1.0 \\ 1.25 \\ 1.45 \\ 0.2 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$a_{11} = 0.45$, $a_{21} = 1.0$, $a_{31} = 1.25$, $a_{41} = 1.45$, $a_{51} = 0.2$

For the second run

$$\begin{pmatrix} 0.5 \\ 1.0 \\ 1.35 \\ 1.6 \\ 0.4 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$a_{12} = 0.5$, $a_{22} = 1.0$, $a_{32} = 1.35$, $a_{42} = 1.6$, $a_{52} = 0.4$

For the third run

$$\begin{pmatrix} 0.55 \\ 1.0 \\ 1.55 \\ 1.9 \\ 0.8 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$a_{13} = 0.55$, $a_{23} = 1.0$, $a_{33} = 1.55$, $a_{43} = 1.9$, $a_{53} = 0.8$

For the fourth run

$$\begin{pmatrix} 0.6 \\ 1.0 \\ 1.8 \\ 2.5 \\ 1.2 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$a_{14} = 0.6$, $a_{24} = 1.0$, $a_{34} = 1.8$, $a_{44} = 2.5$, $a_{54} = 1.2$

For the fifth run

$$\begin{pmatrix} 0.65 \\ 1.0 \\ 2.0 \\ 3.0 \\ 1.8 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$a_{15} = 0.65$, $a_{25} = 1.0$, $a_{35} = 2.0$, $a_{45} = 3.0$, $a_{55} = 1.8$

Substituting the values of the constants, we have [A] matrix

$$\begin{pmatrix} 0.45 & 0.5 & 0.55 & 0.60 & 0.65 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 1.25 & 1.35 & 1.55 & 1.8 & 2.0 \\ 1.45 & 1.6 & 1.9 & 2.5 & 3.0 \\ 0.2 & 0.4 & 0.8 & 1.2 & 1.8 \end{pmatrix}$$

The [A] matrix is further used to calculate the real proportion [Z] by applying eqn. [15]

Therefore, for A_{12}

$$\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{pmatrix} = \begin{pmatrix} 0.45 & 0.5 & 0.55 & 0.60 & 0.65 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 1.25 & 1.35 & 1.55 & 1.8 & 2.0 \\ 1.45 & 1.6 & 1.9 & 2.5 & 3.0 \\ 0.2 & 0.4 & 0.8 & 1.2 & 1.8 \end{pmatrix} * \begin{pmatrix} 0.5 \\ 0.5 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$Z_1 = (0.45*0.5) + (0.5*0.5) = 0.475$
$Z_2 = (1.0*0.5) + (1.0*0.5) = 1.0$
$Z_3 = (1.25*0.5) + (1.35*0.5) = 1.3$
$Z_4 = (1.45*0.5) + (1.6*0.5) = 1.525$
$Z_5 = (0.2*0.5) + (0.4*0.5) = 0.3$

For A_{13}

$$\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{pmatrix} = \begin{pmatrix} 0.45 & 0.5 & 0.55 & 0.60 & 0.65 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 1.25 & 1.35 & 1.55 & 1.8 & 2.0 \\ 1.45 & 1.6 & 1.9 & 2.5 & 3.0 \\ 0.2 & 0.4 & 0.8 & 1.2 & 1.8 \end{pmatrix} * \begin{pmatrix} 0.5 \\ 0 \\ 0.5 \\ 0 \\ 0 \end{pmatrix}$$

Z_1 = (0.45*0.5) + (0.55*0.5) = 0.5
Z_2 = (1.0*0.5) + (1.0*0.5) = 1.0
Z_3 = (1.25*0.5) + (1.55*0.5) = 1.4
Z_4 = (1.45*0.5) + (1.9*0.5) = 1.675
Z_5 = (0.2*0.5) + (0.8*0.5) = 0.5

For A_{14}

$$\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{pmatrix} = \begin{pmatrix} 0.45 & 0.5 & 0.55 & 0.60 & 0.65 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 1.25 & 1.35 & 1.55 & 1.8 & 2.0 \\ 1.45 & 1.6 & 1.9 & 2.5 & 3.0 \\ 0.2 & 0.4 & 0.8 & 1.2 & 1.8 \end{pmatrix} * \begin{pmatrix} 0.5 \\ 0 \\ 0 \\ 0.5 \\ 0 \end{pmatrix}$$

Z_1 = (0.45*0.5) + (0.6*0.5) = 0.525
Z_2 = (1.0*0.5) + (1.0*0.5) =1.0
Z_3 = (1.25*0.5) + (1.8*0.5)= 1.525
Z_4 = (1.45*0.5) + (2.5*0.5) = 1.975
Z_5 = (0.2*0.5) + (1.2*0.5) = 0.7

For A_{15}

$$\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{pmatrix} = \begin{pmatrix} 0.45 & 0.5 & 0.55 & 0.60 & 0.65 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 1.25 & 1.35 & 1.55 & 1.8 & 2.0 \\ 1.45 & 1.6 & 1.9 & 2.5 & 3.0 \\ 0.2 & 0.4 & 0.8 & 1.2 & 1.8 \end{pmatrix} * \begin{pmatrix} 0.5 \\ 0 \\ 0 \\ 0 \\ 0.5 \end{pmatrix}$$

Z_1 = (0.45*0.5) + (0.65*0.5) = 0.55
Z_2 = (1.0*0.5) + (1.0*0.5) = 1.0
Z_3 = (1.25*0.5) + (2.0*0.5) = 1.625
Z_4 = (1.45*0.5) + (3.0*0.5) = 2.225
Z_5 = (0.2*0.5) + (1.8*0.5) = 1.0

For A_{23}

$$\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{pmatrix} = \begin{pmatrix} 0.45 & 0.5 & 0.55 & 0.60 & 0.65 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 1.25 & 1.35 & 1.55 & 1.8 & 2.0 \\ 1.45 & 1.6 & 1.9 & 2.5 & 3.0 \\ 0.2 & 0.4 & 0.8 & 1.2 & 1.8 \end{pmatrix} * \begin{pmatrix} 0 \\ 0.5 \\ 0.5 \\ 0 \\ 0 \end{pmatrix}$$

Z_1 = (0.5*0.5) + (0.55*0.5) = 0.525
Z_2 = (1.0*0.5) + (1.0*0.5) = 1.0
Z_3 = (1.35*0.5) + (1.55*0.5) = 1.575
Z_4 = (1.6*0.5) + (l. 9*0.5) = 2.05
Z_5 = (0.4*0.5) + (0.8*0.5) = 0.8

For A_{24}

$$\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{pmatrix} = \begin{pmatrix} 0.45 & 0.5 & 0.55 & 0.60 & 0.65 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 1.25 & 1.35 & 1.55 & 1.8 & 2.0 \\ 1.45 & 1.6 & 1.9 & 2.5 & 3.0 \\ 0.2 & 0.4 & 0.8 & 1.2 & 1.8 \end{pmatrix} * \begin{pmatrix} 0 \\ 0.5 \\ 0 \\ 0.5 \\ 0 \end{pmatrix}$$

Z_1 =(0.5*0.5) + (0.6*0.5) = 0.55
Z_2 = (1.0*0.5) + (1.0*0.5) = 1.0
Z_3 = (1.35*0.5) + (1.8*0.5) = 1.575
Z_4 = (1.6*0.5) + (2.5*0.5) = 2.05
Z_5 = (0.4*0.5) + (1.2 *0.5) = 0.8

For A_{25}

$$\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{pmatrix} = \begin{pmatrix} 0.45 & 0.5 & 0.55 & 0.60 & 0.65 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 1.25 & 1.35 & 1.55 & 1.8 & 2.0 \\ 1.45 & 1.6 & 1.9 & 2.5 & 3.0 \\ 0.2 & 0.4 & 0.8 & 1.2 & 1.8 \end{pmatrix} * \begin{pmatrix} 0 \\ 0.5 \\ 0 \\ 0 \\ 0.5 \end{pmatrix}$$

$Z_1 = (0.5*0.5) + (0.65*0.5) = 0.575$
$Z_2 = (1.0*0.5) + (1.0*0.5) = 1.0$
$Z_3 = (1.35*0.5) + (2.0*0.5) = 1.675$
$Z_4 = (1.6*0.5) + (3.0*0.5) = 2.3$
$Z_5 = (0.4*0.5) + (1.8*0.5) = 1.1$

For A_{34}

$$\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{pmatrix} = \begin{pmatrix} 0.45 & 0.5 & 0.55 & 0.60 & 0.65 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 1.25 & 1.35 & 1.55 & 1.8 & 2.0 \\ 1.45 & 1.6 & 1.9 & 2.5 & 3.0 \\ 0.2 & 0.4 & 0.8 & 1.2 & 1.8 \end{pmatrix} * \begin{pmatrix} 0 \\ 0 \\ 0.5 \\ 0.5 \\ 0 \end{pmatrix}$$

$Z_1 = (0.55*0.5) + (0.6*0.5) = 0.575$
$Z_2 = (1.0*0.5) + (1.0*0.5) = 1.0$
$Z_3 = (1.55*0.5) + (1.8*0.5) = 1.675$
$Z_4 = (1.9*0.5) + (2.5*0.5) = 2.2$
$Z_5 = (0.8*0.5) + (1.2*0.5) = 1.0$

For A_{35}

$$\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{pmatrix} = \begin{pmatrix} 0.45 & 0.5 & 0.55 & 0.60 & 0.65 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 1.25 & 1.35 & 1.55 & 1.8 & 2.0 \\ 1.45 & 1.6 & 1.9 & 2.5 & 3.0 \\ 0.2 & 0.4 & 0.8 & 1.2 & 1.8 \end{pmatrix} * \begin{pmatrix} 0 \\ 0 \\ 0.5 \\ 0 \\ 0.5 \end{pmatrix}$$

$Z_1 = (0.55*0.5) + (0.65*0.5) = 0.6$
$Z_2 = (1.0*0.5) + (1.0*0.5) = 1.0$
$Z_3 = (1.55*0.5) + (2.0*0.5) = 1.775$
$Z_4 = (1.9*0.5) + (3.0*0.5) = 2.45$
$Z_5 = (0.8*0.5) + (1.8*0.5) = 1.3$

For A_{45}

$$\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{pmatrix} = \begin{pmatrix} 0.45 & 0.5 & 0.55 & 0.60 & 0.65 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 1.25 & 1.35 & 1.55 & 1.8 & 2.0 \\ 1.45 & 1.6 & 1.9 & 2.5 & 3.0 \\ 0.2 & 0.4 & 0.8 & 1.2 & 1.8 \end{pmatrix} * \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0.5 \\ 0.5 \end{pmatrix}$$

$Z_1 = (0.6*0.5) + (0.65*0.5) = 0.625$
$Z_2 = (1.0*0.5) + (1.0*0.5) = 1.0$
$Z_3 = (1.8*0.5) + (2.0*0.5) = 1.95$
$Z_4 = (2.5*0.5) + (3.0*0.5) = 2.90$
$Z_5 = (1.2*0.5) + (1.8*0.5) = 1.55$

The values calculated are summarized in the matrix table shown below.

Table 1. Matrix Table for Scheffe's (5, 2) - Lattice Polynomial

ACTUAL						PSEUDO				
Z1	Z2	Z3	Z4	Z5	RESPONSE	X1	X2	X3	X4	X5
0.45	1	1.25	1.45	0.2	Y1	1	0	0	0	0
0.5	1	1.35	1.6	0.4	Y2	0	1	0	0	0
0.55	1	1.55	1.9	0.8	Y3	0	0	1	0	0
0.6	1	1.8	2.5	1.2	Y4	0	0	0	1	0
0.65	1	2	3	1.8	Y5	0	0	0	0	1

0.475	1	1.3	1.525	0.3	Y12	0.5	0.5	0	0	0
0.5	1	1.4	1.675	0.5	Y13	0.5	0	0.5	0	0
0.525	1	1.525	1.975	0.7	Y14	0.5	0	0	0.5	0
0.55	1	1.625	2.225	1	Y15	0.5	0	0	0	0.5
0.525	1	1.45	1.75	0.6	Y23	0	0.5	0.5	0	0
0.55	1	1.575	2.05	0.8	Y24	0	0.5	0	0.5	0
0.575	1	1.675	2.3	1.1	Y25	0	0.5	0	0	0.5
0.575	1	1.675	2.2	1	Y34	0	0	0.5	0.5	0
0.6	1	1.775	2.45	1.3	Y35	0	0	0.5	0	0.5
0.625	1	1.9	2.75	1.5	Y45	0	0	0	0.5	0.5

2.4.2 Mixture Proportion of Control Points Showing Actual and Pseudo Components

The [A] matrix is further used to obtain the corresponding proportions for the control points by applying eqn. [15] with the pseudo-components set by the sum to one of the mixture components constraint.

Control points for A_1

$$\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{pmatrix} = \begin{pmatrix} 0.45 & 0.5 & 0.55 & 0.60 & 0.65 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 1.25 & 1.35 & 1.55 & 1.8 & 2.0 \\ 1.45 & 1.6 & 1.9 & 2.5 & 3.0 \\ 0.2 & 0.4 & 0.8 & 1.2 & 1.8 \end{pmatrix} * \begin{pmatrix} 0.25 \\ 0.25 \\ 0.25 \\ 0.25 \\ 0 \end{pmatrix}$$

Z_1 = (0.45*0.25) + (0.5*0.25) + (0.55*0.25) + (0.6*0.25) = 0.525

Z_2 = (1.0*0.25) + (1.0*0.25) + (1.0*0.25) + (1.0*0.25) = 1.0

Z_3 = (1.25*0.25) + (1.35*0.25) + (1.55*0.25) + (1.8*0.25) = 1.4875

Z_4= (1.45*0.25) + (1.6*0.25) + (1.9*0.25) + (2.5*0.25) = 1.8625

Z_5 = (0.2*0.25) + (0.4*0.25) + (0.8*0.25) + (1.2*0.25) = 0.65

Control points for A_2

$$\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{pmatrix} = \begin{pmatrix} 0.45 & 0.5 & 0.55 & 0.60 & 0.65 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 1.25 & 1.35 & 1.55 & 1.8 & 2.0 \\ 1.45 & 1.6 & 1.9 & 2.5 & 3.0 \\ 0.2 & 0.4 & 0.8 & 1.2 & 1.8 \end{pmatrix} * \begin{pmatrix} 0.25 \\ 0.25 \\ 0.25 \\ 0 \\ 0.25 \end{pmatrix}$$

Z_1 = (0.45*0.25) + (0.5*0.25) + (0.55*0.25) + (0.65*0.25) = 0.5375

Z_2 = (1.0*0.25) + (1.0*0.25) + (1.0*0.25) + (1.0*0.25) = 1.0

Z_3 = (1.25*0.25) + (1.35*0.25) + (1.55*0.25) + (2.0*0.25) = 1.5375

Z_4 = (1.45*0.25) + (1.6*0.25) + (1.9*0.25) + (3.0*0.25) = 1.9875

Z_5 = (0.2*0.25) + (0.4*0.25) + (0.8*0.25) + (1.8*0.25) = 0.8

Control points for A_3

$$\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{pmatrix} = \begin{pmatrix} 0.45 & 0.5 & 0.55 & 0.60 & 0.65 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 1.25 & 1.35 & 1.55 & 1.8 & 2.0 \\ 1.45 & 1.6 & 1.9 & 2.5 & 3.0 \\ 0.2 & 0.4 & 0.8 & 1.2 & 1.8 \end{pmatrix} * \begin{pmatrix} 0.25 \\ 0.25 \\ 0 \\ 0.25 \\ 0.25 \end{pmatrix}$$

Z_1= (0.45*0.25) + (0.5*0.25) + (0.60*0.25) + (0.65*0.25) = 0.55

Z_2 = (1.0*0.25) + (1.0*0.25) + (1.0*0.25) + (1.0*0.25) = 1.0

Z_3 = (1.25*0.25) + (1.35*0.25) + (1.8*0.25) + (2.0*0.25) = 1.6

Z_4 = (1.45*0.25) + (1.6*0.25) + (2.5*0.25) + (3.0*0.25) = 2.1375

Z_5= (0.2*0.25) + (0.4*0.25) + (1.2*0.25) + (1.8*0.25) = 0.9

Control points for A_4

$$\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{pmatrix} = \begin{pmatrix} 0.45 & 0.5 & 0.55 & 0.60 & 0.65 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 1.25 & 1.35 & 1.55 & 1.8 & 2.0 \\ 1.45 & 1.6 & 1.9 & 2.5 & 3.0 \\ 0.2 & 0.4 & 0.8 & 1.2 & 1.8 \end{pmatrix} * \begin{pmatrix} 0.25 \\ 0 \\ 0.25 \\ 0.25 \\ 0.25 \end{pmatrix}$$

Z_1 = (0.45*0.25) + (0.55*0.25) + (0.60*0.25) + (0.65*0.25) = 0.5625

Z_2 = (1.0*0.25) + (1.0*0.25) + (1.0*0.25) + (1.0*0.25) = 1.0
Z_3 = (1.25*0.25) + (1.55*0.25) + (1.8*0.25) + (2.0*0.25) = 1.65
Z_4 = (1.45*0.25) + (1.9*0.25) + (2.5*0.25) + (3.0*0.25) = 2.2125
Z_5 = (0.2*0.25) + (0.8*0.25) + (1.2*0.25) + (1.8*0.25) = 1.0

Control points for A_5

$$\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{pmatrix} = \begin{pmatrix} 0.45 & 0.5 & 0.55 & 0.60 & 0.65 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 1.25 & 1.35 & 1.55 & 1.8 & 2.0 \\ 1.45 & 1.6 & 1.9 & 2.5 & 3.0 \\ 0.2 & 0.4 & 0.8 & 1.2 & 1.8 \end{pmatrix} * \begin{pmatrix} 0 \\ 0.25 \\ 0.25 \\ 0.25 \\ 0.25 \end{pmatrix}$$

Z_1 = (0.50*0.25) + (0.55*0.25) + (0.60*0.25) + (0.65*0.25) = 0.575
Z_2 = (1.0*0.25) + (1.0*0.25) + (1.0*0.25) + (1.0*0.25) = 1.0
Z_3 = (1.35*0.25) + (1.55*0.25) + (1.8*0.25) + (2.0*0.25) = 1.675
Z_4 = (1.6*0.25) + (1.9*0.25) + (2.5*0.25) + (3.0*0.25) = 2.25
Z_5 = (0.4*0.25) + (0.8*0.25) + (1.2*0.25) + (1.8*0.25) = 1.05

Control points for A_{12}

$$\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{pmatrix} = \begin{pmatrix} 0.45 & 0.5 & 0.55 & 0.60 & 0.65 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 1.25 & 1.35 & 1.55 & 1.8 & 2.0 \\ 1.45 & 1.6 & 1.9 & 2.5 & 3.0 \\ 0.2 & 0.4 & 0.8 & 1.2 & 1.8 \end{pmatrix} * \begin{pmatrix} 0.2 \\ 0.2 \\ 0.2 \\ 0.2 \\ 0.2 \end{pmatrix}$$

Z_1 = (0.45*0.2) + (0.5*0.2) + (0.55*0.2) + (0.6*0.2) + (0.65*0.2) = 0.55
Z_2 = (1.0*0.2) + (1.0*0.2) + (1.0*0.2) + (1.0*0.2) + (1.0*0.2) = 1.0
Z_3 = (1.25*0.2) +(1.35*0.2) + (1.55*0.2) + (1.8*0.2) + (2.0*0.2) = 1.59
Z_4 = (1.45*0.2) + (1.6*0.2) + (1.9*0.2) + (2.5*0.2) + (3.0*0.2) = 2.09
Z_5 = (0.2*0.2) + (0.4*0.2) + (0.8*0.2) + (1.2*0.2) + (1.8*0.2) = 0.88

Control points for A_{13}

$$\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{pmatrix} = \begin{pmatrix} 0.45 & 0.5 & 0.55 & 0.60 & 0.65 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 1.25 & 1.35 & 1.55 & 1.8 & 2.0 \\ 1.45 & 1.6 & 1.9 & 2.5 & 3.0 \\ 0.2 & 0.4 & 0.8 & 1.2 & 1.8 \end{pmatrix} * \begin{pmatrix} 0.3 \\ 0.3 \\ 0.3 \\ 0.1 \\ 0 \end{pmatrix}$$

Z_1= (0.45*0.3) + (0.5*0.3) + (0.55*0.3) + (0.6*0.1) = 0.51
Z_2 = (1.0*0.3) + (1.0*0.3) + (1.0*0.3) + (1.0*0.1) = 1.0
Z_3 - (1.25*0.3) + (1.35*0.3) + (1.55*0.3) + (1.8*0.1) = 1.425
Z_4 = (1.45*0.3) + (1.6*0.3) + (1.9*0.3) + (2.5*0.1) = 1.735
Z_5 = (0.2*0.3) + (0.4*0.3) + (0.8*0.3) + (1.2*0.1) = 0.54

Control points for A_{14}

$$\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{pmatrix} = \begin{pmatrix} 0.45 & 0.5 & 0.55 & 0.60 & 0.65 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 1.25 & 1.35 & 1.55 & 1.8 & 2.0 \\ 1.45 & 1.6 & 1.9 & 2.5 & 3.0 \\ 0.2 & 0.4 & 0.8 & 1.2 & 1.8 \end{pmatrix} * \begin{pmatrix} 0.3 \\ 0.3 \\ 0.3 \\ 0 \\ 0.1 \end{pmatrix}$$

Z_1= (0.45*0.3) + (0.5*0.3) + (0.55*0.3) + (0.65*0.1) = 0.515
Z_2 = (1.0*0.3) + (1.0*0.3) + (1.0*0.3) + (1.0*0.1) = 1.0
Z_3 = (1.2*0.3) + (1.4*0.3) + (1.5*0.3) + (1.9*0.1) = 1.42
Z_4 = (1.5*0.3) + (1.7*0.3) + (1.8*0.3) + (3.0*0.1) = 1.80
Z_5 = (0.3*0.3) + (0.5*0.3) + (0.7*0.3) + (2.0*0.1) = 0.65

Control points for A_{15}

$$\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{pmatrix} = \begin{pmatrix} 0.45 & 0.5 & 0.55 & 0.60 & 0.65 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 1.25 & 1.35 & 1.55 & 1.8 & 2.0 \\ 1.45 & 1.6 & 1.9 & 2.5 & 3.0 \\ 0.2 & 0.4 & 0.8 & 1.2 & 1.8 \end{pmatrix} * \begin{pmatrix} 0.3 \\ 0.3 \\ 0 \\ 0.3 \\ 0.1 \end{pmatrix}$$

$Z_1 = (0.45*0.3) + (0.5*0.3) + (0.6*0.3) + (0.65*0.1) = 0.53$
$Z_2 = (1.0*0.3) + (1.0*0.3) + (1.0*0.3) + (1.0*0.1) = 1.0$
$Z_3 = (1.2*0.3) + (1.4*0.3) + (2.0*0.3) + (1.9*0.1) = 1.57$
$Z_4 = (1.5*0.3) + (1.7*0.3) + (2.8*0.3) + (3.0*0.1) = 2.1$
$Z_5 = (0.3*0.3) + (0.5*0.3) + (1.1*0.3) + (2.0*0.1) = 0.77$

Control points for A_{23}

$$\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{pmatrix} = \begin{pmatrix} 0.45 & 0.5 & 0.55 & 0.60 & 0.65 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 1.25 & 1.35 & 1.55 & 1.8 & 2.0 \\ 1.45 & 1.6 & 1.9 & 2.5 & 3.0 \\ 0.2 & 0.4 & 0.8 & 1.2 & 1.8 \end{pmatrix} * \begin{pmatrix} 0.3 \\ 0 \\ 0.3 \\ 0.3 \\ 0.1 \end{pmatrix}$$

$Z_1 = (0.45*0.3) + (0.55*0.3) + (0.6*0.3) + (0.65*0.1) = 0.545$
$Z_2 = (1.0*0.3) + (1.0*0.3) + (1.0*0.3) + (1.0*0.1) = 1.0$
$Z_3 = (1.25*0.3) + (1.55*0.3) + (1.8*0.3) + (2.0*0.1) = 1.58$
$Z_4 = (1.45*0.3) + (1.9*0.3) + (2.5*0.3) + (3.0*0.1) = 2.055$
$Z_5 = (0.2*0.3) + (0.8*0.3) + (1.2*0.3) + (1.8*0.1) = 0.84$

Control points for A_{24}

$$\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{pmatrix} = \begin{pmatrix} 0.45 & 0.5 & 0.55 & 0.60 & 0.65 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 1.25 & 1.35 & 1.55 & 1.8 & 2.0 \\ 1.45 & 1.6 & 1.9 & 2.5 & 3.0 \\ 0.2 & 0.4 & 0.8 & 1.2 & 1.8 \end{pmatrix} * \begin{pmatrix} 0 \\ 0.3 \\ 0.3 \\ 0.3 \\ 0.1 \end{pmatrix}$$

$Z_1 = (0.5*0.3) + (0.55*0.3) + (0.6*0.3) + (0.65*0.1) = 0.560$
$Z_2 = (1.0*0.3) + (1.0*0.3) + (1.0*0.3) + (1.0*0.1) = 1.0$
$Z_3 = (1.35*0.3) + (1.55*0.3) + (1.8*0.3) + (2.0*0.1) = 1.66$
$Z_4 = (1.6*0.3) + (1.9*0.3) + (2.5*0.3) + (3.0*0.1) = 2.19$
$Z_5 = (0.4*0.3) + (0.8*0.3) + (1.2*0.3) + (1.8*0.1) = 0.89$

Control points for A_{25}

$$\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{pmatrix} = \begin{pmatrix} 0.45 & 0.5 & 0.55 & 0.60 & 0.65 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 1.25 & 1.35 & 1.55 & 1.8 & 2.0 \\ 1.45 & 1.6 & 1.9 & 2.5 & 3.0 \\ 0.2 & 0.4 & 0.8 & 1.2 & 1.8 \end{pmatrix} * \begin{pmatrix} 0.1 \\ 0 \\ 0.3 \\ 0.3 \\ 0.3 \end{pmatrix}$$

$Z_1 = (0.45*0.1) + (0.55*0.3) + (0.6*0.3) + (0.65*0.3) = 0.585$
$Z_2 = (1.0*0.1) + (1.0*0.3) + (1.0*0.3) + (1.0*0.3) = 1.0$
$Z_3 = (1.25*0.1) + (1.55*0.3) + (1.8*0.3) + (2.0*0.3) = 1.73$
$Z_4 = (1.45*0.1) + (1.9*0.3) + (2.5*0.3) + (3.0*0.3) = 2.365$
$Z_5 = (0.2*0.1) + (0.8*0.3) + (1.2*0.3) + (1.8*0.3) = 1.16$

Control points for A_{34}

$$\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{pmatrix} = \begin{pmatrix} 0.45 & 0.5 & 0.55 & 0.60 & 0.65 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 1.25 & 1.35 & 1.55 & 1.8 & 2.0 \\ 1.45 & 1.6 & 1.9 & 2.5 & 3.0 \\ 0.2 & 0.4 & 0.8 & 1.2 & 1.8 \end{pmatrix} * \begin{pmatrix} 0.1 \\ 0.3 \\ 0 \\ 0.3 \\ 0.3 \end{pmatrix}$$

$Z_1 = (0.45*0.1) + (0.5*0.3) + (0.6*0.3) + (0.65*0.3) = 0.57$
$Z_2 = (1.0*0.1) + (1.0*0.3) + (1.0*0.3) + (1.0*0.3) = 1.0$
$Z_3 = (1.25*0.1) + (1.35*0.3) + (1.8*0.3) + (2.0*0.3) = 1.67$
$Z_4 = (1.45*0.1) + (1.6*0.3) + (2.5*0.3) + (3.0*0.3) = 2.275$
$Z_5 = (0.2*0.1) + (0.4*0.3) + (1.2*0.3) + (1.8*0.3) = 1.04$

Control points for A_{35}

$$\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{pmatrix} = \begin{pmatrix} 0.45 & 0.5 & 0.55 & 0.60 & 0.65 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 1.25 & 1.35 & 1.55 & 1.8 & 2.0 \\ 1.45 & 1.6 & 1.9 & 2.5 & 3.0 \\ 0.2 & 0.4 & 0.8 & 1.2 & 1.8 \end{pmatrix} * \begin{pmatrix} 0.1 \\ 0.3 \\ 0.3 \\ 0 \\ 0.3 \end{pmatrix}$$

Z_1 = (0.45*0.1) + (0.5*0.3) + (0.55*0.3) + (0.65*0.3) = 0.555
Z_2 = (1.0*0.1) + (1.0*0.3) + (1.0*0.3) + (1.0*0.3) = 1.0
Z_3 - (1.25*0.1) + (1.35*0.3) + (1.55*0.3) + (2.0*0.3) = 1.595
Z_4 = (1.45*0.1) + (1.6*0.3) + (1.9*0.3) + (3.0*0.3) = 2.095
Z_5 = (0.2*0.1) + (0.4*0.3) + (0.8*0.3) + (1.8*0.3) = 0.92

Control points for A_{45}

$$\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{pmatrix} = \begin{pmatrix} 0.45 & 0.5 & 0.55 & 0.60 & 0.65 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 1.25 & 1.35 & 1.55 & 1.8 & 2.0 \\ 1.45 & 1.6 & 1.9 & 2.5 & 3.0 \\ 0.2 & 0.4 & 0.8 & 1.2 & 1.8 \end{pmatrix} * \begin{pmatrix} 0.1 \\ 0.3 \\ 0.3 \\ 0.3 \\ 0 \end{pmatrix}$$

Z_1 = (0.45*0.1) + (0.5*0.3) + (0.55*0:3) + (0.6*0.3) = 0.54
Z_2 = (1.0*0.1) + (1.0*0.3) + (1.0*0.3) + (1.0*0.3) = 1.0
Z_3= (1.2*0.1) + (1.4*0.3) + (1.5*0.3) + (2.0*0.3) = 1.59
Z_4 = (1.5*0.1) + (1.7*0.3) + (1.8*0.3) + (2.8*0.3) = 2.04
Z_5 = (0.3*0.1) + (0.5*0.3) + (0.7*0.3) + (1.1*0.3)= 0.72

The calculated results for the control points are summarized in the table below.

Table 2. Mixture Proportion of Control Points Showing Actual and Pseudo Components

ACTUAL						PSEUDO				
Z1	**Z2**	**Z3**	**Z4**	**Z5**	**RESPONSE**	**X1**	**X2**	**X3**	**X4**	**X5**
0.525	1	1.4875	1.8625	0.65	C1	0.25	0.25	0.25	0.25	0
0.5375	1	1.5375	1.9875	0.8	C2	0.25	0.25	0.25	0	0.25
0.55	1	1.6	2.1375	0.9	C3	0.25	0.25	0	0.25	0.25
0.5625	1	1.65	2.2125	1	C4	0.25	0	0.25	0.25	0.25
0.575	1	1.675	2.25	1.05	C5	0	0.25	0.25	0.25	0.25
0.55	1	1.59	2.09	0.88	C12	0.2	0.2	0.2	0.2	0.2
0.51	1	1.425	1.735	0.54	C13	0.3	0.3	0.3	0.1	0
0.515	1	1.445	1.785	0.6	C14	0.3	0.3	0.3	0	0.1
0.53	1	1.52	1.965	0.72	C15	0.3	0.3	0	0.3	0.1
0.545	1	1.58	2.055	0.84	C23	0.3	0	0.3	0.3	0.1
0.56	1	1.61	2.1	0.9	C24	0	0.3	0.3	0.3	0.1
0.585	1	1.73	2.365	1.16	C25	0.1	0	0.3	0.3	0.3
0.57	1	1.67	2.275	1.04	C34	0.1	0.3	0	0.3	0.3
0.555	1	1.595	2.095	0.92	C35	0.1	0.3	0.3	0	0.3
0.54	1	1.535	1.945	0.74	C45	0.1	0.3	0.3	0.3	0

These mixture proportions formulated would be utilized to mix a five component concrete and cured for 28 days after which their responses would be obtained which is used to generate the mathematical model.

III. MATERIALS AND METHODS

3.1 Materials

The materials for the experiments are a mixture of cement, water, fine and coarse aggregate and palm nut fibre forming a five component mixture. The cement is Dangote cement, a brand of Ordinary Portland Cement. The fine aggregate, whose size ranges from 0.05 - 4.5mm was procured from the local river. Crushed granite of 12.5mm size downgraded to 4.75mm obtained from a local stone market was used in the experimental investigation.The palm nut fibre was obtained from Oboro in Ikwuano L.G.A, Abia state.

3.2 Method

The specimens for the compressive strength were concrete cubes. They were cast in steel mould measuring 150mm*150mm*150mm. The mould and its base were damped together during concrete casting to prevent leakage of mortar. Thin engine oil was applied to the inner surface of the moulds to make for easy removal of the cubes. Batching of all the constituent material was done by weight using a weighing balance of 50kg capacity based on the adapted mix ratios and water cement ratios. A total number of 30 mix ratios were to be used to produce 90 prototype concrete cube. Fifteen (15) out of the 30 mix ratios were as control mix ratios to produce 45 cubes for the conformation of the adequacy of the mixture design for compressive strength of palm nut fibre reinforced concrete. Curing commenced 24hours after moulding. The specimens were removed from the moulds and were placed in clean water for curing. After 28days of curing the specimens were taken out of the curing tank and compressive strength determined. Three concrete cubes were cast for each mixture and cured at 28 days in which the average compressive strength was determined after crushing [26-27].
The compressive strength was then calculated using the formula below:

$$\text{Compressive strength} = \frac{\text{average failure load}(N)}{\text{cross-sectional area}(mm^2)} = \frac{P}{A} \tag{16}$$

IV. RESULTS AND DISCUSSION

4.1 Result of Chemical Analysis of Ordinary Portland cement

TEST REPORT

PROPERTIES TESTED: CEMENT CHEMICAL COMPOSITION-SOUNDNESS, SETTING TIME, CONSISTENCE, LOSS ON IGNITION, INSOLUBLE RESIDUE AND SULPHATE CONTENT

CLIENT NAME/ADDRESS : Alaneme George U (Civil Engineering Department Federal University of Agriculture Umudike)

IDENTIFICATION OF SAMPLE(S) SUBJECTED TO TEST: Dangote Cem I 52.5N

JOB IDENTIFICATION NO : STD/FUAU/2018/01 Date of Sample Analysis: 06-08-2018

PLACE OF TEST(S) (REFERENCE) : ENGINEERING. LAB. ENUGU

RESULT OF ANALYSIS

S/N	TESTED PARAMETER		NIS 444-1:2003 REQUIREMENT (CEM I 52.5N)	TEST RESULT	REMARK
1.	Setting Time [minutes]	Initial	45 minimum	181	Conformed
2.	Soundness [mm]		≤ 10	0.20	Conformed
3.	Loss on Ignition (LOI) [%]		≤ 5.0	4.00	Conformed
4.	Insoluble Residue (IR) [%]		≤ 5.0	2.70	Conformed
5.	Sulphate content (SO_3) [%]		≤ 4.0	2.05	Conformed

Notice: NA denotes Not Applicable, AR denotes Awaiting Result

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Declaration:

Results of tests and measurements contained in this report relate only to the tested items

Results and observations recorded on this page were carried out by me and are correct to the best of my knowledge

Results and observations recorded on this page were checked by me and are correct to the best of my knowledge

STANDARDS ORGANISATION OF NIG. TECHNICAL COORDINATOR

Usar Emmanuel Signature

AUG 2018

SIGN

Fig 2:chemical analysis result of ordinary Portland cement.

4.2 Physical properties of aggregates.

Table 3:Physical properties of aggregates.

Physical and Mechanical Properties	Crushed Stone Aggregate	palm nut fibre	fine aggregates
Maximum size (mm)	12.5	25	4.75
Water absorption (%)	0.22	21.8	-
Specific gravity	2.68	1.24	2.62
Fineness Modulus	6.82	6.57	3.72

4.2.1. Particle Size Distribution of Fine and Coarse Aggregate

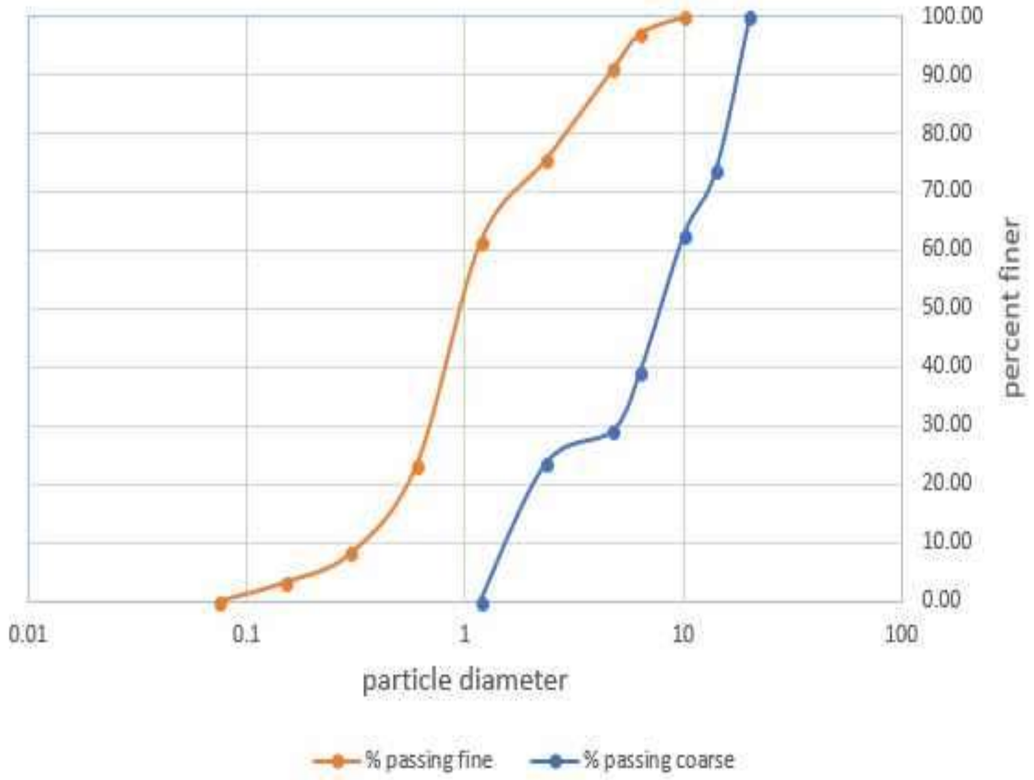

Fig 3: Grain Size Distribution of Studied Materials

4.3 Compressive Strength (Response, Yi)

The results of the compressive strength of the palm nut fibre reinforced concrete (PFRC)was gotten in the laboratory after 28 days curing. The laboratory results is shown in table 4 and table 5 for the control points.

Table 4.The Results of the Compressive Strength (Response, Yi) Based on a 28-Days Strength

Response Symbol	Replicate	Average Weight (Kg)	Volume (m^3)	Average Bulk Density (Kg/m^3)	Crushing Load (N)	Cross Sectional Area (m^2)	Compressive Strength (N/mm^2)	Average Compressive Strength (N/mm^2)
Y1	A	8.20	0.003375	2430.62	608000	22500	27.02	30.39
Y1	B				745000		33.11	
Y1	C				698000		31.02	
Y2	A	7.98	0.003375	2363.46	678000	22500	30.13	28.61
Y2	B				615000		27.33	
Y2	C				638000		28.36	
Y3	A	7.83	0.003375	2320.99	589000	22500	26.18	24.87

Y3	B				564000		25.07	
Y3	C				526000		23.38	
Y4	A	7.59	0.003375	2247.90	429000	22500	19.07	17.25
Y4	B				387400		17.22	
Y4	C				348000		15.47	
Y5	A	7.63	0.003375	2261.73	418000	22500	18.58	18.44
Y5	B				420000		18.67	
Y5	C				407000		18.09	
Y12	A	8.12	0.003375	2406.91	697000	22500	30.98	29.84
Y12	B				626000		27.82	
Y12	C				691000		30.71	
Y13	A	8.03	0.003375	2379.26	634000	22500	28.18	27.41
Y13	B				597000		26.53	
Y13	C				619000		27.51	
Y14	A	7.87	0.003375	2331.85	577000	22500	25.64	25.66
Y14	B				591000		26.27	
Y14	C				564000		25.07	
Y15	A	8.09	0.003375	2396.05	539000	22500	23.96	23.78
Y15	B				548000		24.36	
Y15	C				518000		23.02	
Y23	A	8.03	0.003375	2379.26	728000	22500	32.36	31.53
Y23	B				719000		31.96	
Y23	C				681000		30.27	
Y24	A	7.52	0.003375	2229.14	549000	22500	24.40	24.93
Y24	B				593000		26.36	
Y24	C				541000		24.04	
Y25	A	7.71	0.003375	2283.46	454000	22500	20.18	19.61
Y25	B				403000		17.91	
Y25	C				467000		20.76	
Y34	A	7.97	0.003375	2362.47	569000	22500	25.29	25.67
Y34	B				585000		26.00	
Y34	C				579000		25.73	
Y35	A	8.07	0.003375	2391.11	487000	22500	21.64	20.92
Y35	B				448000		19.91	
Y35	C				477000		21.20	
Y45	A	7.64	0.003375	2262.72	425000	22500	18.89	18.30
Y45	B				402000		17.87	
Y45	C				408000		18.13	

Table 5.The 28th Day Compressive Strength Values and their Corresponding Density for the Control Points.

Response Symbol	Replicate	Average Weight (Kg)	Volume (m^3)	Average Bulk Density Kg/m^3	Crushing Load (N)	Cross Sectional Area (m^2)	Compressive Strength (N/mm^2)	Average Compressive Strength (N/mm^2)
C1	A	7.96	0.003375	2357.53	527000	22500	23.42	22.73
C1	B				513000		22.80	
C1	C				494000		21.96	
C2	A	8.12	0.003375	2406.91	501000	22500	22.27	23.30
C2	B				583000		25.91	
C2	C				489000		21.73	
C3	A	7.78	0.003375	2304.20	423000	22500	18.80	20.98
C3	B				507000		22.53	
C3	C				486000		21.60	
C4	A	8.00	0.003375	2369.38	649000	22500	28.84	27.59
C4	B				621000		27.60	
C4	C				592000		26.31	
C5	A	8.06	0.003375	2387.16	501000	22500	22.27	22.80
C5	B				527000		23.42	
C5	C				511000		22.71	
C12	A	7.94	0.003375	2353.58	606000	22500	26.93	26.22
C12	B				594000		26.40	
C12	C				570000		25.33	
C13	A	8.07	0.003375	2390.12	651000	22500	28.93	27.88
C13	B				604000		26.84	
C13	C				627000		27.87	
C14	A	8.12	0.003375	2405.93	642000	22500	28.53	27.70
C14	B				627000		27.87	
C14	C				601000		26.71	
C15	A	7.91	0.003375	2342.72	477000	22500	21.20	23.42
C15	B				586000		26.04	
C15	C				518000		23.02	
C23	A	7.96	0.003375	2358.52	581000	22500	25.82	26.21
C23	B				579000		25.73	
C23	C				609000		27.07	
C24	A	7.98	0.003375	2365.43	419000	22500	18.62	18.90
C24	B				473000		21.02	
C24	C				384000		17.07	
C25	A	7.85	0.003375	2324.94	409000	22500	18.18	20.58
C25	B				454000		20.18	
C25	C				526000		23.38	
C34	A	7.91	0.003375	2344.69	491000	22500	21.82	20.65
C34	B				487000		21.64	
C34	C				416000		18.49	
C35	A	8.05	0.003375	2385.19	519000	22500	23.07	24.77
C35	B				554000		24.62	

C35	C				599000		26.62	
C45	A	7.84	0.003375	2323.95	489000	22500	21.73	19.76
C45	B				443000		19.69	
C45	C				402000		17.87	

4.4 Regression Equation for Compressive Strength

From eqn. [14], the coefficients of the Scheffe's second degree polynomial were determined as follows;

Table 6. The Coefficients of the Scheffe's Second Degree Polynomial for the compressive Strength.

β1	β2	β3	β4	β5	β12	β13	β14	β15	β23	β24	β25	β34	β35	β45
30.39	28.61	24.87	17.25	18.44	1.36	-0.89	7.37	-2.55	19.14	8.02	-15.64	18.45	-2.96	1.80

Substituting the values of these coefficients into Eqn. [13] yields

$\hat{Y} = 30.39X_1 + 28.61X_2 + 24.87X_3 + 17.25X_4 + 18.44X_5 + 1.36X_1X_2 - 0.89X_1X_3 + 7.37X_1X_4 - 2.55X_1X_5 + 19.14X_2X_3 + 8.02X_2X_4 - 15.64X_2X_5 + 18.45X_3X_4 - 2.96X_3X_5 + 1.80X_4X_5$ (17)

Eqn. [17] is the improved model for the optimization of the compressive strength of palm nut fibre.

4.5 Replication Variance

Mean responses, Y and the variances of replicates Si^2 in Table 6 were obtained from Eqns. Below

$$Y = \frac{\sum_{i=1}^{n} Y_i}{n}; \quad (18)$$

$$Si^2 = \left[\frac{1}{n-1}\right]\left[\sum Y_i^2 - \left[\frac{1}{n(\sum Y_i)^2}\right]\right] \quad (19)$$

Where 1≤ i ≤ n. The eqn. is expanded as follows;

$$Si^2 = \left[\frac{1}{n-1}\right]\left[\sum_{i=1}^{n}[Y_I - Y]^2\right] \quad (20)$$

Where Yi = responses; Y = mean of the responses for each control point; n = number of parallel observations at every point; n – 1 = degree of freedom; Si^2 = variance at each design point.

For all the design points, number of degrees of freedom [28-30],

$$V_e = (\sum N) - 2 \quad (21)$$

$$= 30 - 2 = 28$$

Where N is the number of points

$Sy^2 = 69.61/28 = 2.485901$

Where Si^2 is the variance at each point

Sy = 1.576674

Table 7.Experimental Test Result and the Replication Variance

Response Symbol	Replicate	Response Yi(N/mm2)	$\sum$ yi	Y	$\sum$ yi²	Si²
Y1	A	27.02	91.16	30.39	2788.92	9.57
Y1	B	33.11				
Y1	C	31.02				
Y2	A	30.13	85.82	28.61	2459.17	2.01
Y2	B	27.33				
Y2	C	28.36				
Y3	A	26.18	74.62	24.87	1860.13	1.99
Y3	B	25.07				
Y3	C	23.38				
Y4	A	19.07	51.75	17.25	899.21	3.24
Y4	B	17.22				
Y4	C	15.47				

Y5 Y5 Y5	A B C	18.58 18.67 18.09	55.33	18.44	1020.79	0.10
Y12 Y12 Y12	A B C	30.98 27.82 30.71	89.51	29.84	2676.87	3.06
Y13 Y13 Y13	A B C	28.18 26.53 27.51	82.22	27.41	2254.87	0.68
Y14 Y14 Y14	A B C	25.64 26.27 25.07	76.98	25.66	1975.91	0.36
Y15 Y15 Y15	A B C	23.96 24.36 23.02	71.33	23.78	1697.08	0.47
Y23 Y23 Y23	A B C	32.36 31.96 30.27	94.58	31.53	2984.11	1.23
Y24 Y24 Y24	A B C	24.40 26.36 24.04	74.80	24.93	1868.11	1.55
Y25 Y25 Y25	A B C	20.18 17.91 20.76	58.84	19.61	1158.74	2.26
Y34 Y34 Y34	A B C	25.29 26.00 25.73	77.02	25.67	1977.73	0.13
Y35 Y35 Y35	A B C	21.64 19.91 21.20	62.76	20.92	1314.37	0.81
Y45 Y45 Y45	A B C	18.89 17.87 18.13	54.89	18.30	1004.83	0.28
C1 C1 C1	A B C	23.42 22.80 21.96	68.18	22.73	1550.49	0.54
C2 C2 C2	A B C	22.27 25.91 21.73	69.91	23.30	1639.53	5.17
C3 C3 C3	A B C	18.80 22.53 21.60	62.93	20.98	1327.75	3.77
C4 C4	A B	28.84 27.60	82.76	27.59	2286.04	1.60

C4	C	26.31				
C5 C5 C5	A B C	22.27 23.42 22.71	68.40	22.80	1560.20	0.34
C12 C12 C12	A B C	26.93 26.40 25.33	78.67	26.22	2064.14	0.66
C13 C13 C13	A B C	28.93 26.84 27.87	83.64	27.88	2334.31	1.09
C14 C14 C14	A B C	28.53 27.87 26.71	83.11	27.70	2304.19	0.85
C15 C15 C15	A B C	21.20 26.04 23.02	70.27	23.42	1657.78	5.99
C23 C23 C23	A B C	25.82 25.73 27.07	78.62	26.21	2061.60	0.56
C24 C24 C24	A B C	18.62 21.02 17.07	56.71	18.90	1079.99	3.97
C25 C25 C25	A B C	18.18 20.18 23.38	61.73	20.58	1284.09	6.88
C34 C34 C34	A B C	21.82 21.64 18.49	61.96	20.65	1286.53	3.52
C35 C35 C35	A B C	23.07 24.62 26.62	74.31	24.77	1847.07	3.18
C45 C45 C45	A B C	21.73 19.69 17.87	59.29	19.76	1179.21	3.74

Table 8. Experimental Test Results and Scheffe's Model Test Results

symbols	experimental test results	scheffe model test results
Y1	30.39	30.39
Y2	28.61	28.61
Y3	24.87	24.87
Y4	17.25	17.25
Y5	18.44	18.44
Y12	29.84	29.84
Y13	27.41	27.41
Y14	25.66	25.66
Y15	23.78	23.78
Y23	31.53	31.53
Y24	24.93	24.93
Y25	19.61	19.61
Y34	25.67	25.67

Y35	20.92	20.92
Y45	18.30	18.30
C1	22.73	25.07
C2	23.30	26.04
C3	20.98	22.10
C4	27.59	24.29
C5	22.80	20.89
C12	26.22	23.69
C13	27.88	26.74
C14	27.70	27.30
C15	23.42	23.21
C23	26.21	25.22
C24	18.90	21.04
C25	20.58	22.57
C34	20.65	19.95
C35	24.77	25.20
C45	19.76	23.05

V. Validation and Test of Adequacy ofthe Model

The test of adequacy of the model was done using statistical tool for determining differences among means using hypothesis. The student's t-test and ANOVA method was the statistical tool used; the adequacy of the model was tested against the experimental results of the control points. The predicted values (Y-predicted) for the test control points were obtained by substituting the corresponding values of the pseudo-components X1, X2, X3, X4 and X5 into the improved model equation i.e. Eqn. [15]. These values were compared with the experimental results (Y-observed).

The test for adequacy of the model was done using student's t-test and ANOVA at 95% confidence level on the compressive strength at the control points (that is, C1, C2, C3, C4, C5, C12, C13, C14, C15, C23, C24, C25, C34, C35 and C45). In this test, two hypotheses were set as follows:

5.1 Null Hypothesis

There is no significant difference between the laboratory tests and model predicted strength results.

5.2 Alternative Hypothesis

There is a significant difference between the laboratory test and model predicted strength results.

5.3 Student's t-Test

A two-tail test (inequality) is used to compare the two groups and if t Stat > t Critical two-tail, we reject the null hypothesis. From the result presented in Tables 9, t stat = -0.36331and t critical two-tail = 2.144787so t critical > t stat. Therefore, we accept the null hypothesis.

Table 9.t-Test for the Compressive strength

compressive				
symbols	**lab**	**model**	**lab-model**	**(lab-model)^2**
C1	22.73	25.07	-2.34	5.49
C2	23.30	26.04	-2.74	7.51
C3	20.98	22.10	-1.12	1.25
C4	27.59	24.29	3.30	10.88
C5	22.80	20.89	1.91	3.67
C12	26.22	23.69	2.53	6.41
C13	27.88	26.74	1.15	1.31
C14	27.70	27.30	0.40	0.16
C15	23.42	23.21	0.21	0.04
C23	26.21	25.22	0.99	0.97
C24	18.90	21.04	-2.14	4.56
C25	20.58	22.57	-1.99	3.95
C34	20.65	19.95	0.70	0.49
C35	24.77	25.20	-0.42	0.18
C45	19.76	23.05	-3.29	10.80
Total			**-2.84**	**57.69**

$$t_{stat} = \frac{\sum(\mathbf{lab-model})}{\sqrt{\frac{(15*\sum((\mathbf{lab-model})^2)-(\sum(\mathbf{lab-model}))^{\wedge}2}{(15-1)}}} = \frac{(-2.84)}{\sqrt{\frac{(15*57.69)-(-2.84^2)}{(15-1)}}} = \mathbf{-0.36331}$$

α = 0.05 and 0.025 for to tail (t-distribution table).

$t_{critical}$ = **2.144787**
$t_{stat} < t_{critical}$

5.4 Analysis of Variance

If F > F crit, we reject the null hypothesis of the analysis of variance.From the result presented in Table 11, F = 0.038285 and F crit = 4.195972 so F crit> F Therefore, we do not reject null hypothesis.Therefore, the difference between the experiment result and the model result was not significant. Hence, the model is adequate for use in predicting the probable flexural strength when the mix ratio is known and vice-versa.

Table 10.ANOVA for the Compressive strength

compressive				
symbols	**lab**	**model**	**lab^2**	**model^2**
C1	22.73	25.07	516.47	628.43
C2	23.30	26.04	543.06	678.31
C3	20.98	22.10	440.07	488.25
C4	27.59	24.29	760.94	589.86
C5	22.80	20.89	519.84	436.19
C12	26.22	23.69	687.60	561.19
C13	27.88	26.74	777.38	714.78
C14	27.70	27.30	767.50	745.27
C15	23.42	23.21	548.60	538.82
C23	26.21	25.22	686.83	636.12
C24	18.90	21.04	357.35	442.68
C25	20.58	22.57	423.44	509.22
C34	20.65	19.95	426.50	397.96
C35	24.77	25.20	613.57	634.80
C45	19.76	23.05	390.57	531.28
total	**353.50**	**356.34**	**8459.73**	**8533.16**

N= total scores = 30
K = 2
Df_b =K-1 =1
Df_w = N-K = 28

$$SS_b = \frac{(\Sigma(\mathbf{lab}))^2}{15} + \frac{(\Sigma(\mathbf{model}))^2}{15} - \frac{((\Sigma(\mathbf{lab}))+(\Sigma(\mathbf{model})))^2}{30}$$

$$= \frac{(353.50))^2}{15} + \frac{(356.34))^2}{15} - \frac{((353.50)+(356.34))^2}{30}$$

$$= \mathbf{0.27}$$

$$SS_w = \left(\Sigma(\mathbf{lab}^2\right) + \left(\Sigma(\mathbf{model}^2\right) - \frac{(\Sigma(\mathbf{lab}))^2}{15} + \frac{(\Sigma(\mathbf{model}))^2}{15}$$

$$= 8459.73 + 8533.16 - \frac{(353.50))^2}{15} + \frac{(356.34))^2}{15}$$

$$= \mathbf{197.06}$$

$$MS_b = \frac{SS_b}{df_b} = \frac{0.27}{1} = \mathbf{0.27}$$

$$MS_w = \frac{SS_w}{df_w} = \frac{197.06}{28} = \mathbf{7.04}$$

$$F = \frac{MS_b}{MS_w} = \frac{0.27}{7.04} = \mathbf{0.038285}$$

$F_{critical}$= **4.195972** (F-distribution) [df_{btw} =1 df_{wth} = 28]

Table 11: Anova

Summary						
Groups	**Count**	**Sum**	**Average**	**Variance**		
Lab	15	353.4963	23.56642	9.220254		
Model	15	356.3394	23.75596	4.855368		
ANOVA						
Source of Variation	**SS**	**df**	**MS**	**F**	**P-value**	**F crit**
Between Groups	0.269443	1	0.269443	0.038285	0.846285	4.195972
Within Groups	197.0587	28	7.037811			
Total	197.3281	29				

5.5 Discussion of Results

Using Scheffe's simplex model the values of the compressive strength were obtained. The model gave highest compressive strength of 31.53Nmm^2 corresponding to mix ratio of 0.525**:**1.0**:**1.45**:**1.75**:**0.6 for water, cement, fine and coarse aggregate and palm nut fibre respectively. This further showed that the improved value of compressive strength was achieved by the addition of about 11.3% by weight of the palm nut fibre as a fifth component to the concrete mix with a water-cement ratio of 0.525. The lowest strength was found to be 17.25Nmm^2 corresponding to mix ratio of 0.6**:**1.0**:**2.0**:**2.8**:**1.1.This further showed that the minimum value of compressive strength was achieved by the addition of about 14.67% by weight of the palm nut fibre as a fifth component to the concrete mix with a water-cement ratio of 0.65.The fibre inclusion, in general, significantlyimproved the compressive strength and ductility of matrices. The bond of the natural fibres in composites is very satisfactory; the fibre inclusion greatly enhances the impact strength of composites [31-32].
Using the model, the compressive strength of all points in the simplex can be derived.Fibers are used in concrete to control cracking due to plastic shrinkage and to drying shrinkage. They also reduce the permeability of concrete and thus reduce bleeding of water. Some types of fibers produce greater impact, abrasion and shatter–resistance in concrete [27-28].

VI. CONCLUDING NOTES

- Scheffe's second degree polynomial was used to formulate a model for predicting the compressive strength of Palm nut fibre concrete. This model could predict the compressive strength of the Palm nut fibre concrete cubes if the mix ratios are known and vice versa.
- The adequacy of the model was tested using student's t-test and Fisher's test (ANOVA); the result of the test shows a good correlation between the model and control results.The strengths predicted by the models are in good agreement with the corresponding experimentally observed results.
- Since the fibers were added on volume basis, aspect ratio did not influence the densities of the mixes used. Therefore, the slump values were 10–12 mm and no difficulties were faced during casting of specimens and compaction. Both the slump and density decreased with increase in the percentage of fibers. The fibers play a role in influencing the workability and density of fresh and hardened concrete.
- The optimum attainable compressive strength (response) predicted by the model at the 28th day within the factor space was 31.53Nmm^2 with a mix proportion corresponding to mix ratio 0.525**:**1.0**:**1.45**:**1.75**:**0.6 for water, cement, fine and coarse aggregate and palm nut fibre respectively. The minimum strength was found to be 17.25Nmm^2 corresponding to mix ratio of 0.6**:**1.0**:**1.8**:**2.5**:**1.2.
- There is a saving in cost of when some percentage of palm nut fibre are used and very important in the area of waste management and Concrete made with palm nut fibre are lighter than the normal concrete.

NOTATIONS
q =number of components
k = degree of dimensional space
Xi = proportion of ith components of mixtures
m = degree of the scheffe polynomial
X_1 = fraction of water cement ratio
X_2 = fraction of ordinary Portland cement
X_3 = fraction of fine aggregate
X_4= fraction of coarse aggregate
X_5 = fraction of palm nut fibre
n = degree of polynomial regression
Z = actual components

X = pseudo components
Y_1, Y_2, Y_3, Y_4, Y_5, Y_{12}, Y_{13}, Y_{14}, Y_{15}, Y_{23}, Y_{24}, Y_{25}, Y_{34}, Y_{35}, Y_{45} = responses from treatment mixture proportions
C_1, C_2, C_3, C_4, C_5, C_{12}, C_{13}, C_{14}, C_{15}, C_{23}, C_{24}, C_{25}, C_{34}, C_{35}, C_{45} = responses from control mixture proportions
β_1, β_2, β_3, β_4, β_5, β_{12}, β_{13}, β_{14}, β_{15}, β_{23}, β_{24}, β_{25}, β_{34}, β_{35}, β_{45} = model coefficients
Y = optimized compressive strength of palm nut fibre concrete

MATLAB PROGRAM FOR THE COMPRESSIVE STRENGHT OF PALM NUT FIBRE

```
symsX1X2X3X4X5
Z1=[0.45 1 1.2 1.5 0.3];
Z2=[0.5 1 1.4 1.7 0.5];
Z3=[0.55 1 1.5 1.8 0.7];
Z4=[0.6 1 2 2.8 1.1];
Z5=[0.65 1 1.9 3 2];
A=[Z1' Z2' Z3' Z4' Z5'];
disp('matrix A =');
disp(A)
%PSEUDO COMPONENTS
X12=[0.5 0.5 0 0 0];
X13=[0.5 0 0.5 0 0];
X14=[0.5 0 0 0.5 0];
X15=[0.5 0 0 0 0.5];
X23=[0 0.5 0.5 0 0];
X24=[0 0.5 0 0.5 0];
X25=[0 0.5 0 0 0.5];
X34=[0 0 0.5 0.5 0];
X35=[0 0 0.5 0 0.5];
X45=[0 0 0 0.5 0.5];
%PSEUDO COMPONENTS CONTROL
C1=[0.25 0.25 0.25 0.25 0];
C2=[0.25 0.25 0.25 0 0.25];
C3=[0.25 0.25 0 0.25 0.25];
C4=[0.25 0 0.25 0.25 0.25];
C5=[0 0.25 0.25 0.25 0.25];
C12=[0.2 0.2 0.2 0.2 0.2];
C13=[0.3 0.3 0.3 0.1 0];
C14=[0.3 0.3 0.3 0 0.1];
C15=[0.3 0.3 0 0.3 0.1];
C23=[0.3 0 0.3 0.3 0.1];
C24=[0 0.3 0.3 0.3 0.1];
C25=[0.1 0 0.3 0.3 0.3];
C34=[0.1 0.3 0 0.3 0.3];
C35=[0.1 0.3 0.3 0 0.3];
C45=[0.1 0.3 0.3 0.3 0];
A12=A*X12';
A13=A*X13';
A14=A*X14';
A15=A*X15';
A23=A*X23';
A24=A*X24';
A25=A*X25';
A34=A*X34';
A35=A*X35';
A45=A*X45';
CT1=A*C1';
CT2=A*C2';
CT3=A*C3';
CT4=A*C4';
CT5=A*C5';
CT12=A*C12';
```

```
CT13=A*C13';
CT14=A*C14';
CT15=A*C15';
CT23=A*C23';
CT24=A*C24';
CT25=A*C25';
CT34=A*C34';
CT35=A*C35';
CT45=A*C45';

table=[Z1; Z2; Z3;Z4; Z5; A12';A13';A14';A15';A23';A24';A25';A34';A35';A45'];
disp('5,2 scheffe ratio')
disp(table)

control=[CT1'; CT2'; CT3';CT4'; CT5'; CT12';CT13';CT14';CT15';CT23';CT24';CT25';CT34';CT35';CT45'];
disp('control scheffe ratio')
disp(control)

% RESPONSE (COMPRESSIVE STRENGTH)
Y1=30.39;
Y2=28.61;
Y3=24.87;
Y4=17.25;
Y5=18.44;
Y12=29.84;
Y13=27.41;
Y14=25.66;
Y15=23.78;
Y23=31.53;
Y24=24.93;
Y25=19.61;
Y34=25.67;
Y35=20.92;
Y45=18.30;

%CONTROL RESPONSE
ct1=22.73;
ct2=23.30;
ct3=20.98;
ct4=27.59;
ct5=22.80;
ct12=26.22;
ct13=27.88;
ct14=27.70;
ct15=23.42;
ct23=26.21;
ct24=18.90;
ct25=20.58;
ct34=20.65;
ct35=24.77;
ct45=19.76;

% MODEL RELATIONSHIP
B1=Y1;
B2=Y2;
B3=Y3;
B4=Y4;
B5=Y5;
```

```
B12=4*Y12-2*Y1-2*Y2;
B13=4*Y13-2*Y1-2*Y3;
B14=4*Y14-2*Y1-2*Y4;
B15=4*Y15-2*Y1-2*Y5;
B23=4*Y23-2*Y2-2*Y3;
B24=4*Y24-2*Y2-2*Y4;
B25=4*Y25-2*Y2-2*Y5;
B34=4*Y34-2*Y3-2*Y4;
B35=4*Y35-2*Y3-2*Y5;
B45=4*Y45-2*Y4-2*Y5;
coefficient1=double([B1 B2 B3 B4 B5 B12 B13 B14 B15 B23 B24 B25 B34 B35 B45]');
y=(B1*X1)+(B2*X2)+(B3*X3)+(B4*X4)+(B5*X5)+(B12*X1*X2)+(B13*X1*X3)+(B14*X1*X4)+(B15*X1*
X5)+(B23*X2*X3)+(B24*X2*X4)+(B25*X2*X5)+(B34*X3*X4)+(B35*X3*X5)+(B45*X4*X5);
disp('y =')
disp (y)
disp('coefficient')
disp(coefficient1)
M1=ct1;
M2=ct2;
M3=ct3;
M4=ct4;
M5=ct5;
M12=4*ct12-2*ct1-2*ct2;
M13=4*ct13-2*ct1-2*ct3;
M14=4*ct14-2*ct1-2*ct4;
M15=4*ct15-2*ct1-2*ct5;
M23=4*ct23-2*ct2-2*ct3;
M24=4*ct24-2*ct2-2*ct4;
M25=4*ct25-2*ct2-2*ct5;
M34=4*ct34-2*ct3-2*ct4;
M35=4*ct35-2*ct3-2*ct5;
M45=4*ct45-2*ct4-2*ct5;

coefficient2=double([M1 M2 M3 M4 M5 M12 M13 M14 M15 M23 M24 M25 M34 M35 M45]');
m=(M1*X1)+(M2*X2)+(M3*X3)+(M4*X4)+(M5*X5)+(M12*X1*X2)+(M13*X1*X3)+(M14*X1*X4)+(M15
*X1*X5)+(M23*X2*X3)+(M24*X2*X4)+(M25*X2*X5)+(M34*X3*X4)+(M35*X3*X5)+(M45*X4*X5);
disp('control model =')
disp (m)
disp('coefficient')
disp(coefficient2)
y1=subs(y,[X1 X2 X3 X4 X5],[1 0 0 0 0]);
y2=subs(y,[X1 X2 X3 X4 X5],[0 1 0 0 0]);
y3=subs(y,[X1 X2 X3 X4 X5],[0 0 1 0 0]);
y4=subs(y,[X1 X2 X3 X4 X5],[0 0 0 1 0]);
y5=subs(y,[X1 X2 X3 X4 X5],[0 0 0 0 1]);
y12=subs(y,[X1 X2 X3 X4 X5],X12);
y13=subs(y,[X1 X2 X3 X4 X5],X13);
y14=subs(y,[X1 X2 X3 X4 X5],X14);
y15=subs(y,[X1 X2 X3 X4 X5],X15);
y23=subs(y,[X1 X2 X3 X4 X5],X23);
y24=subs(y,[X1 X2 X3 X4 X5],X24);
y25=subs(y,[X1 X2 X3 X4 X5],X25);
y34=subs(y,[X1 X2 X3 X4 X5],X34);
y35=subs(y,[X1 X2 X3 X4 X5],X35);
y45=subs(y,[X1 X2 X3 X4 X5],X45);

%control equation substitution
m1=subs(m,[X1 X2 X3 X4 X5],C1);
m2=subs(m,[X1 X2 X3 X4 X5],C2);
```

```
m3=subs(m,[X1 X2 X3 X4 X5],C3);
m4=subs(m,[X1 X2 X3 X4 X5],C4);
m5=subs(m,[X1 X2 X3 X4 X5],C5);
m12=subs(m,[X1 X2 X3 X4 X5],C12);
m13=subs(m,[X1 X2 X3 X4 X5],C13);
m14=subs(m,[X1 X2 X3 X4 X5],C14);
m15=subs(m,[X1 X2 X3 X4 X5],C15);
m23=subs(m,[X1 X2 X3 X4 X5],C23);
m24=subs(m,[X1 X2 X3 X4 X5],C24);
m25=subs(m,[X1 X2 X3 X4 X5],C25);
m34=subs(m,[X1 X2 X3 X4 X5],C34);
m35=subs(m,[X1 X2 X3 X4 X5],C35);
m45=subs(m,[X1 X2 X3 X4 X5],C45);

table2=double([y1 y2 y3 y4 y5 y12 y13 y14 y15 y23 y24 y25 y34 y35 y45]');
disp('model response')
disp(table2)

table3=double([m1 m2 m3 m4 m5 m12 m13 m14 m15 m23 m24 m25 m34 m35 m45]');
disp('model control response')
disp(table3)

%trial test run
Test=[0.1 0.2 0.4 0.1 0.2];
double(subs(y,[X1 X2 X3 X4 X5],Test))
```

Trial test ans =
25.7420

Conflict of Interests

There are no recorded conflicts of interests in this research work. We also affirm that the content of this work is original and has followed the journal template.Compliance with Ethical Standards was strictly observed.

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