American Journal of Engineering Research (AJER)
e-ISSN: 2320-0847 p-ISSN : 2320-0936
Volume-8, Issue-6, pp-124-134
www.ajer.org
Research Paper Open Access

# FDS Bias at Tailored Durability Test Specification of Dynamically Excited Components under Typical Off-Road Vehicle Mission

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***ABSTRACT :*** *Tailored bench tests are vibration experiments specifically developed for a product in order to reproduce the severity of a customized application, generally within an accelerated time.*
*When structures are dynamically excited, the response is extremely sensitive to input(s). Tailored test parameters, (a) input degree of freedom, (b) magnitude, (c) frequency content and (d) exposure time, must be determined not only from mission characteristics, but also considering component's modal properties.*
*Fatigue Damage Spectrum (FDS) index has been used to promptly quantify mission severity. It has a straightforward calculation since it is based solely in input environment characteristics. Structures dynamics is represented by a single degree of freedom (SDoF) system of variable natural frequency.*
*There are however, two questionable assumptions during PSD computation from FDS: (a) election among SDoF responses, displacement or velocity, as stress proportional parameter and (b) flat frequency, white noise base driven acceleration.*
*Herein, the durability of a cantilever beam specimen is investigated under a typical vehicle off-road application and under its correspondent synthetized tailored test. Numerical results demonstrates that tailored test damage is overrated during PSD specification. Therefore, in order to achieve the reference damage and validate the structures durability performance, exposure time should be increased.*
***KEYWORDS*** *Durability; Tailored Test; FDS; PSD; Dynamic Response.*

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Date of Submission: 28-05-2019 Date of acceptance: 10-06-2019

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## I. INTRODUCTION

Fatigue is responsible for around 90% of all mechanical service failures [1]. It is a localized cyclic process that induce cumulative material damage, resulting in crack nucleation, propagation, and final structure fracture. Naturally, durability is a customer perceived quality aspect. Automotive industry must therefore assure reliable methods for development and validation of its products. Test of dynamically excited components are particularly challenging since relative small input changes magnitude and/or degree or freed could lead to decisive modification in structures' response [2].

According to Wang et al. [3], random vibration test is considered the most effective one to validate component's durability with the concern of resonant fatigue. There are four key parameters for tailored test specification: 1. degree(s) of freedom (DoF), 2. magnitude, 3. frequency content and 4. exposure time (test duration). Concretely though, ground vehicles application environment is extremely vast. Track irregularities and driving maneuvers are widely variable, abruptly modifying input loads and imposing outlier events at an unpredictable incidence. Characterize an off-road mission by directly calculating its PSD, results in a drastically reduced spectrum, since high amplitude outliers' contribution is averaged over windowing process, traditionally adopted and recommended [4] to minimize leakage error during frequency domain transformation.

Inspired by Biot's [5] earthquake researches, Lalanne [6] proposed an ingenious severity index called Fatigue Damage Spectrum (FDS). It helps to assess three important aspects of mission severity. First, a single degree system (SDoF) is used to predict dynamic response. Secondly, the entire time history response is considered for damage accumulation prediction. Therefore, even if sporadic, high amplitude outliers' contribution is definitively registered, due to logarithmic relationship between damage and cycle amplitude. And third, it has the capacity to quantify damage potential to different structures attached to a common base, even if they have distinct dynamic properties. Take for instance the components attached to a truck's chassis: battery case, fuel tanks, mudguards, radiator, bumper, electronic controlling units, pneumatic pipeline. They have very different masses and stiffness. Therefore, distinct dynamic behavior. Experimentally measure each component's response, make the task almost unfeasible due to resources (equipment and time) limitation. Significant effort may therefore be saved if, instead of multiple dedicated instrumentation, durability test could be specified only monitoring their common input base.

Despite recent durability studies have been based on FDS [7,8], McNeill [9] showed that two questionable assumptions are required to determine tailored test PSD from FDS: a. election among SDoF responses, displacement or velocity, as stress proportional parameter to calculate damage [3,8,10–17]; b. the input is assumed flat over frequency range, likewise white noise excitation environment. Such condition is only rarely empirically satisfied. Herein it is investigated the correlation between durability results of a cantilever beam specimen subject to a typical off-road vehicle mission and its relative synthetized tailored test. Finite elements structural analysis (FEA) was combined to ordinary FDS approach in order to correct errors from the quoted theoretical assumptions.

## II. VEHICLE OFF-ROAD LOAD ENVIRONMENT

Real acceleration, imposed by off-road vehicle application, was considered as reference validation mission. Accelerometer (A1) was placed at truck's rear axle (vehicle unsprung mass) to record wheel hub typical mission profile. Additional sensors (A2 and A3) were also incorporated to monitor structure's response for numerical model correlation and validation. The instrumentation points are illustrated at Fig. 1.

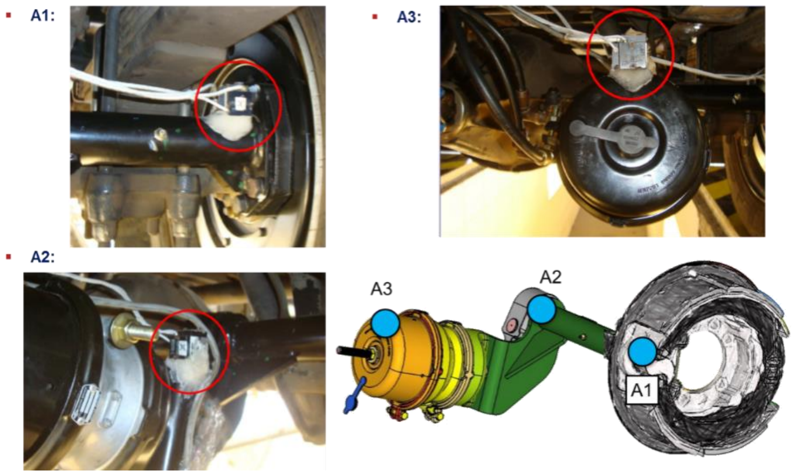


**Fig.1. Pneumatic brake chamber support crack and acceleration measurement points**

Off-road is a worldwide ordinary application for ground vehicles. It is a chaotic, unpredictable environment composed by a widely variable obstacles, mixing random (irregular track) and deterministic events such as: pot holes, ribs and vehicle chassis torsion. Each one contributes for damage accumulation with its own peculiarity and incidence frequency. The recorded data was resampled and scaled to match mechanical resistance of the studied essay specimen, a thin cantilever beam. The time history and its spectrum content is presented at Fig. 2. The frequency domain transformation was performed using Hamming windows with 5 different sizes and 50% of overlap. Table 1 summarizes the principal statistical parameters to characterize input mission.

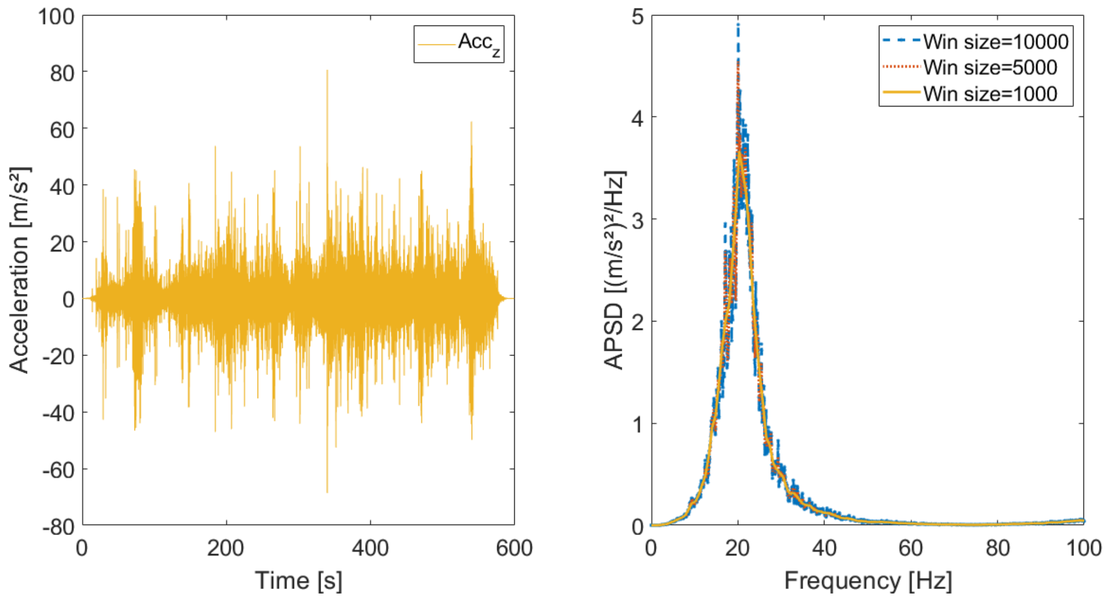


**Fig.2. Typical vehicle off-road mission: time history and spectrum**

**Table 1. Principal statistics: reference mission input accelerations**

| | **Acc Z** |
|---|---|
| Max [m/s²] | 80.6 |
| Min[m/s²] | -68.6 |
| RMS[m/s²] | 6.08 |
| Kurtosis | 11.1 |
| Skewness | 7.98e-02 |
| Crest | 13.3 |

Clearly, the mission is not flat over considered frequency range. As typical in vehicle data, the frequency spectrum is dominated by sprung/unprung natural frequencies. Moreover, considering three different windows sizes: 1000, 5000 and 10000 points, Fig. 2, it was possible to notice that the flatness assumption is not even valid within half power band width.

Inspection of time history principal statistics, kurtosis and crest at Table 1, also evince that outliers events are mixed with random track inputs. Crest factor is the ratio between signal's absolute maximum peak and its RMS (equivalent to standard deviation, since it oscillates around zero). Kurtosis is a descriptor of the shape of probability distribution of a signal. The kurtosis of any univariate normal distribution is 3 [18]. Distributions with kurtosis less than 3 are said to be platykurtic, it means the distribution produces fewer and less extreme events than normal Gaussian distribution does. On contrary, distributions with kurtosis greater than 3 are said to be leptokurtic. This is indeed the case of the measured off-road mission, where there are peaks significantly higher (crest=10.4 – 16.2) than predicted limits for a normal distribution data. For instance, the peak response for a purely random Gaussian excitation can be approximately computed by multiplying the RMS response by $\sqrt{2 \cdot \ln(f_n \cdot T)}$ [9], therefore crest would be 4.5 (considering a signal of 600s and $f_n = 50$Hz. Which is a value approximately 3 times lower than recorded level. As second exercise, once again assuming Gaussian distribution, as described by equation 1, the acceleration peak equal to or higher than 80.6m/s², maximum value of input reference mission, Table 1, has the insignificant probability of 2.02e-38%.

$$Normal\ pdf(x) = \frac{1}{SD \cdot \sqrt{2\pi}} \cdot e^{\frac{-(x-\mu)^2}{2 \cdot SD^2}} \qquad (1)$$

## III. DURABILITY SPECIMEN: CANTILEVER BEAM

It's a thin beam of 1.0mm thickness, presented at Fig. 3, made of SAE 1020, with different masses at each tip and anchored by asymmetric point to impose distinct dynamic characteristics to longer and shorter spans.

Two beam slices, 30x30mm, were bolted at shorter span tip. At the longer span, the only added weight was the M6 bolt and nut, totalizing 7.05e-3kg.

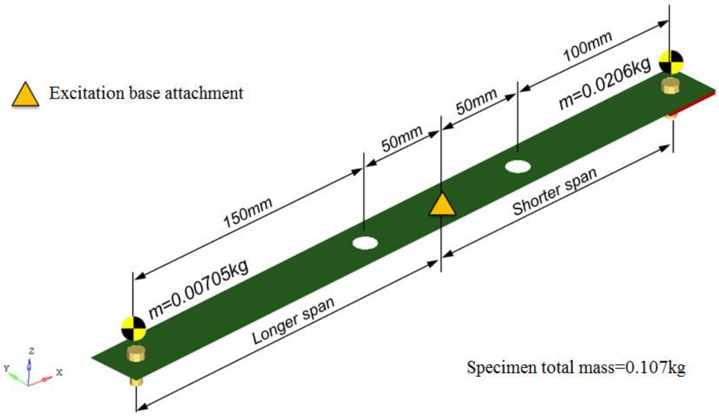


**Fig. 3. Cantilever beam specimen**

Fig. 4 describes the considered specimen's material SN curve. It was considered mechanical properties defined in FKM Guideline [19] for C22 steel at normalized condition, which has similar ultimate stress (430MPa) to essay's material.

**SAE 1020 - Mechanical Properties**

| | |
|---|---|
| Tensile Strength ($\sigma_{ult}$) [MPa] | 430 |
| Yield Stress [MPa] | 240 |
| Ultimate Elongation [%] | 23 |
| Young's Modulus [GPa] | 210 |
| Poisson's Ratio | 0.3 |
| Density [kg/m$^3$] | 7900 |
| Fatigue strength exponent (b) | -0.2 |
| Stress range intercept (SR1) [MPa] | 6181 |

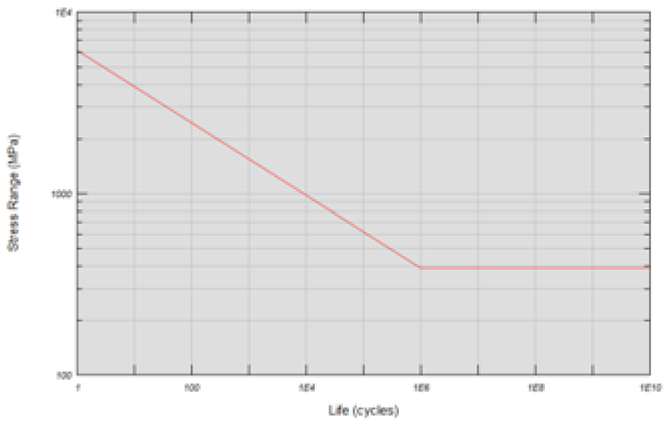


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**Fig. 4. SAE 1020 mechanical properties**

Stress concentration holes were placed equidistant from input point and are subjected to distinct dynamic conditions: lighter mass in a longer span opposing heavier mass in a shorter span.

## IV. DURABILITY SPECIMEN: CANTILEVER BEAM

Dynamic response is strictly related with structure's modal parameters. Therefore, it is essential to reliably determine its mode shapes and natural frequencies. A FE model was prepared mostly with 2D first order shell elements. The bolts at either tips were represented by first order 3D hexahedron and triangular prism elements. The first two modal modes are presented at Fig. 5 and Fig. 6.

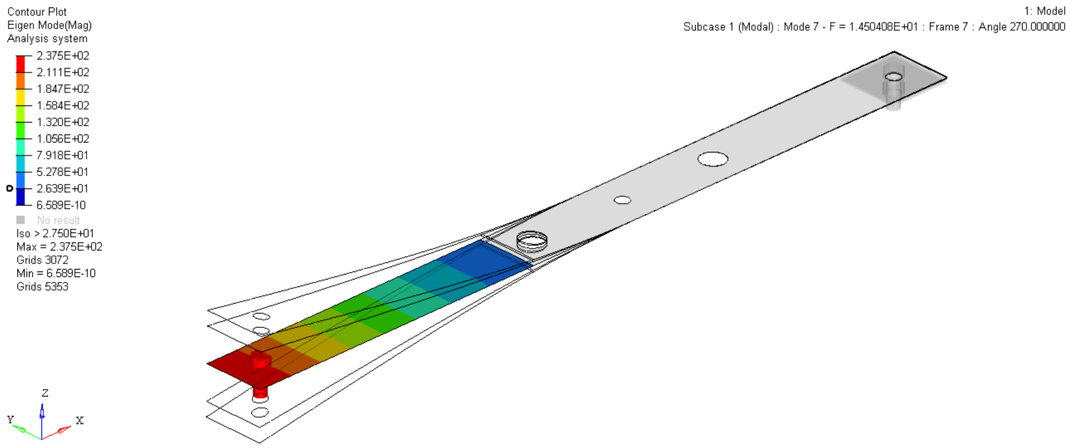


**Fig. 5. 1st Mode shape: Longer span bending @ 14.5Hz**

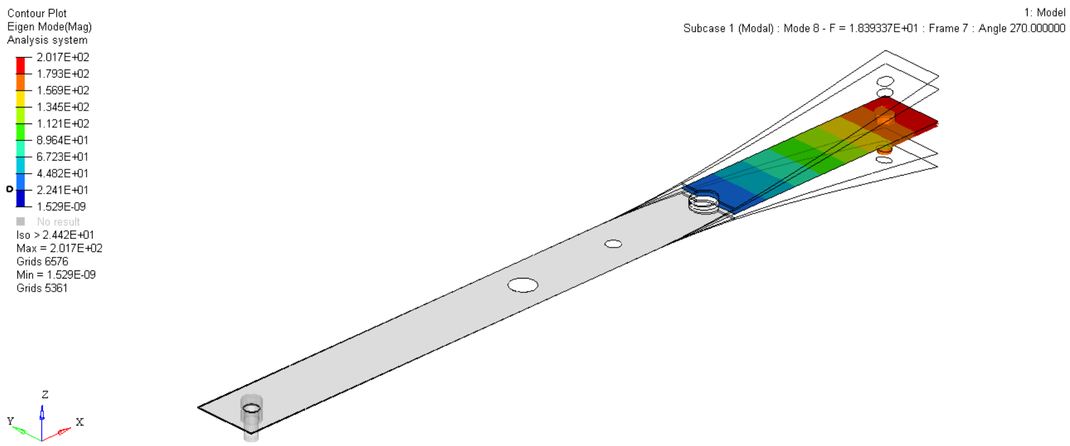


**Fig. 6. 2nd Mode shape: Shorter span bending @ 18.4Hz**

Modal analysis also provide the sensibility of each mode shape to inputs DoFs. Fig. 7 shows the reaction forces at anchorage point of the two principal mode shapes. Practically, both are excited exclusive by vertical (z) input.

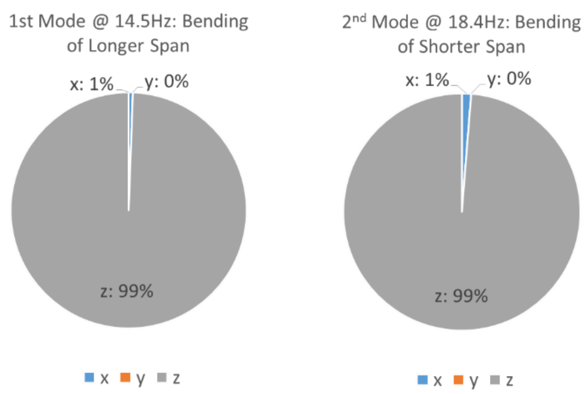


**Fig. 7. Reaction forces of Cantilever beam a) 1st normal mode: 14.5Hz b) 2nd normal mode: 18.4Hz**

## V. THE FATIGUE DAMAGE SPECTRUM

The Fatigue Damage Spectrum (FDS) index is computed based on dynamic response of a base excited SDoF system [20]. In order to predict damage potential of a given test environment, SDoF's natural frequency is iteratively varied to cover the entire frequency range of interest. Damage is then accumulated based on response's time history. However, since SDoF is constituted by a rigid mass, there is no stresses or strains to really compute damage from SN or Epsilon-N fatigue approaches. Controversially, one of the available time history response, displacement or velocity, must be elect as stress proportional parameter to proceed with severity prediction. Lalanne [6], Lee [17], Cho [7] and Halfpenny [20] adopt displacement for FDS computation. While McNeil [9] and other authors [12–14,21], use velocity instead. The only consensus is that acceleration is not appropriate for damage level prediction.

Efficiency to deal with digital data from empirical acquisition is determinant for FDS approach feasibility. Several loops are necessary to sweep along interested frequency range for each SDoF natural frequency ($f_n$) iteration. Consider, for instance, a test made with a sinusoidal input with 1,000,000 points. Matlab's built-in "ode45" function took, on average of 100 runs, 20.5 seconds in a computer with 8 cpus and 16 GB of RAM memory to find SDoF velocity response. Using FIR digital filter transfer function [9] to iteratively solve the same situation, only 0.83 seconds was required. The computational cost was decreased by almost 25 times. Such efficiency is crucial to deal with long and irregular inputs as it is the case for grounded vehicles environment. The relatively small, 600 seconds (10 minutes), off-road data acquisition herein considered is constituted by 360,000 points, since the considered sample frequency was 600Hz.

The transfer function between base acceleration and velocity response is given by equation 2 with a=1.

$$H(s) = \frac{{\omega_n}^a}{s^2+2\,\zeta\,\omega_n\,s+{\omega_n}^2} \qquad (2)$$

The correspondent FIR proposed by McNeil [9] and herein adopted for FDS calculation is presented at equation 3.

$$\widetilde{H}(z) = \frac{b_0+b_1 z^{-1}+b_2 z^{-2}}{a_0+a_1 z^{-1}+a_2 z^{-2}} \qquad (3)$$

where:

$\omega_d = \omega_n\sqrt{1-\zeta^2}$

$a_0 = 1$

$a_1 = -2C$

$a_2 = E^2$

$$b_0 = \frac{f_s}{{\omega_n}^2}\left[2\,\zeta\,(C-1) + \frac{2\,\zeta^2-1}{\sqrt{1-\zeta^2}}\,S + T_s\omega_n\right]$$

$$b_1 = \frac{f_s}{{\omega_n}^2}\left[-2\,C\,T_s\omega_n + 2\,\zeta\,(1-E^2) - \frac{2\,(2\,\zeta^2-1)}{\sqrt{1-\zeta^2}}\,S\right]$$

$$b_2 = \frac{f_s}{{\omega_n}^2}\left[E^2(T_s\omega_n + 2\,\zeta) - 2\,\zeta\,C + \frac{2\,\zeta^2-1}{\sqrt{1-\zeta^2}}\,S\right]$$

$E = e^{-\zeta\,\omega_n T_s}$

$K = \omega_d/f_s$

$C = E\cos(K)$

$S = E\sin(K)$

$f_s$ = sample frequency

Note that to determine SDoF output displacements instead of velocity, it is only necessary to divide transfer function (H) by $\omega_n$. Analogously, to compute output accelerations, function H must be multiplied by $\omega_n$. Digital FIR filters become unstable if input signal frequency content approximates sampling Nyquist frequency. Therefore, to avoid instability, input signal is up-sampled to at least 10 times the maximum natural frequency of the analyzed system.

Once SDoF response is determined, FDS calculation proceed with rainflow counting, using WAFO algorithm [22] and then damage accumulation by combining Palmgren [23] and Miner [24] rule with SN material law.

Once FDS is determined, an associated synthetized tailored test input could be determined back using equations 4 and 5. However, equation 5 assume the response is dominated by SDoF of natural frequency $f_n$ uniformly excited over the entire frequency range, id. est. flat white noise base driven acceleration.

$$\sigma_{pv}{}^{2} = \frac{1}{2}\left[\frac{FDS(f)}{f_n \cdot T \cdot \Gamma(1+b/2)}\right]^{\frac{2}{b}} \quad (4)$$

$$\sigma_{pv} \approx \sqrt{\frac{PSD(f)\,Q}{8\,\pi\,f_n}} \rightarrow PSD(f) = \frac{8\cdot\pi\cdot f_n}{Q}\sigma_{pv}{}^{2} \quad (5)$$

Where:

T: Exposure time
Γ: Gamma function
$\sigma_{pv}$: stress proportional parameter (SDoF response velocity)
b: material Basquin coefficient
$f_n$: natural frequency
Q: quality factor = $1/(2 \cdot \zeta)$
ζ: modal damping ratio

If the flatness assumption is not valid within the interested input frequency, the integration can be iteratively performed to calculate $\sigma_{pv}$ taking into account PSD shape, Equation 6.

$$\sigma_{pv} = \sqrt{\sum_{i=1}^{N} \frac{\frac{1}{(2\,\pi\,f_n)^2}}{[1-r_i{}^2]^2+\left[\frac{r_i}{Q}\right]^2} PSD(f_i)\Delta f_i} \quad (6)$$

Where:

$f_i$: ith frequency sample of input PSD
$r_i = f_i/f_n$

However, only Equation 5 is invertible [9]. Which means that it is only possible to directly determine the input PSD from the FDS assuming flatness input.

If input is already given by a PSD, it is possible to determine FDS directly from frequency domain. Therefore, in this case, it is not necessary to use the presented FIR, just Equation 7.

$$FDS(f_n) = \frac{f_n\,T}{c} k^b \left[2\,\sigma_{pv}{}^2\right]^{b/2} \Gamma\left(1+\frac{b}{2}\right) \quad (7)$$

where:

c: material SN proportionality constant
k: Proportional constant between stress and pseudo velocity

Constants c and k can be set to unit value since proportionality does not affect the use of the FDS for environment comparison.

## VI. TAILORED DURABILITY TEST

In order to understand the consequences of FDS theoretical assumptions, (a) election among SDoF responses, displacement or velocity, as stress proportional parameter and (b) input flatness over frequency range, in tailored durability test, the proposed cantilever beam specimen was numerically subjected to both inputs: original measured acceleration and synthetized PSD derived from environment FDS.

Off-Road FDS was computed from 1 to 100Hz, refer to Fig. 8.

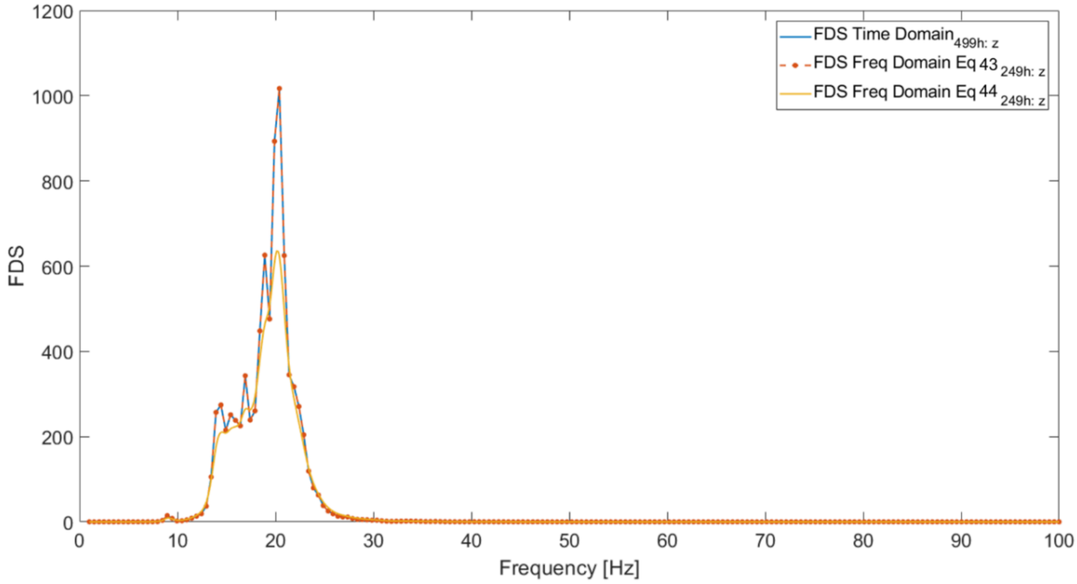


**Fig. 8. Mission Fatigue Damage Spectrum (FDS) calculated at time and frequency domains**

The synthetized PSD, presented at Fig. 9, was determined throughout equations 4 and 5 aiming an acceleration factor of 2. Which means, achieve equivalent damage level to original mission within half of the time.

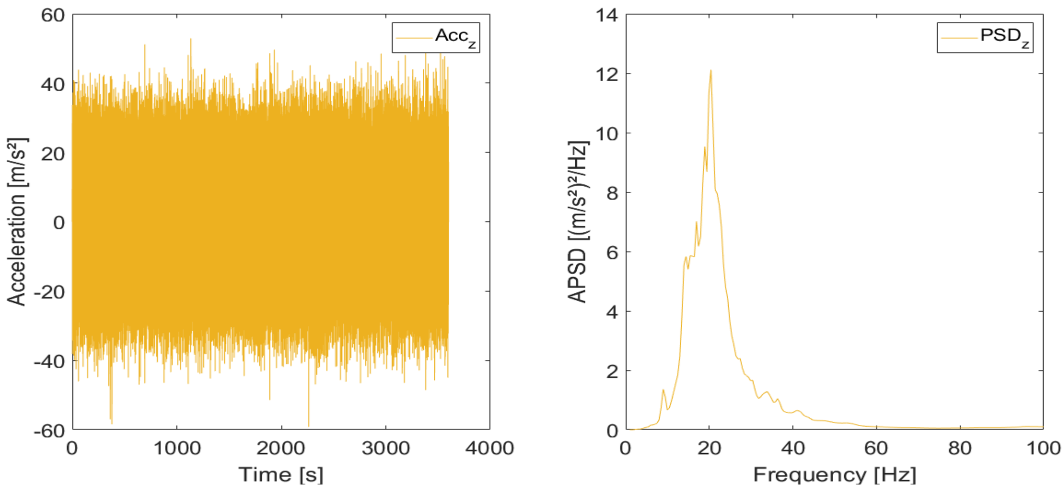


**Fig. 9. Tailored test input PSD: derived from mission FDS throughout Equations 5 and 6**

Since PSD is inherently random, signal peaks might vary from sample to sample. However, as a stationary signal, its principal statistics characteristics, presented at Table 2, are preserved. The peak response for Gaussian random excitation can be approximated by multiplying the RMS response by $\sqrt{2 \cdot \ln(f \cdot T)}$ [9]. Therefore, the estimated crest for a signal of 3600s and $f = 50$Hz is 4.9, very much aligned with the value calculated from time history presented below.

**Table 2. Principal statistics: cantilever beam synthetized input accelerations**

| | **Acc Z** |
|---|---|
| Max [m/s²] | 52.1 |
| Min [m/s²] | -53.9 |
| RMS [m/s²] | 10.9 |
| Kurtosis | 3.01 |
| Skewness | -5.43E-04 |
| Crest | 4.92 |

Once input PSD had been determined, equations 7 was used to calculate back FDS, considering both $\sigma_{pv}$ equations 5 and 6. Obviously, FDS calculated from frequency domain using equation 5 is identical to the one calculated from time domain. Input PSD was actually determined throughout the same equation.

However, from Fig. 8, it is possible to notice a significant difference in magnitude when FDS was computed from frequency domain using equations 6. Notably, when the FDS is computed without flatness assumption, considering integration of the entire PDS profile, the expected damage was not achieved within the proposed time. Equation 5 overrates the damage content of PSD due to flatness assumption, resulting in an input signal with less magnitude than what would be required to impose the reference damage within intended time.

Proceeding with durability prediction, but now at the actual evaluated structure instead of SDoF estimation, Fig. 10 compares the accumulated damages during original vehicle mission and the synthetized input. The damage was monitored at the critical specimen spot: concentration hole of the shorter span. Since real input "miss" spectrum content, id est, it is not a white noise environment, the derived tailored test ended up imposing 15% less damage than targeted value.

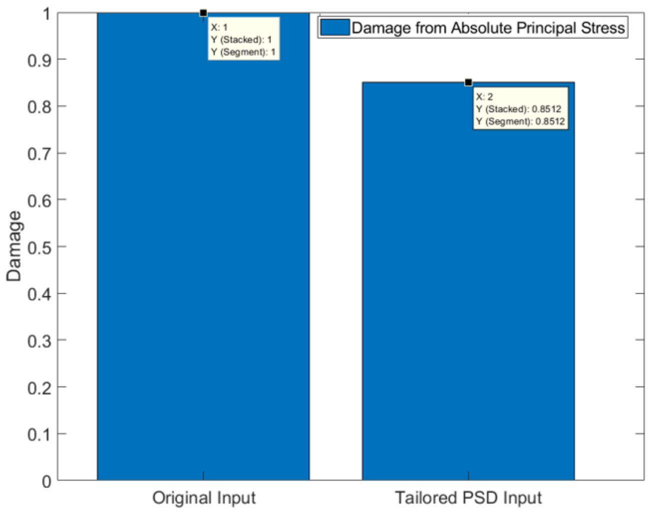


**Fig. 10. Damage comparison between original and synthetized inputs at stress concentration hole of the shorter span**

Therefore, in order to achieve a reliable tailored test, exposure time needed to be extended to offset test severity biased error from FDS. While under application condition it is expected a durability life of 500hours, at tailored synthetized condition, instead of the desired acceleration factor of 2, required 293 hours to achieve the same damage level. Fig. 11 and Fig. 12 present the numerically predicted durability life for both evaluated test environments.

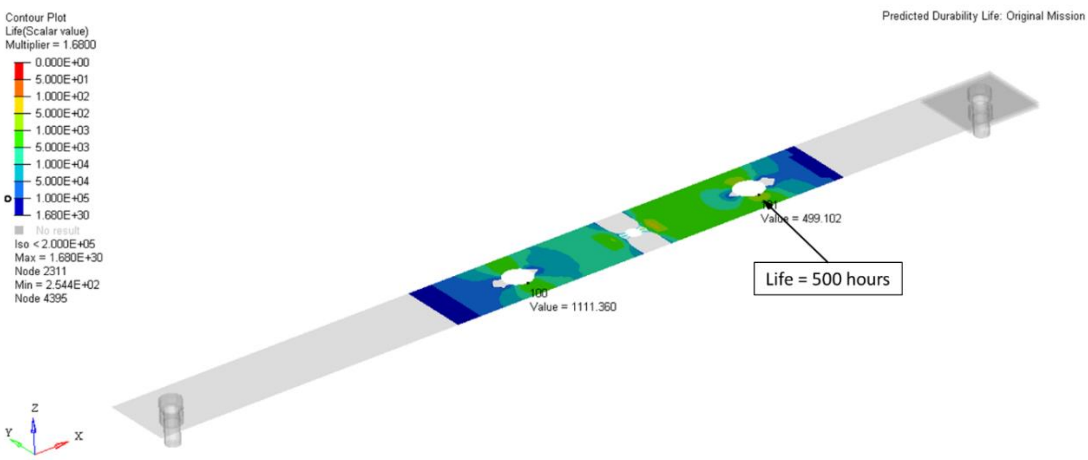

**Fig. 11. Predicted durability life under original mission input: 500hours**

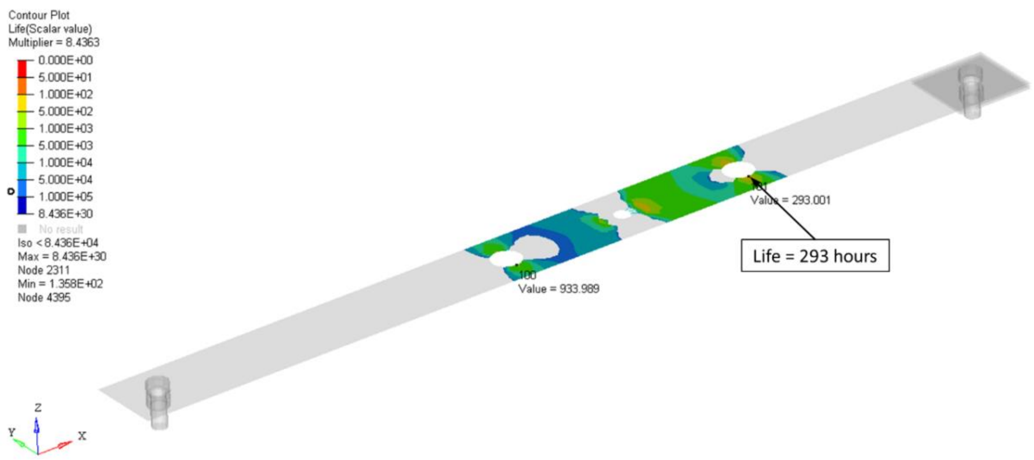


**Fig. 12. Predicted durability life under tailored accelerated synthetized test: 293hours**

For the longer span, the acceleration factor was even lower, -16%. Around the natural frequency of the longer span, 14.5Hz, the flatness assumption holds even weaker. From Fig. 2, it is possible to notice that off road mission spectrum content is in fact an accentuated increasing slope around such frequency.

## VII. CONCLUSIONS

FDS plays a central role in mission severity characterization and tailored test specification for dynamically excited components. However, it requires questionable assumptions that induce errors that, even though might be within expected durability life variation, should be quantified and somehow corrected to enhance validation test correlation.

Herein, numerical FE structural analysis and durability prediction were performed to evince that the achieved acceleration factor on the tailored durability test is significantly affected by reference mission frequency spectrum content.

In the investigated uniaxial case, even numerically preserving every durability parameter, material property and surface finishing, resulted in an underrated synthetized tailored test, missing 15% of the targeted damage.

FDS is also limited to properly address multiaxial load environment. When it is the case, the stresses' tensor are the resultant of the multiple dynamic response of each input DoF. Since FDS is based on SDoF systems, each input DoF must be individually assessed. The superposed FDS, would never represent the damage computed from the resultant multiaxial stress tensor, since damage is logarithmic related to time history amplitudes. The linear requirement necessary for superposition is violated. Therefore, since in practice, structures are generally simultaneously excited, a more drastic correction would be required. Such aspect should be further investigated in future works.

The validation time was not reduced by half as intended. But, even requiring more time than expected, tailored durability test optimized validation time in 1.7. Mission leptokurtic characteristic was essential to achieve such optimization. The lower kurtosis of tailored test indicates that extreme events that were scarce under original mission, became more frequent at the synthetized PSD. Instead, if input signal was platykurtic, a longer time would be required to match reference damage, since the peaks incidence would be higher at original mission, turning validation test optimization unfeasible.

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Ismael Coutinho" FDS Bias at Tailored Durability Test Specification of Dynamically Excited Components under Typical Off-Road Vehicle Mission" American Journal of Engineering Research (AJER), vol.8, no.06, 2019, pp.124-134