American Journal of Engineering Research (AJER)
e-ISSN: 2320-0847 p-ISSN : 2320-0936
Volume-8, Issue-6, pp-08-13
www.ajer.org
Research Paper Open Access

Modified Chain-Ladder Reserve Estimates: Inclusion of a Tail Factor Estimatied using Sherman's Method

Media Rahmawati[1], Isnani Darti[2], Marjono[3]
[1,2,3]*(Department of Mathematics, Faculty of Science, Brawijaya University, Malang, Indonesia)*
Corresponding Author: Media Rahmawati

ABSTRACT : *Claim reserves are future obligations of a non-life insurance company classified as liabilities. The outstanding claim reserve is unknown until the company settles it. The company needs to estimate the total amount of fund in order to meet its liability. A claim that occurred but has not yet settled or reported (IBNR) is total debt owned by the company to registered claimants who have covered loss but it has not reported to the company yet. The accuracy of IBNR value estimation plays an important role because it affects company's stabilities in some aspects. We modified the basic CL method by considering inclusion of LDTF. We used curve-fitting Sherman's method to predict LDTF. The goal of this study is to forecast the claim IBNR estimation using our modified Chain-Ladder-Sherman's method (MCL-S) and then we calculate its mean squared error using elaborated MCL formula to see its perform. Result shows that MCL-S method produced higher standard error for every accident year.*

KEYWORDS *IBNR, Modified Chain-Ladder-Sherman's method, standard error.*

I. INTRODUCTION

The main duty of insurance company, either general or life, is to provide protection against uncertainty concerning loss to their clients. Total loss or total claim, in general insurance, is considered as sum of reported claim and claim that incurred but have not reported called IBNR. The reported claim known by the insurance company, but IBNR needs to be forecasted by actuaries. The process of estimating IBNR called loss reserving.

Loss reserving is one of the main activities in insurance that plays an important role. The accuracy of IBNR estimation reserve affects three main aspects in company: management internal, investor, and regulator [1]. Inaccurate estimation may leads to misstated balance sheet. Loss reserving for general insurance usually based on aggregated data model in a run-off triangle. In practice, there is a long tradition of actuaries calculating reserve estimates according to deterministic methods without explicitly referenced to a stochastic model. The most widely known of claim reserving method is basic Chain-Ladder method (CL) [2], and Bornhuetter and Ferguson method (BF) [3], but BF is limited to work optimally only in small frequent data claim with high amount at each reported claim. Researchers have been improving CL method from year to year significantly. The basic CL assumption is that there are development factors $\mathrm{f}_1, \ldots, \mathrm{f}_{\mathrm{I}-1} > 0$ [2]. Mostly actuaries assumed that claim IBNR is fully developed in the latest year I, but it is not true because development factor hasn't been closed enough to 1.00. There are possibilities for claims occurred after the eldest maturity in a given run-off triangle.

CAS Tail Factor Working Party in 2017, discussed about the inclusion of Loss Development Tail Factors (LDTF) in forecasting IBNR [4]. Predicting LDTF, which is an average of age-to-age factors, is the most important process in forecasting total claims for every development year (DY) other than that, they formed a portion of the loss development to each of accident years (AY). We use LDTF to control the development of estimated claim IBNR in below diagonal run-off triangle.

In this paper, we are going to modify basic CL method by including tail factors. We use the class of Sherman's curve-fitting method as our forecasting technique to estimate LDTF [4], and then use the result to squared the triangle. We also modified our run-off triangle to adapt Mack's Chain-Ladder method, so that it will be possible to see the performance of our model. In the next section, we will calculate the Mean Squared Error (MSE). The standard error of Sherman's ultimate claim formula will be have done by elaborating the formula of MSE given by [5]. Actuary normally uses MSE to see the performance estimation of total ultimate claim.

II. MODEL ASSUMPTION

We follow the notation as in [2]. General form of cumulative run-off triangle is presented bellow.

Table 1: Cumulative Run-Off Triangle

AY	DY						
	1	2	$\cdots$	k	$\cdots$	$I-1$	I
1	$C_{1,1}$	$C_{1,2}$	$\cdots$	$C_{1,k}$	$\cdots$	$C_{1,I-1}$	$C_{1,I}$
2	$C_{2,1}$	$C_{2,2}$	$\cdots$	$C_{2,k}$	$\cdots$	$C_{2,I-1}$	
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$		
I	$C_{i,1}$	$C_{2,2}$	$\cdots$	$C_{i,k}$			
$\vdots$	$\vdots$	$\vdots$	$\vdots$				
$I-1$	$C_{i-1,1}$	$C_{i-1,2}$					
I	$C_{I,1}$						

We use cumulative run-off triangle sized $I \times I$ with considering indexes $1 \le i \le I$ for accident years (AY) and indexes $0 \le k \le I$ for development years (DY) under consideration. $C_{i,k}$ denotes the cumulative payment for claim with AY i and DY k. We assume that $C_{i,k}$ where $i \in \{1,2,\dots,I\}$ and $k \in \{1,2,\dots,I-i+1\}$ have been observed and $C_{i,k}$ where $i \in \{1,2,\dots,I\}$ and $k \in \{I-i+1,\dots,I\}$ are claims that we are going to predict. The payment for claim used in this paper is restricted to be non-negative. The basic chain ladder algorithm consists of the stepwise prediction rule

$$\hat{f}_k = \frac{\hat{C}_{i,k+1}}{\hat{C}_{i,k}}, \tag{1}$$

for $i \in \{1,2,\dots,I\}$ and $k \in \{1,2,\dots,I-1\}$ and remember that $\hat{C}_{i,I+1-i} = C_{i,I+1-i}$.

III. INCLUSSION OF SHERMAN'S METHOD

We extended eq. (1) to include tail factor. Actuaries use tail factors to estimate the additional development that will occur after $k = I$ or after the eldest maturity in a given loss development triangle. Along with survey that had been done and published in [4] several method has been purposed, but among all those methods, we tend to choose Sherman's curve-fitting due to their simplicity and good perform.

The basic idea of this method is to explore some relationship between the development factors at various DY, and use that relationship as an main assumption to fit a curve to the development factors. Then the projected development factors in DY, which covered by the tail factor, can be multiplied together to provide an estimate of the tail factor.

Sherman's method use inverse power curve $1 + ak^b$, where k represents DY in $\hat{f}_k$. Let

$$\hat{f}_k = 1 + ak^b, \tag{2}$$

we get

$$\ln(\hat{f}_k - 1) = \ln a + b\ln(k) = \hat{f}_k^* \tag{3}$$

for $k \in \{1,2,\dots,I-1\}$. Where b is a slope of regression equation and a refers to exponential of intercept form. In this paper, we denote the extrapolation of $\hat{f}_k$ by $\hat{f}_k^*$.

From eq. (3), we will forecast LDTF using

$$\hat{f}_k^{exp} = \exp \hat{f}_k^* + 1, \tag{4}$$

$$\hat{f}^{tail} = \prod_{k=1}^{n+1} \hat{f}_k^{exp} \tag{5}$$

$n \in \{1,\dots,100\}$. The number n is desired number of extrapolation iteration. For instance, we choose $n = k - I + 1$ to calculate $\hat{f}_{56}$. The basic idea of our MCL-S is that we differentiate our development factors into two before calculating the MSE and standard error s.e..

$$\hat{f}_k = \frac{\sum_{j=1}^{I-k} C_{j,k+1}}{\sum_{j=1}^{I-k} C_{j,k}}, \qquad \text{for } 1 \le k \le I-2 \tag{6}$$

and

$$\hat{f}_k = \frac{\sum_{j=1}^{I-k} C_{j,k+1}}{\sum_{j=1}^{I-k} C_{j,k}} \times \hat{f}^{tail}, \qquad \text{for } k = I-1 \tag{7}$$

Finally to calculate $\hat{C}_i^{ult}$, we use

$$\hat{C}_{i,ult} = C_{i,I+1-i} \times \hat{f}_{I+1-i} \times \ldots \times \hat{f}_{I-1}. \tag{8}$$

Estimation of claim IBNR can be calculated by

$$\hat{R}_i^{cl} = \hat{C}_{i,ult} - C_{i,I+1-i}. \tag{9}$$

For $1 \in \{1,2,\ldots,I\}$. Total claim reserve IBNR can be predicted by

$$\hat{R}^{cl} = \sum_{i=1}^{I} \hat{R}_i^{cl}. \tag{10}$$

IV. MEAN SQUARED ERROR AND STANDARD ERROR OF CHAIN-LADDER SHERMAN'S METHOD

We modified run-off triangle in order to cope modified model. To simplify our calculation in the next step, we change the $\hat{C}_{i,I}$ by $\hat{C}_{i,ult}$ in run-off triangle, we get

Table 2: Cumulative Run-Off Triangle For Modified Method

AY	DY						
	1	2	$\cdots$	k	$\cdots$	$I-1$	Ult
1	$C_{1,1}$	$C_{1,2}$	$\cdots$	$C_{1,k}$	$\cdots$	$C_{1,I-1}$	$C_{1,ult}$
2	$C_{2,1}$	$C_{2,2}$	$\cdots$	$C_{2,k}$	$\cdots$	$C_{2,I-1}$	
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$		
I	$C_{i,1}$	$C_{2,2}$	$\cdots$	$C_{i,k}$			
$\vdots$	$\vdots$	$\vdots$	$\vdots$				
$I-1$	$C_{i-1,1}$	$C_{i-1,2}$					
I	$C_{I,1}$						

We provide our method with Mean Squared Error (MSE) for each AY. We extend the formula given by [2] part 3 to get our new MSE model. The $\mathrm{mse}(\hat{C}_{i,ult})$ of the estimator $\hat{C}_{i,ult}$ of $C_{i,ult}$ is defined to be

$$\mathrm{mse}(\hat{C}_{i,ult}) = E\left(\left(\hat{C}_{i,ult} - C_{i,ult}\right)^2 \middle| D\right), \tag{11}$$

hence

$$\mathrm{mse}(\hat{R}_i) = E\left(\left(\hat{R}_i - R_i\right)^2 \middle| D\right) = E\left(\left(\hat{C}_{i,ult} - C_{i,ult}\right) \middle| D\right) = \mathrm{mse}(\hat{C}_{i,ult}), \tag{12}$$

where $D = \{C_{i,k} | I + k \leq I + 1\}$ is the set off all observed data given by run-off triangle. For the rest of our method, we will follow Mack's (1993). In addition, to calculate $\mathrm{s.e.}\hat{C}_{i,ult}$, we substitute $\hat{f}_{ult}$ with $\hat{f}^{tail}$ that we have already estimated using Sherman's method, so that $\hat{f}_{ult}$ and $\hat{f}^{tail}$ are interchangeable.

As a plausibility consideration, we will able to find an index $k < I, ult = I$. Therefore, to simplify our model we assume there are three conditions of $\hat{f}^{tail}$.

- $\hat{f}^{tail} > \hat{f}_k > \hat{f}_{k-1}$; $\mathrm{s.e.}(\hat{f}^{tail}) > \mathrm{s.e.}(\hat{f}_k) > \mathrm{s.e.}(\hat{f}_{k-1})$ (13)
- $\hat{f}_k > \hat{f}^{tail} > \hat{f}_{k-1}$; $\mathrm{s.e.}(\hat{f}_k) > \mathrm{s.e.}(\hat{f}^{tail}) > \mathrm{s.e.}(\hat{f}_{k-1})$ (14)
- $\hat{f}_k > \hat{f}_{k-1} > \hat{f}^{tail}$; $\mathrm{s.e.}(\hat{f}_k) > \mathrm{s.e.}(\hat{f}_{k-1}) > \mathrm{s.e.}(\hat{f}^{tail})$ (15)

for $\mathrm{s.e.}(F_{i,ult})$, we still use the same inequality presented in Mack's (1999).

$$\mathrm{s.e.}(F_{i,k-1}) > \mathrm{s.e.}(F_{i,ult}) > \mathrm{s.e.}(F_{i,k}). \tag{16}$$

We briefly follow Mack's (1993) for the parameter $\hat{f}_{k-1}, \hat{f}_k, \mathrm{s.e.}(\hat{f}_{k-1}), \mathrm{s.e.}(\hat{f}_k)$, $\mathrm{s.e.}(F_{i,k-1})$, and $\mathrm{s.e.}(F_{i,k})$. As stated in [7] about the possibility of $\mathrm{s.e.}(F_{i,ult})$, it is reasonable to calculate our $\mathrm{s.e.}(F_{i,ult})$ as follows

$$\mathrm{s.e.}(F_{i,ult}) = \frac{\mathrm{s.e.}(F_{i,m}) + \mathrm{s.e.}(F_{i,n})}{2} \tag{17}$$

m and n are respectively left and right index of DY where our $F_{i,ult}$ belongs to.

Finally, the formula for the total reserve of all accident years with its inclusion of tail factors given by

$$\left(\mathrm{s.e.}\left(\sum_{i=I+1-k}^{I}\hat{C}_{i,k+1}\right)\right)^2 = \left(\mathrm{s.e.}\left(\sum_{i=I+2-k}^{I}\hat{C}_{i,k}\right)\right)^2 \cdot \hat{f}_k^2 + \sum_{i=I+1-k}^{I}\hat{C}_{i,k}^2 \cdot \left(\mathrm{s.e.}\left(F_{i,k}\right)\right)^2 + \left(\sum_{i=I+1k}^{I}\hat{C}_{i,k}\right)^2 \cdot \left(\mathrm{s.e.}\left(\hat{f}_k\right)\right)^2 \tag{18}$$

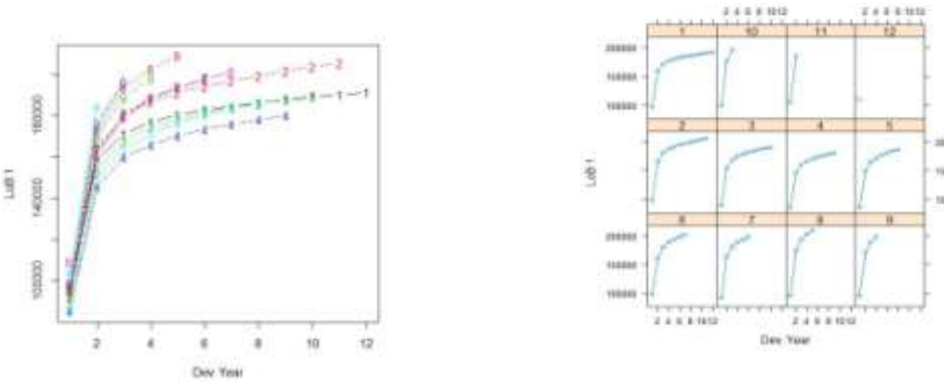
V. RESULT

We use secondary data taken from [6] for LoB 1.

Table 3: Cumulative Run-Off Triangle Line of Business 1 (in 1.000)

Acc. Year	Development Year											
	1	2	3	4	5	6	7	8	9	10	11	12
1	96.118	62.683	12.439	5.828	3.328	2.452	1.768	1.605	1.385	1.395	1.195	1.139
2	97.350	67.534	14.914	7.036	4.371	2.929	2.649	2.375	2.400	2.096	1.824	
3	89.018	64.518	14.403	6.731	3.580	3.218	2.351	2.127	2.073	1.900		
4	85.127	60.353	14.336	5.942	4.561	3.181	2.453	2.220	2.057			
5	85.516	63.582	14.535	7.881	4.874	3.718	3.450	2.656				
6	96.833	63.843	18.436	9.165	5.407	3.984	3.530					
7	91.886	71.474	17.451	7.713	5.111	4.230						
8	94.711	78.518	20.093	9.453	6.433							
9	95.071	74.690	18.750	9.653								
10	98.998	77.219	20.088									
11	104.434	79.948										
12	109.294											

Our model is fully calibrated. The visualizing of data presented bellow



(a) in one graph **(b) each AY**

Fig. 1. Visualization of Cumulative run-off Triangle

From Fig. 1, we can see that for each claims that reported or settled follows exponential growth distribution. It means that in the early of DY, the claims came to company tends to decrease in frequency and value for older DY. Our first step is calculating $\hat{f}_k$.

Table 4: Development Factors $\mathbf{k \in \{1, 2, \ldots, I-1\}}$

	Development Year k pada LoB 1										
	1	2	3	4	5	6	7	8	9	10	11
	$1 \to 2$	$2 \to 3$	$3 \to 4$	$4 \to 5$	$5 \to 6$	$6 \to 7$	$7 \to 8$	$8 \to 9$	$9 \to 10$	$10 \to 11$	$11 \to 12$
$\hat{f}_k$	1,738	1,102	1,044	1,026	1,018	1,015	1,012	1,011	1,009	1,008	1,006

Using eq. (3), we get

Table 4: $\mathbf{\hat{f}_k^* = \ln(\hat{f}_k - 1)}$

Development Year *k* pada LoB 1											
	1 $1\to2$	2 $2\to3$	3 $3\to4$	4 $4\to5$	5 $5\to6$	6 $6\to7$	7 $7\to8$	8 $8\to9$	9 $9\to10$	10 $10\to11$	11 $11\to12$
$\hat{f}_k$	1,738	1,102	1,044	1,026	1,018	1,015	1,012	1,011	1,009	1,008	1,006
$\hat{f}_k - 1$	0,738	0,102	0,044	0,026	0,018	0,015	0,012	0,011	0,009	0,008	0,006
$\hat{f}_k^*$	- 0,303	- 2,278	- 3,128	- 3,654	- 3,992	- 4,227	- 4,433	- 4,551	-4,673	-4,868	-5,118

Let $\hat{f}_k^* = \beta_0 + \beta_1 x_k$ where $\hat{f}_k^*$ is dependent variable, and $x_k \in DY$ as independent variable. β_0 is intercept of regression formula and β_1 is coefficient of x_k. Using Ordinary Least Square (OLS) technique and eq. (3), we get $\hat{f}_k^* = -1{,}4949 - 0{,}3755x_k$, the Log-linear extrapolation of $\hat{f}_k$ visualization presents bellow

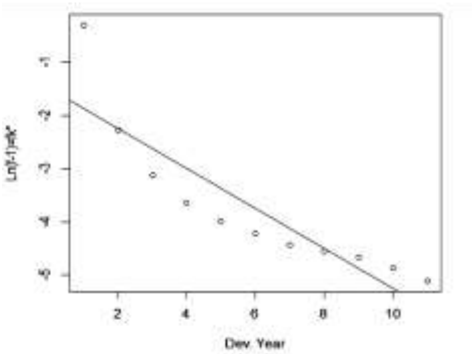


Fig. 2. Extrapolation Log-Linear of Development Factor

Table 4: Result of $\hat{f}_k^{exp}$ with $n = 100$

	$\hat{f}_k^{exp}$							
$(n=1; k=12)$	1,002477	1,001702	1,001169	1,000803	1,000552	1,000379	1,000260	1,000179
$(n=9; k=20)$	1,000123	1,000084	1,000058	1,000040	1,000027	1,000019	1,000013	1,000009
$(n=17; k=28)$	1,000006	1,000004	1,000003	1,000002	1,000001	1,000001	1,000001	1,000000
	...	...	...	...	...		$(n=100; k=112)$	1,000000

Then, from selected $\hat{f}_k^{exp}$ we use eq. (5) to find $\hat{f}^{tail}$, $k \in \{12,13,\ldots,112\}$.
$\hat{f}^{tail} = \prod_{k=12}^{112} \hat{f}_k^{exp} = 1{,}007939.$

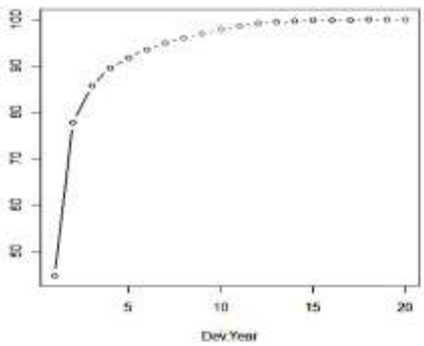


Fig. 3. Expected Development of Claim IBNR

Fig. 3 presents the visualization of developed claim IBNR until $k = 20$. Actuaries use that to see the development of claim IBNR, whether the claim is fully developed or it needs another development process. we decided to bound our development factor until $k = 20$. From Fig. 3 we can see that our chosen development factor is close enough to 100%.

Table 5: Full Cumulative Run-Off Triangle Line of Business 1 (in 1.000)

Acc. Year	Development Year 1	2	3	4	5	6	7	8	9	10	11	ULT
1	96.118	158.801	171.240	177.068	180.396	182.848	184.616	186.221	187.606	189.001	190.196	192.854
2	97.350	164.884	179.798	186.834	191.205	194.134	196.783	199.158	201.558	203.654	205.478	208.350
3	89.018	153.536	167.939	174.670	178.250	181.468	183.819	185.946	188.019	189.919	191.379	194.054
4	85.127	145.480	159.816	165.758	170.319	173.500	175.953	178.173	180.230	181.913	183.312	185.874
5	85.516	149.098	163.633	171.514	176.388	180.106	183.556	186.212	188.178	189.936	191.396	194.071
6	96.833	160.676	179.112	188.277	193.684	197.668	201.198	203.588	205.738	207.659	209.256	212.180
7	91.886	163.360	180.811	188.524	193.635	197.865	200.754	203.138	205.283	207.201	208.794	221.536
8	94.711	173.229	193.322	202.775	209.208	213.072	216.183	218.750	221.060	223.125	224.841	227.983
9	95.071	169.761	188.511	198.164	203.292	207.047	210.070	212.565	214.809	216.816	218.483	221.536
10	98.998	176.217	196.305	204.905	210.208	214.090	217.216	219.796	222.117	224.191	225.915	229.072
11	104.434	184.382	203.270	212.175	217.666	221.686	224.923	227.594	229.998	232.146	233.931	237.200
12	109.294	190.004	209.468	218.645	224.303	228.446	231.781	234.534	237.011	239.225	241.064	244.433

Here is our final result of Chain-Ladder Sherman's method.

Table 6: Estimated Reserve Claim IBNR (in 1.000)

i	$C_{i,I+1-i}$	Basic Chain-Ladder			Modified Chain-Ladder Sherman		
		IBNR	s.e. (%)	s. e. $\hat{C}_{i,I}$	IBNR	s.e. (%)	s. e. $\hat{C}_{i,ult}$
1	191.335	0,000	-	-	1.519,009	-	-
2	205.478	1.230,517	37,88%	78.297,21	2.872,576	43,44%	90.507,06
3	189.919	2.606,313	23,93%	46.073,08	4.135,771	29,12%	56.508,46
4	180.230	4.179,832	16,74%	30.871,84	5.644,862	20,22%	37.583,69
5	186.212	6.330,644	13,42%	25.840,63	7.859,240	16,23%	31.497,76
6	201.198	9.310,995	10,87%	22.880,81	10.982,226	12,49%	26.501,31
7	197.865	12.179,088	10,33%	21.692,27	13.847,628	11,43%	24.198,64
8	209.208	16.978,984	8,77%	19.831,06	18.775,683	8,99%	20.495,64
9	198.164	21.627,188	8,13%	17.873,39	23.372,110	8,35%	18.498,27
10	196.305	30.962,882	7,36%	16.719,00	32.767,162	7,78%	17.821,81
11	184.382	50.949,580	6,90%	16.231,52	52.817,878	7,56%	17.932,31
12	109.294	133.213,489	6,35%	15.404,94	135.138,756	7,29%	17.819,15
Total		289.569,514	4,02%	102.088,32	309.727,902	4,21%	107.747,31

VI. CONCLUSION

We have extended basic Chain-Ladder method by including Sherman's curve fitting method to forecast LDTF. This extension was done by extrapolating the development factor. This extended model has the advantage that the actuary can predict the claim that occurred after the oldest DY. The issue we faced here was to predict the s.e. for the first AY.

The difference between two estimated total claim IBNR given by basic Chain-Ladder and Modified Chain-Ladder Sherman is 20.158,388. MCL-S method produced higher standard error for each AY. Our method produce the same pattern in s.e, which decreased in older AY.

REFERENCES

[1]. Friedland, J. Estimating Unpaid Claims Using Basic Techniques. In Casualty Actuarial Society, vol. 201 (2010).

[2]. Mack, T. Distribution-Free Calculation Of The Standard Error Of Chain Ladder Reserve Estimates. Astin bulletin, vol. 23(2),,pp.213–225 (1993).

[3]. Friedland, J. Estimating unpaid claims using basic techniques. In : Casualty Casualty Actuarial Society, vol. 3, pp. 84–131 (2010).

[4]. Ashab and Working, C. T. F. The estimation of loss development tail factors: A summary report. In: Casualty Actuarial Society Forum. Citeseer, vol. 201 (2017).

[5]. Mack, T. The standard error of chain ladder reserve estimates: Recursive calculation and inclusion of a tail factor, ASTIN Bulletin: The Journal of the IAA vol. 29, pp.361–366 (1999).

[6]. Wüthrich, M. V. Non-Life Insurance: Mathematics & Statistics. https://ssrn.com/abstract=2319328, accessed 3th February 2019. (2017).

[7]. Mack, T. Distribution-Free Calculation of The Standard Error Of Chain Ladder Reserve Estimates. Astin bulletin, vol. 23(2), pp. 213–225. (1993).

Media Rahmawati" Modified Chain-Ladder Reserve Estimates: Inclusion of a Tail Factor Estimatied using Sherman's Method" American Journal of Engineering Research (AJER), vol.8, no.06, 2019, pp.08-13