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Arithmetic Sequence with Multiple Reasons

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ABSTRACT: *The study of the mathematical series provides an advance in the concepts of analysis and probability, by contributing with the predictions and simulations for diverse performances. Serious mathematics saves time and energy in operations, so their study provides the construction of a solid statistical forecast. Nevertheless, the arithmetic series is limited to studying the effects of a single reason influencing the projections of the data. However, they exceed the calculations when the reasons become numerous. A real series could present other determinant factors for the simulation, which during its construction are essential in the generation of the information, making necessary a historical construction of data capable of absorbing these particularities. The purpose of this study is to elucidate this type of simulation, approaching a series with a number of finite ratios that we will call the sequence. This study presents three formulas that allows the construction of arithmetic series with multiple reasons, in which a single algorithm can simulate a whole set of interactions.*
KEYWORDS*-Theory of Numbers, Modular Arithmetic, Sequences, Progressions, Reasons.*

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I INTRODUCTION

In this paper we will discuss the study of a particularity of the numerical sequences, a set of numbers that represent a function whose domain is natural. Thus, we determine that:

$sequence = f(x), where\ x \in \mathbb{N}$

In this way, the sequence obeys a pattern or order of relation established by the function construction [11]. We must also conceptualize that the sequences can be finite, when in their construction it is possible to establish the last term, and infinite, when the last term is indefinite or impossible to determine [4]. We will study in this article the infinite sequences.

Within the infinite sequences, we have two types to consider. Those that increase their ratio to each term, constructed by a condition established by the function, and those that do not alter their reason. The sequences that keep their reason constant, are given the special name of progressions. These progressions are divided into two models, arithmetic and geometric.

Arithmetic progression, or simply (AP), is the sequence in which each next term is a result of the previous term, plus the ratio of the sequence. The geometric progression (GP), however, differs because the next term is the result of the product of its predecessor by the ratio of the sequence.

These concepts were presented by the mathematician Karl f. Gauss and will be improved within this study in new procedures and approaches. Arithmetic progression will be used, which will henceforth be referred to as AP, going through an introduction of considerable advances in its structure.

II SEQUENCE WITH MULTIPLE REASONS

By adopting a progression that has a finite number of reasons, where they may be real or complex, they allow us to work with small amounts of influential terms, as well as large clusters of data. Therefore, in taking a finite number, one can observe that at some point the different reasons are closed, allowing the construction of a new cycle. Having this characteristic, the series behaves with the principles of modular arithmetic.

Modular arithmetic, a concept that improved by Gauss and implemented by Fermat in his little theorem, presents a structure known as clock calculator, widely used in the construction of data encryption, developed in 1977 by Rivest, Shamir and Adleman, known with key algorithm public or RSA, presenting in [5] [9] [10]. Encryption enables the secure use of the Internet for purchases, transmission of information, access to bank accounts, as well as the acquisition of shares and crypto-coins [6].

The term clock calculator presents a modular sum by allowing the transformation of numbers into other numerical bases, imprisoning a result within a cycle. We have the computational language, the bits, that hold in module 2, our numbering system that uses module 10, and infinite possibilities that allow us to use diverse modules in the cryptography, when approaching a characterization of a specific value in a known module. In general we have:

$X = Y\, mod(p),$

That is to say, that the number X is now represented by the number Y, in a periodic cycle of p hours.

III DETERMINING THE GENERAL TERM OF AT AP

When we adopt at AP, in which the reasons are, even if in great finite quantity, we will have at one moment a closing cycle and another initiating. Taking an as a general form for each term, we have a1 $a_1 = first\ term$, $a_2 = second\ term$ and$a_3 = thirt\ term$, and so on. We will also consider$r_1 = first\ ratio$, $r_2 = secondratio$, etc. The construction of AP follows a linear addition of the ratios, that is:

$a_1 = a_1$
$a_2 = a_1 + r_1$
$a_3 = a_2 + r_2$
$a_4 = a_3 + r_3$
$\vdots$
$a_k = a_{k-1} + r_{k-1},$
$wherekrepresentsthe\ number\ of\ reasons.$

When the number of distinct reasons comes to end, we have a resumption in the sequence, assuming the same substitution order as presented above. So, we have the cycle closed when we reach the value of k in the ratios. Soon our clock calculator will always have module k.

Each turn in the clock will be represented by y, and so we can present the first equation that will be used in the deduction:

$$y = \frac{n}{k} \quad (1)$$

Where n is the desired term within the sequence and k that isthe number of ratios. However, you must enter another variable x, which represents the rest of this division or the values marked on the watch's display, where the hands run. So, we can write:

$$x = n - yk \quad (1.1)$$

Thus, this formula is a variation of (1), which will be adopted in some moments.

From this stage, our approach will take two distinct paths that will meet at the end, which are the times when the division is exact and the cycle is closed, determined by $x = 0$. As well as the non-exact divisions in which we $x > 0$.

Since $y = 1$ is a determinant index of the cycles of our clock, we have a given turn; $y = 2$, two complete turns and so on, we can conclude on, $x = 0$ which makes $n = yk$.

In constructing the first cycle, we present the following rationale:

$a_1 = a_1$
$a_2 = a_1 + r_1$
$a_3 = (a_2) + r_2 \Rightarrow a_1 + r_1 + r_2$
$a_4 = (a_3) + r_3 \Rightarrow a_1 + r_1 + r_2 + r_3$
$a_5 = (a_4) + r_4 \Rightarrow a_1 + r_1 + r_2 + r_3 + r_4$
$\vdots$

$$a_k = a_1 + \sum_{i=1}^{k-1} r_k , \qquad (2)$$

At the moment we have the closing of the first cycle, and when we observe the second part of [2], it is noticed that the construction depends exclusively on the parcels of the reasons. Therefore, with each cycle we have a further addition of the values of the ratios.

The next term, in which the final ratior_k is used, the new cycle is recalculated and the values reassembled the clock and the pointer traversed the same paths previously covered in the first cycle, making $a_{k+1} = a_1 mod(k)$, that is, the same initial position with the value of the module added to it. In this way we need to define the value of module k of our cycle:

$$R = \sum_{i=1}^{k} r_k (3)$$

The value R represents the sum of all the ratios; in this perspective, we have that the second cycle will have its construction governed by the following equation:

$$a_n = a_k + R$$

Therefore, the second cycle is composed of the values of a_k added once the sum of the ratios R, the third will be the value of a_k added to twice the sum of the ratios R, which leads us to deduce:

Table 01: Construction of the cycles of a sequence

Cycle	How to determine
y=1	$a_n = a_k$
y=2	$a_n = a_k + R$
y=3	$a_n = a_k + 2R$
y=4	$a_n = a_k + 3R$
$\vdots$	$\vdots$
y=n	$a_n = a_k + (y-1)R$

Thus, we have the final equation for the general term of atAP, with multiple reasons in closed cycles, where the searched term n is a multiple of the number of reasons k:

$$a_n = a_k + (y-1)R \quad (4)$$

Now we will consider $x > 0$, that is: $x = n - yk$. Therefore, we have that y is a complete cycle and the rest x indicates the position of the term after the cycle. For this we will consider that $y < 1$ means that the first cycle has not yet been completed, implying $n < k$ and the value of x represents the same position in the clock, then $x = n$. Soon:

$$for x = 1 \Rightarrow a_1$$
$$for x = 2 \Rightarrow a_2$$
$$for x = 3 \Rightarrow a_3$$

Then:

$$a_n = a_x$$

By the same reasoning presented in the deduction of (2), we conclude:

$$a_x = a_1 + \sum_{i=1}^{x-1} r_x, \qquad (5)$$

To define in general, $y = 1$, in this case, the value of $n > k$, indicating that we will have a division, obtaining a value for y and another for x, representing the rest. Now we have the closed loop, returning the use of the value of R presented in (3).

The first value after the cycle closes will be, as in a clock, the length of the module added to the initial term of the series, as well as the second will be the sum of the module and the second term, and so on, ensuring that the positioning on the clock will be similar to those already presented in the first cycle.

Since x is always a present value in the equation, we can assume that each cycle is directly connected to the value of the module, where we arrive at the deduction of (6):

$$a_n = a_x + yR \quad (6)$$

In this way, y shows us how many laps we have already covered in the calculating clock, summarizing this point as follows:

$$if x = 0, be\ used: \ a_n = a_k + (y-1)R (4)$$
$$if x > 0, be\ used: \ a_n = a_x + yR \ (6)$$

IV DETERMINING THE SUM OF THE TERMS OF AT AP

As seen earlier, we will consider the sum of terms of at AP with two situations, the first, before the cycle closes and the second after that, to construct an equation that satisfies both conditions.

In this case, if $y = 0$, then $n < k$, the division of (1) becomes impossible, which leads us to conclude that:

$$S_n = \sum_{i=1}^{x} a_n, \qquad where n = x \ (7)$$

We will call this sum of B,

$$B = \sum_{i=1}^{x} a_n \rightarrow S_n = B \ (8)$$

Thus, a methodology will be developed to calculate B, represented as follows:

$a_1 = a_1$
$a_2 = a_1 + r_1$
$a_3 = a_1 + r_1 + r_2$
$a_4 = a_1 + r_1 + r_2 + r_3$
$a_5 = a_1 + r_1 + r_2 + r_3 + r_4$
$\vdots$
$a_n = a_1 + r_1 + r_2 + r_3 + r_4 + \cdots r_{(n-1)}$
$\vdots$

$$\sum a_n = na_1 + (n-1)r_1 + (n-2)r_2 + (n-3)r_3 + (n-4)r_4 + \cdots 1r_{(n-1)}$$

Since x = n, we have:

$$\sum a_x = xa_1 + (x-1)r_1 + (x-2)r_2 + (x-3)r_3 + (x-4)r_4 + \cdots 1r_{(n-1)}$$

The $n-1$ index is understood to be the last reason before closing the cycle, and we will now call it i, as a variable that will be the index of both the ratio repetition factor and the ratio itself:

$$\sum a_x = xa_1 + (x-1)r_1 + (x-2)r_2 + (x-3)r_3 + (x-4)r_4 + \cdots 1r_i$$

$\vdots$

$$\sum a_x = xa_1 + \sum_{i=1}^{x-1} (x-i)r_i$$

By (8):

$$B = xa_1 + \sum_{i=1}^{x-1} (x-i)r_i \ (9)$$

When we have the value of $x = k$, the clock will complete the first round, then we will have $y = 1$ and $x = 0$, so we will define another marker that we will call A, where the variable becomes k. With the closed cycle we will have $k = 1$, being able to consider by (7) and (8) that:

$$A = \sum_{i=1}^{k} a_n \ (10)$$

Analogously to the procedure presented for marker B, resulting from (9), we will have:

$$\sum a_k = ka_1 + (k-1)r_1 + (k-2)r_2 + (k-3)r_3 + (k-4)r_4 + \cdots 1r_i$$

$\vdots$

$$\sum a_k = ka_1 + \sum_{i=1}^{k-1} (k-i)r_i$$

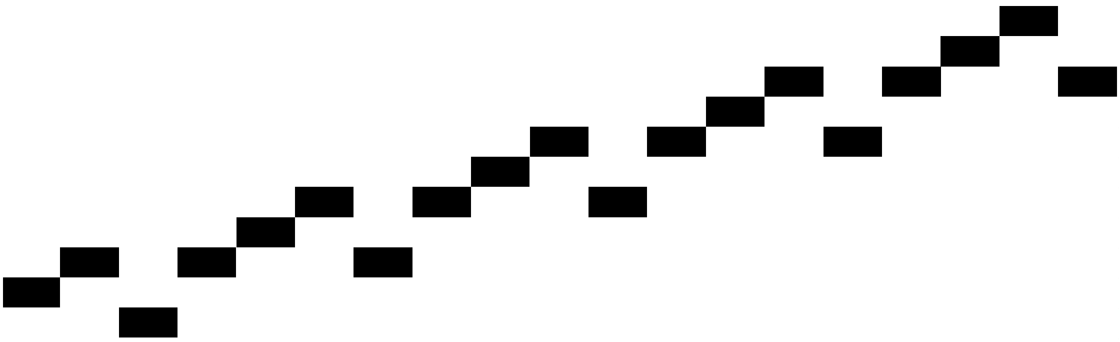
By (10):

$$A = ka_1 + \sum_{i=1}^{k-1} (k-i)r_i \ (11)$$

Considering only the first revolution of the clock, we could adopt $S_n = A$, but it is necessary to analyze the infinite cycles that may exist, requiring a more comprehensive reasoning; as shown above, when closing the cycle, values or pointers on the clock start to go through the same paths prior to closing, however, in relation to the sum of the ratios, each x factor of the term must be connected to a conditioned value R, so each term is in what we shall call the upper step, with the height of this step as the value of R.

To insert these values into the equation, we have to define items such as the number of laps in the cycle, which we will call Ay, plus the number of terms that walk within the plots to reach the next cycle, ie B, in addition to a lifting factor of steps of this ladder, this being the direct product of the height R by the number of cycles and a position in the cycle x, that is xRy.

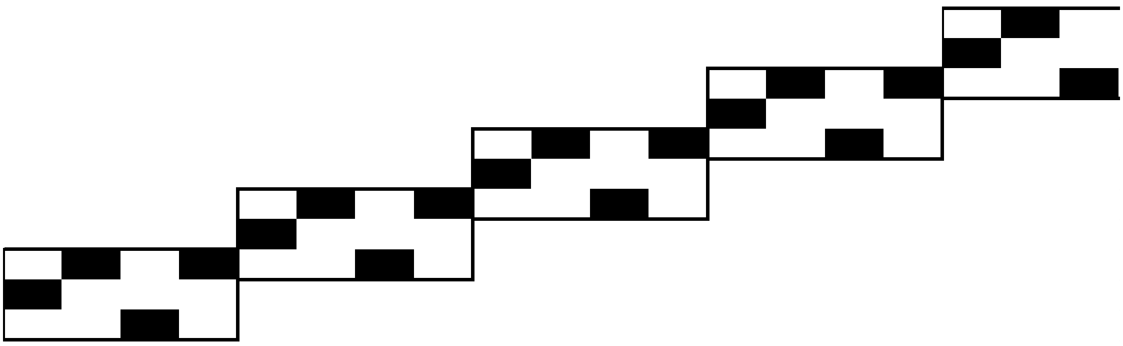
Consequently, the equation that corrects the cycle and its positioning is $Ay + B + xRy$, when defining each step in its positioning. Fig. 01 presents the simulation of a PA with 4 reasons, being$r_1$positive,r_2 positive, $r_3$negative e r_4 positive, being$R = 2$, allowing a better view of the concept presented.



a_1 a_2 a_3 a_4 a_5 a_6 a_7 a_8 a_9 a_{10} a_{11} a_{12} a_{13} a_{14} a_{15} a_{16} a_{17} a_{18} a_{19}

figure 01: simulation of a series with 4 reasons, being R=2

Notice that the values oscillate at the same time that they rise in a certain linearity. Separating each group of four terms k, which represent the cycle of the ratios, we have steps formed as if they were a ladder, shown in Fig. 02:



a_1 a_2 a_3 a_4 a_5 a_6 a_7 a_8 a_9 a_{10} a_{11} a_{12} a_{13} a_{14} a_{15} a_{16} a_{17} a_{18} a_{19}

figure 02: separation in cycles (steps)

It may be noted that the closed loop represented by A forms the basis of each step, in addition to that the height of the step is given by R. Each point represents the same value of the previous cycle but is at a height which is represented by the product of the step number (cycle) y, its own height which is given by R and its positioning which is x, then comes the term xRy, shown above. Each block is A, the number of blocks of the base is given by Ay.

Each point outside the closed loop is given by the sum of its values, represented by B. When determining a term outside the closed loop, presented bya_{17}, a_{18} or a_{19}, it is necessary to have the initial value of the base containing it Ay, its positioning xRy and the sum of the off-cycle terms to itself, that is$B$, then any of them within their incomplete cycle is given by $Ay + B + xRy$.

The cycles that were previous to the sum, that is, the complete cycles will be the area occupied by them. Fig.03 presents this visually:

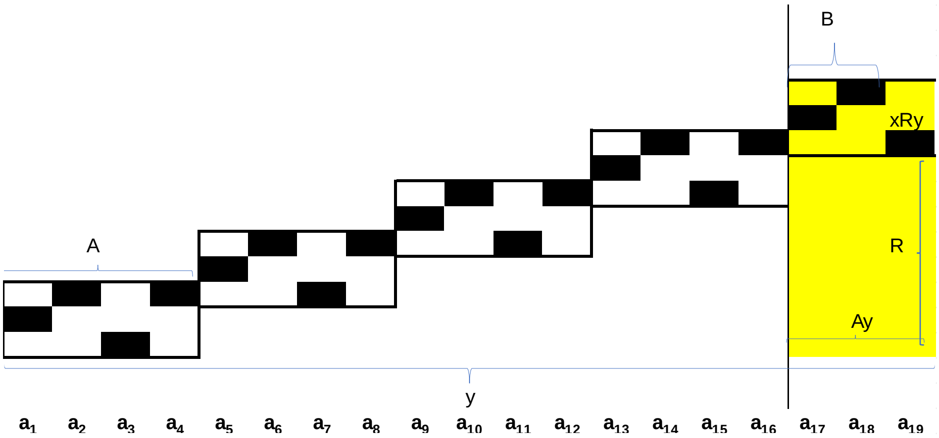


figure 03: the elements of each step

Note that in this way, the region occupied by the yellow area (Ya) of Fig. 03, represents a part of the ladder where the cycle is not closed, that is, without full step, it will be the result of the factors of the base by the values already covered by the end term, therefore,

$Ya = Ay + B + xRy.$ (12)

To find the value of the white region, which represents the number of complete steps, it can be seen that it has a triangular shape, so in a very intuitive way we have that the region occupied by these values represents the area of the triangle that the contains. Fig. 04 will allow you to view this property:

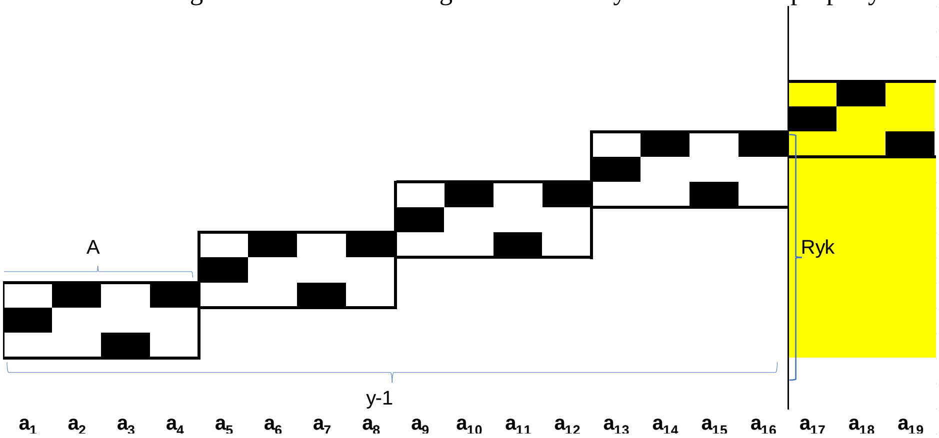


Figure 04: the elements of the white region

The base of the triangular region is given by $y - 1$ times, its height is given by the product of step height R, by the cycle where they are y, and the quantity of ratios or values in each cycle k, thus being the area of the region white (Wa) is expressed by:

$$Wa = \frac{bh}{2}$$

$$Wa = \frac{(y-1)Ryk}{2}$$

For a better presentation, we will adopt:

$$Wa = \frac{kRy(y-1)}{2} \quad (13)$$

Finally, to find the sum of the finite terms of a PA, we must add the area of the two regions, the sum of the parts of (12) and (13) being:

$$S_n = Wa + Ya$$

$$S_n = \frac{kRy(y-1)}{2} + Ay + B + xRy \quad (14)$$

Note that (14) is composed of two parts, the first one represented by the ratio, works with the closed cycles and the second with the terms within a non-complete cycle. When the cycle is closed the second part will admit null value and the first one will present the result.

V CONCLUSION

The use of the mathematical series has a great variety of applications, in all cases, the traditional equations that allow only a growth rate are used. If a simulation with multiple reasons is required, this simulation was compromised by the limitation of the available equations.

It is believed that with the use of these equations presented here, they can contribute substantially to the advancement in the studies of these series, providing a gain in time and precision in the future calculations.

Given an arithmetic progression (AP) with multiple reasons, one should adopt the following procedure: either n the number of terms or term sought, k the number of reasons, R the sum of the ratios, $y = \frac{n}{k}$ and x the rest of this division, for the general term we have:

If $x = 0$ be used:

$$a_n = a_k + (y-1)R \quad (4)$$

If $x > 0$ be used:

$$a_n = a_x + yR (6)$$

For the sum of the finite terms of at AP in these conditions use:

$$S_n = \frac{kRy(y-1)}{2} + Ay + B + xRy \ (14)$$

Being:

$$B = xa_1 + \sum_{i=1}^{x-1}(x-i)r_i \quad (9)$$

and

$$A = ka_1 + \sum_{i=1}^{k-1}(k-i)r_i \quad (11)$$

With these equations it will be possible to predict any term within a simulation or sequence with several influencing factors (reasons), as well as to calculate the sum of all terms up to the desired term.

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