

The Characteristics of Dynamic Functioning Regimes For A Synchronous Machine

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ABSTRACT: In this paper, we analyse the dynamics of a synchronous machine electromagnetic processes from the roots of characteristic equation, and we also study the problems related to simplifying differential equations.

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I. INTRODUCTION

Modelling synchronous generators when they operate in stationary or dynamic regime is currently widely used due to obvious advantage which modelling the excitation generator load system offers in different operating conditions of the generator.

Study on the model of the whole system behaviour allows the determination of actual functioning of both steady and transient regime for different electric charges of the generator.

The manner in which are modelled in matlab/simulink both the proper generator and the excitation system and its electrical charge is not unique, even if the mathematical model of the generator remains the same.

Knowing the internal angle (often called "load angle") of a high power generator is important for its use in practice.

An accessible method of calculating the generator internal angle is to use the newer package simpower-systems of matlab/simulink computing environment. However, there are other methods for determining the internal angle through simulation methods not using the medium^{[1].....[4]}. This is why the system model for determining the internal angle was designed and implemented. Furthermore, it was described a friendly mode for presenting simulation results.

To verify the suggested method for determining the internal angle, we can apply a simple experimental method called "stroboscopic method"^[5] with the aid of which it can be measured the internal angle when the electric charge is present and the generator works in a steady regime.

In this paper, we analyse the dynamics of a synchronous machine electromagnetic processes from the roots of characteristic equation, and we also study the problem related to simplifying differential equations.

II. ANALYSIS OF ROOTS FOR SYNCHRONOUS MACHINE CHARACTERISTIC EQUATION WITH SUPPLY OF EXCITATION WINDING FROM E.M.F SOURCE

The character of dynamics properties change in a synchronous machine with rotor variation in rotation speed is well known. The dynamic properties can be studied by the roots of characteristic equation. That is why we analyse the behaviour of characteristic equation roots in function of rotor rotation speed.

The transient function of synchronous machine state variables is a linear combination of exponential time functions in which characteristic equation roots are the time scale parameters.

The roots of characteristic equation can be either real or complex numbers. Negative real roots

$P_1 = \delta_1$ create damping function $\exp(\delta_1 t)$. Complex roots $P_1 = \delta_1 + j\Omega_1$ create oscillating transient processes:

$$\exp(P_1 t) = \exp(\delta_1 t) \cdot [\cos(\Omega_1 t) + j \sin(\Omega_1 t)]$$

From the expression $|\Omega_1/\delta_1|$ that is ‘‘oscillating factor’’ we can determine the damping oscillating speed.

If $|\Omega_1/\delta_1| < 1$, then the oscillating character is not considerable.

The description of characteristic equation is better done using the matrix forms. For a synchronous machine whose excitation winding is supplied from e.m.f source, the characteristic equation is:

$$Y(P) = \det(L \cdot P + A) = 0$$

The equations system for a synchronous machine can be represented by a single matrix equation:

$$L \cdot P I = -A I + U$$

Where $U^T = [u_{1d} u_{1q} \quad 0 \quad 0 \quad u_f]$; $I^T = [i_{1d} i_{1q} i_{2d} i_{2q} i_f]$

$$L = \begin{bmatrix} L_1 + L_{dd} & 0 & L_{dd} & 0 & L_{dd} \\ 0 & L_1 + L_{qq} & 0 & L_{qq} & 0 \\ L_{dd} & 0 & L_{2d} + L_{dd} & 0 & L_{dd} \\ 0 & L_{qq} & 0 & L_{2q} + L_{qq} & 0 \\ L_{dd} & 0 & L_{dd} & 0 & L_f + L_{dd} \end{bmatrix}$$

$$A = \begin{bmatrix} R_1 - \omega L_q & 0 & -\omega L_{qq} & 0 & 0 \\ \omega L_d R_1 & \omega L_{dd} & 0 & \omega L_{dd} & 0 \\ 0 & 0 & R_{2d} & 0 & 0 \\ 0 & 0 & 0 & R_{2q} & 0 \\ 0 & 0 & 0 & 0 & R_f \end{bmatrix}$$

The characteristic equation is :

$$Y(P) = \det(LP + A) = 0$$

The matrices A and L have 5th order, therefore the characteristic polynome Y (P) also has 5th order, and the characteristic equation $Y(P) = 0$ has 5 roots.

The plots of dependences real and imaginary root parts of characteristic equation $Y(P) = 0$ on rotor rotation speed are shown on figure 1a.

One of the roots of characteristic equation δ_0 is real. The absolute value $|\delta_0|$ of that root is little. We can assume that δ_0 is related with dynamic electromagnetic processes in excitation winding. These processes occur slowly because the excitation winding active resistance is little.

From figure 1a it appears that for little values of speed $\omega^* < \omega_{s1}^*$, the dynamics in synchronous machine have an aperiodic character. The value ω_{s1}^* can be evaluated from formula

$$\omega_{s1}^* = (R_{2d}/L_{dd} + R_{2q}/L_{qq})/2 \cdot \omega_0$$

For rotation speeds $\omega_{s1}^* < \omega^* < \omega_{s2}^*$ the solution of characteristic equation creates complex-conjugate roots:

$$P_{\pm 1} = \delta_1 \pm j\Omega_1; P_{\pm 2} = \delta_2 \pm j\Omega_2.$$

The factor of oscillations $|\Omega_1/\delta_1|$ for the first root is high, that is why the solution initiated by $P_{\pm 1}$ are slowly damped. The oscillation factor $|\Omega_2/\delta_2|$ for the second root is little and the solution initiated by $P_{\pm 2}$ are quickly damped.

The plots of roots as function of rotor rotation speed are shown in figure 1b.

For rotation speeds $\omega^* > \omega_{s2}^*$, the characteristic equation gives two complex-conjugate roots

$$P_{\pm 1} = \delta_1 \pm j\Omega_1 \text{ and two real roots } \delta_{21}, \delta_{22}.$$

The real parts satisfy the condition

$$\delta_0 < \delta_1 < \delta_{21} < \delta_{22}.$$

Thus, for $\omega^* > \omega_{s2}^*$ the components of transient processes created by the roots δ_{21} and δ_{22} are quickly damped than for others.

Finally, the analysis of roots shows that roots δ_0 and $P_{\pm 1}$ have decisive influence on synchronous machine dynamic processes.

From the roots properties, we can assume that when supplying excitation winding with e.m.f source, we can make approximation of differential equation of 3rd order.

III. ANALYSIS OF SYNCHRONOUS MACHINE ROOTS OF CHARACTERISTIC EQUATION WHEN SUPPLYING THE EXCITATION WINDING WITH CURRENT SOURCE

We assume that the current in excitation winding is stabilised. Then we can consider that the excitation winding is supplied from a current source. And the equations of synchronous machine look as follows:

$$L_N \cdot P I_N = -A_N \cdot I_N + U_N$$

$$\text{Where } L_N = \begin{bmatrix} L_d & 0 & L_{dd} & 0 \\ 0 & L_q & 0 & L_{qq} \\ -L_{dd} & 0 & L_{2d} + L_{dd} & 0 \\ 0 & L_{qq} & 0 & L_{2q} + L_{qq} \end{bmatrix}; U_N = \begin{bmatrix} u_{1d} \\ u_{1q} \\ -\omega \cdot L_{dd} \\ 0 \\ 0 \end{bmatrix}$$

$$A_N = \begin{bmatrix} R_1 - \omega L_q & 0 & -\omega L_{qq} \\ \omega \cdot L_d R_1 \omega \cdot L_{dd} & 0 & 0 \\ 0 & 0 & R_{2d} & 0 \\ 0 & 0 & 0 & R_{2q} \end{bmatrix}; I_N = \begin{bmatrix} i_{1d} \\ i_{1q} \\ i_{2d} \\ i_{2q} \end{bmatrix}$$

The characteristic equation for a synchronous machine whose excitation winding is supplied from current source is:

$$Y_N(P) = \det(L_N P + A_N) = 0$$

The matrix A_N has 4th order, and the characteristic polynomial $Y_N(P)$ also has 4th order therefore the characteristic equation $Y_N(P) = 0$ has four roots.

In the figure 2a we represent the dependences real and imaginary parts of roots $P_{1,2}$ of characteristic equation $Y(P) = 0$ on speed ω^* with supply of excitation winding from current source

From figure 2a, it appears that equations have complex-conjugate roots for any rotor rotation speed.

$$P_{\pm 1} = \delta_1 \pm j\Omega_1; P_{\pm 2} = \delta_2 \pm j\Omega_2.$$

The following conditions are also observed.

$$\delta_1 < \delta_2; |\Omega_1| > |\Omega_2|.$$

Finally, the analysis of roots shows that the complex conjugate roots $P_{\pm 1}$ play a decisive influence on dynamic processes in synchronous machine.

IV. SIMPLIFICATION OF VOLTAGES DIFFERENTIAL EQUATIONS IN A SYNCHRONOUS MACHINE STATOR WINDING

Stator currents I_1 will be searched considering that energy losses do not exist in rotor windings. In that case currents in synchronous machine windings are represented I_1'', I_2'', I_f'' and are called superconductors.

We consider the following equations:

$$\begin{aligned} U_1 &= R_1 I_1'' + L_1 \cdot P I_1'' + \omega \cdot E \cdot L_1 I_1'' + L_0 \cdot P I_0'' + \omega \cdot E \cdot L_0 \cdot I_0'' \\ 0 &= L_2 \cdot P I_2'' + L_0 \cdot P I_0''; \\ 0 &= L_f \cdot P I_f'' + L_0 \cdot P I_0'' \end{aligned}$$

$$\text{Where } I_0'' = I_1'' + I_2'' + I_f''$$

$$\text{Thus } I_2'' = I_2(0) - [(L_2 + L_0) \cdot L_f + L_2 \cdot L_0]^{-1} \cdot L_0 \cdot L_f \cdot I_1'';$$

$$I_f'' = I_f(0) - [(L_2 + L_0) \cdot L_f + L_2 \cdot L_0]^{-1} \cdot L_2 \cdot L_f \cdot I_1''.$$

Where $I_2(0), I_f(0)$ are initial currents values of rotor windings.

We consider $I_2(0) = 0$ and $I_f(0) = U_f / R_f$

$$\text{Thus } U_1 - E_f = R_1 I_1'' + \omega \cdot E \cdot L_0 \cdot I_1'' + L_0 \cdot P I_1'' \quad (1)$$

Where $L_0'' = \text{diag}(L_d'', L_q'')$ – diagonal matrix of supertransient inductances; $E_f = \omega \cdot E \cdot L_0 I_f(0)$.

Finally, in a developed form, equation (1) has the following aspect:

$$\begin{aligned} u_{1d} &= R_1 \cdot i_{1d}'' + L_d'' \cdot P i_{1d}'' - \omega \cdot L_q'' \cdot i_{1q}''; \\ u_{1q} - E_f &= R_1 i_{1q}'' + L_q'' \cdot P i_{1q}'' + \omega \cdot L_d'' \cdot i_{1d}'' \end{aligned} \quad (2)$$

$$\text{Where } E_f = \omega \cdot L_{dd} \cdot i_f(0)$$

The characteristic polynomial of equations (2)

$$Y_B(P) = P^2 + (R_1/L_d'' + R_1/L_q'') \cdot P + R_1^2/(L_d'' \cdot L_q'') + \omega^2$$

The roots of characteristic equation $Y_B(P) = 0$ are $P_{1,2} = \delta \pm j\Omega$, where $\Omega = \sqrt{\omega^2 + \delta^2} \approx \omega$

The plots of dependence of real part δ and imaginary part Ω for roots $P_{1,2}$ of characteristic equation $Y(P) = 0$ on speed ω are shown in figure 1a and figure 2a by discontinuous lines. Transient characteristics of currents, created by equation (2) for $\omega^* = 0,7$ and $u_{id}'' = u_{1q}'' = \sqrt{2}$ are shown on figure 3.

Finally, for given initial values that are determined by current captors, i_d'' and i_q'' are well predicted in modulation period by equations (2).

V. SIMPLIFICATION OF VOLTAGE DIFFERENTIAL EQUATION IN SYNCHRONOUS MACHINE EXCITATION WINDING

We search equations that characterize dynamic processes in excitation winding. We assume that damping loops have superconductive properties: $R_{2d} = 0, R_{2q} = 0$.

We have $0 = (L_{2d} + L_{dd}) \cdot Pi_{2d} + L_{dd} \cdot Pi_f + L_{dd} \cdot Pi_{1d}$
 And $Pi_{2d} = -[L_{dd}/(L_{2d} + L_{dd})] \cdot P(i_{1d} + i_f)$.
 Or $R_f \cdot i_f + (L_f + L'_d) \cdot Pi_f + L'_d \cdot Pi_{1d} = u_f$ (3)
 Where $L'_d = (1/L_{2d} + 1/L_{dd})^{-1}$

Mutual inductance between excitation winding and shortloop damping winding of rotor.

The simplified equation (3) is used for design of electromagnetic processes dynamics in rotor excitation winding.

VI. CONCLUSIONS

Dynamics processes with supply of excitation winding from e.m.f source and little rotor rotation speed have aperiodical character. For high rotation speed, dynamic processes have oscillating character.

When we supply excitation winding from current source, dynamic processes in the whole rotation speeds diapason variation will have oscillating character.

The oscillating character of dynamic processes in stator winding can be approximated by differential equations system of second order (2). The dynamic processes in excitation winding can be approximated by first order differential equation (3).

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FIGURES

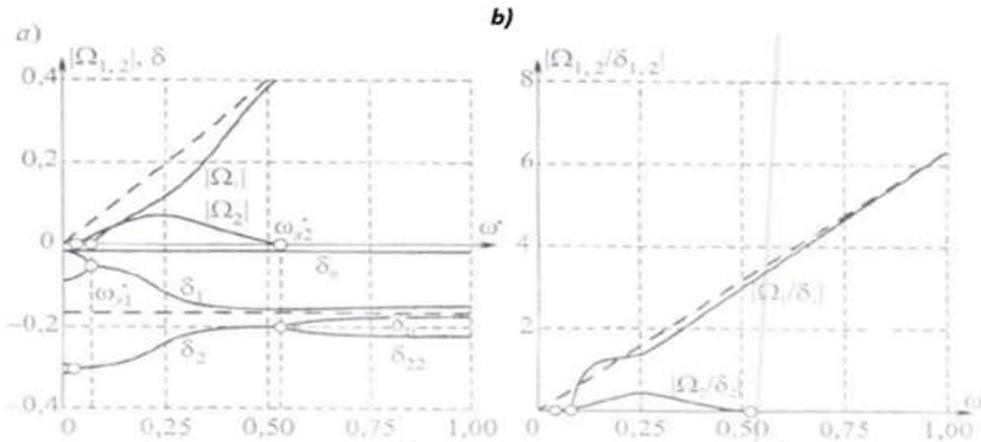


Figure 1: Dependences on Speed: a) real and imaginary parts of characteristic equation roots; b) damping factors (with supply of excitation winding from e.m.f source)

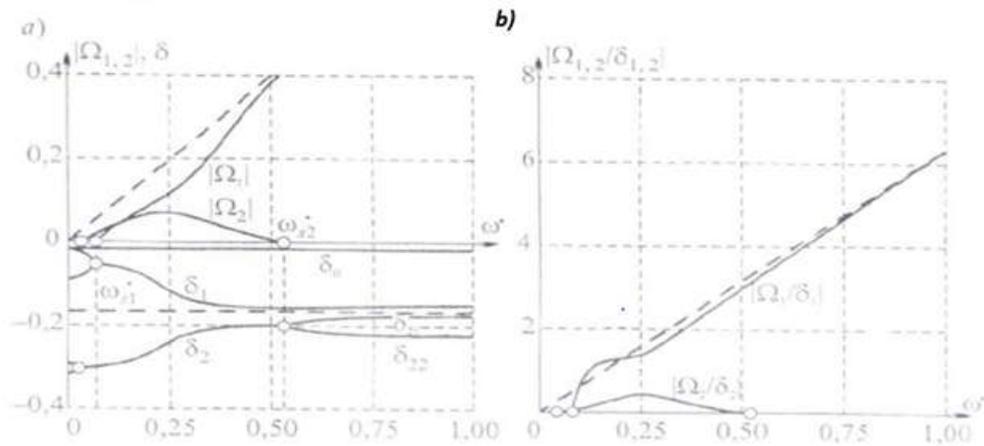


Figure 2: Dependences on speed: **a)** real and imaginary parts of characteristic equation roots; **b)** damping factors (with supply of excitation winding from current source)

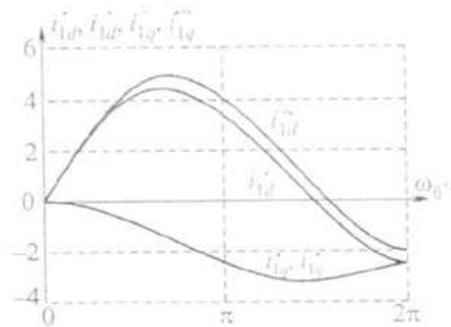


Figure 3: Transient characteristics of currents created by equations for $\omega^* = 0,7$ and $u_{1d}^* = u_{1q}^* = \sqrt{2}$

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