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Quotient Labeling of Some Ladder Graphs

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ABSTRACT : *Let G be a finite, non-trivial, simple and undirected graph with vertex set V and an edge set Eof order n and size m. For an one-one assignment f : V(G)→{1, 2, ... n}, A Quotient labelingf* : E(G)→ {1, 2, ... , n} is defined by* $f^*(uv)=\left\lfloor\frac{f(u)}{f(v)}\right\rfloor$ *where f(u) > f(v), then the edge labels need not be distinct. The maximum value of f*(E(G)) is known as* $q_l(f^*)$*, the q-labeling number.The quotient labeling number* $Q_L(G)$ *is the minimum value among* $q_l(f^*)$*. In this paper the quotient labeling number for a family ofladder graphs like open ladder, closed ladder, open triangular ladder, closed triangular ladder, slanting ladder, step ladder, open diagonal ladder are calculated.*

KEYWORDS:*closed ladder, open ladder, triangular ladder, slanting ladder, step ladder, open diagonal ladder, Mobius ladder..*

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I. INTRODUCTION

Graph labeling is an assignment of set of integers to the set of vertices, edges or both based on certain conditions. In 1967, Alex Rosa introduced the graph labeling problems. Graph labeling problems are useful family of mathematical models applied in many areas such as radar, missile guidance, radio frequency modulation, circuit designing and many more.Every year an updated survey comes about various types of labeling by J.A. Gallian [1]. From the survey, various types of labeling analyzed and introduced a new type of labeling called quotient labeling [6]. For notations and terminology we follow [2].

II. PRELIMINARIES

All graphs considered in this paper are finite, simple, non-trivial and undirected graphs. The definitions and terminologies that we are using in this paper are followed. The following definition that are relevant to this paper are used.

Definition:A ladder graph [3] L_n is defined by $L_n = P_n \times K_2$ where P_n is a path with n vertices and x denotes the Cartesian product and K_2 is a complete graph with two-vertices.

Definition: An Open ladder[8] $O(L_n)$, $n \geq 2$ is obtained from two paths of length n-1 with $V(G) = \{u_i, v_i : 1 \leq i \leq n\}$ and $E(G) = \{u_iu_{i+1}, v_iv_{i+1} : 1 \leq i \leq n-1\} \cup \{u_iv_i : 2 \leq i \leq n-1\}$.

Definition:A slanting ladder SL_n [5] is the graph obtained from two paths $u_1u_2 \dots u_n$ and $v_1v_2 \dots v_n$ by joining each u_i with v_{i+1}, $1 \leq i \leq n-1$.

Definition:A triangular ladder [7] TL_n, $n \geq 2$ is a graph obtained from L_n by adding the edges u_iv_{i+1}, $1 \leq i \leq n-1$. The vertices of L_n are u_i and v_i. u_i and v_i are the two paths in the graph L_n where $i = \{1, 2 \dots n\}$.

Definition: An open Triangular ladder [81] $O(TL_n)$, $n \geq 2$ is obtained from an open ladder $O(L_n)$ by adding the edges u_iv_{i+1} for $1 \leq i \leq n-1$.

Definition: Let P_n be a path on n vertices denoted by (1,1),(1,2),...,(1,n) and with n-1 edges denoted by $e_1, e_2, \dots, e_{n-1}$ where e_i is the edge joining the vertices (1,i) and (1,i+1). We erect a ladder with no of steps equal to n-(i-1). The number of steps include the edge e_i also where $i = \{1, 2 \dots n-1\}$. The graph obtained is called a step ladder graph [9] and is denoted by $S(T_n)$, where n denotes the number of vertices in the base.

Definition: An open diagonal ladder [8] $O(DL_n)$ is obtained from a diagonal ladder graph by removing the edges u_iv_i, for i = 1 and n.

Definition: A Mobius ladder graph M_n [4] is a graph obtained from the ladder $P_n \times P_2$ by joining the opposite end points of the two copies of P_n.

Definition:[6] Let G (V, E) be a finite, non-trivial, simple and undirected graph of order n and size m. For an one-one assignment $f : V(G) \rightarrow \{1, 2, \ldots. n\}$, A Quotient labeling $f^* : E(G) \rightarrow \{1, 2, \ldots, n\}$ is defined by $f^*(uv) = \left\lfloor \frac{f(u)}{f(v)} \right\rfloor$ where $f(u) > f(v)$, then the edge labels need not be distinct. The maximum value of $f^*(E(G))$ is known as $q_l(f^*)$, the q-labeling number. The Quotient Labeling Number $Q_L(G)$ is the minimum value among $q_l(f^*)$.

III. MAIN RESULT

Lemma: 3.1 The quotient labeling number of a ladder graph L_n is 3.

Proof: Let $G = L_n$be a ladder graph on 2n vertices with $V(G) = \{u_i, v_i : 1 \leq i \leq n\}$ and
$E(G) = \{u_i u_{i+1}, v_i v_{i+1} : 1 \leq i \leq n-1\} \cup \{u_i v_i : 1 \leq i \leq n\}$.
Define f: $V(G) \rightarrow \{1,2,\ldots.2n\}$ as follows: $f(u_i) = 2i$ for $1 \leq i \leq n$ $f(v_i) = 2i-1$ for $1 \leq i \leq n$ For the above vertex labeling we get $f^*(E(G)) = \{1, 2, 3\}$
Therefore the maximum value of $f^*(E(G))$ is equal to 3. Then $q_l(f^*) = 3$. Since minimum degree $\delta(G) = 2$ and maximum degree $\Delta(G) = 3$. Therefore $q_l(f^*)$ can take the value 3 or 4.
Here $q_l(f^*) = 3$ and is minimum. Hence $Q_L(L_n) = 3$.

Lemma: 3.2 The quotient labeling number of an open ladder graph $O(L_n)$ is 2.

Proof: Let $G = O(L_n)$be an open ladder graph on 2n vertices with
$V(G) = \{u_i, v_i : 1 \leq i \leq n\}$ and
$E(G) = \{u_i u_{i+1}, v_i v_{i+1} : 1 \leq i \leq n-1\} \cup \{u_i v_i : 2 \leq i \leq n-1\}$.
Define f: $V(G) \rightarrow \{1, 2, \ldots. 2n\}$ as follows: $f(u_i) = i$ for i = 1, 2.
$f(u_3) = 4$,
$f(v_1) = 5, f(v_2) = 3$
$f(v_i) = 2i$ for $3 \leq i \leq n$
$f(u_i) = 2i-1$ for $4 \leq i \leq n$For the above vertex labeling we get $f^*(E(G)) = \{1, 2\}$
Therefore the maximum value of $f^*(E(G))$ is equal to 2. Then $q_l(f^*) = 2$. Since minimum degree $\delta(G) = 1$ and maximum degree $\Delta(G) = 3$.Therefore $q_l(f^*)$ can take the value 2 or 3 or 4.
Here $q_l(f^*) = 2$ and is minimum.
Hence $Q_L(O(L_n)) = 2$.

Example: 3.3 The quotient labeling of an open ladder graph $O(L_{11})$ is shown below.

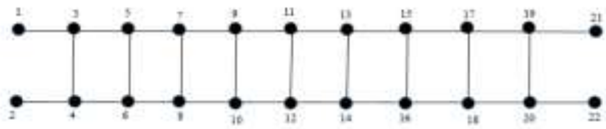

Fig.1. Quotient labeling of $O(L_{11})$

Theorem: 3.4 The quotient labeling number of (i) a ladder graph L_n is 3 (ii) an open ladder graph $O(L_n)$ is 2.

Proof: Case (i) Let $G = L_n$ be a ladder graph.
The Proof follows Lemma 3.1.
Case (ii) Let $G = O(L_n)$be an open ladder graph.
The Proof follows Lemma 3.3.

Theorem: 3.5 The quotient labeling number of a slanting ladder graph SL_n is 2.

Proof: let $G = SL_n$ be a slanting ladder graph on 2n vertices with $V(G) = \{v_i, u_i : 1 \leq i \leq n\}$ and
$E(G) = \{(v_i u_{i+1}), (v_i v_{i+1}), (u_i u_{i+1}) : 1 \leq i \leq n-1\}$. Define f: $V(G) \rightarrow \{1,2,\ldots.,2n\}$ as follows:
$f(u_1) = 1$, $f(u_i) = 2(i-1)$ for $2 \leq i \leq n$. $f(v_i) = 2i+1$ for
$1 \leq i \leq n-1$. $f(v_n) = 2n$.
For the above vertex labeling $f^*(E(G)) = \{1, 2\}$. Therefore the
maximum value of $f^*(E(G))$ is equal to 2.
Then $q_l(f^*) = 2$. But in G, $\delta(G) = 1$ and $\Delta(G) = 3$. Therefore $q_l(f^*)$ can take the value 2 or 3 or 4.
Here $q_l(f^*) = 2$ and it is minimum.
Hence $Q_L(SL_n) = 2$.

Example: 3.6 The quotient labeling of any slanting ladder SL_{10} is shown below.

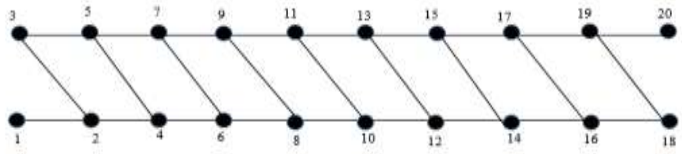

Fig.2. Quotient labeling of SL_{10}

Lemma: 3.7 The quotient labeling number of a triangular ladder TL_n, $n\geq 2$ is 3.
Proof: let G = TL_n be any triangular ladder graph on 2n vertices with
$V(G)= \{u_i, v_i : 1 \leq i \leq n\}$ and
$E(G) = \{u_i u_{i+1}, v_i v_{i+1} : 1 \leq i \leq n-1\} \cup \{u_i v_i : 1 \leq i \leq n\} \cup \{v_i u_{i+1} : 1 \leq i \leq n-1\}$.
Define f: V(G) $\rightarrow \{1,2,\ldots,2n\}$ as follows:
f(ui)= 2i-1 for $1\leq i\leq n$
$f(v_i)=2i$ for $1\leq i\leq n$.
For the above vertex labeling f*(E(G)) = {1, 2, 3}.Therefore the maximum value of f*(E(G)) is equal to 3. Then $q_l(f^*)=3$. But in G, the minimum degree $\delta(G)=2$ and maximum degree $\Delta(G)=4$.
Therefore $q_l(f^*)$ can take the value 3 or 4 or 5. Here $q_l(f^*)=3$ and it is minimum.
Hence $Q_L(TL_n)=3$.

Example: 3.8 The quotient labeling of the triangular ladder TL_{10} is shown below.

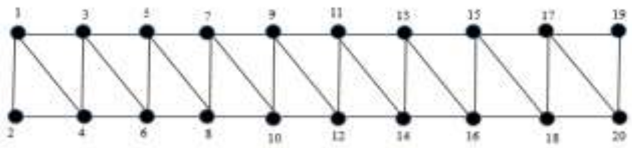

Fig.3. Quotient labeling of TL_{10}

Lemma: 3.9 The quotient labeling number of any open triangular ladder $O(TL_n)$, $n\geq 2$ is 2.
Proof: let G = $O(TL_n)$ be any triangular ladder graph on 2n vertices with
$V(G) = \{u_i, v_i : 1 \leq i \leq n\}$ and $E(G) = \{u_i u_{i+1}, v_i v_{i+1} : 1 \leq i \leq n-1\} \cup \{u_i v_i : 2 \leq i \leq n-1\} \cup \{v_i u_{i+1} : 1 \leq i \leq n-1\}$. Define f: V(G) $\rightarrow \{1, 2, \ldots 2n\}$ as follows:
$f(u_i) = i$ for i =1,2.
$f(u_i) = 2i-1$ for $3\leq i\leq n$.
$f(v_1) = 3$,
$f(v_i)=2i$ for $2\leq i\leq n$. For the above vertex labeling f*(E(G)) = {1, 2}.
Therefore the maximum value of f*(E(G)) is equal to 2.
Then $q_l(f^*)=2$. But in G, the minimum degree $\delta(G)=1$ and maximum degree $\Delta(G)=4$.
Therefore $q_l(f^*)$ can take the value 2 or 3 or 4 or 5.
Here $q_l(f^*)=2$ and it is minimum.
Hence $Q_L(O(TL_n))=2$.

Example: 3.10 The quotient labeling of an open triangular ladder $O(TL_{10})$ is shown below.

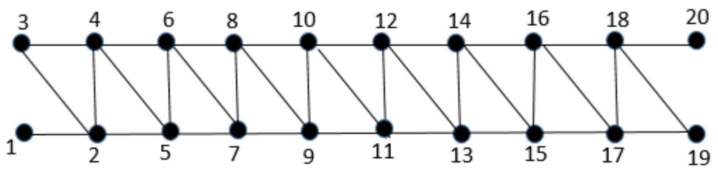

Fig.4. Quotient labeling of $O(TL_{10})$

Theorem: 3.11 The quotient labeling number of (i) a triangular ladder TL_n is 3, (ii) an open triangular ladder $O(TL_n)$ is 2.
Proof: Case (i) Let G =TL_n be a triangular ladder graph.

The Proof follows Lemma 3.7.
Case (ii) Let G = $O(TL_n)$be an open triangular ladder graph.
The Proof follows Lemma3.9.

Theorem: 3.12 The quotient labeling number of a step ladder graph $S(T_n)$ is 3.
Proof: Let G = $S(T_n)$ be a step ladder graph with n-(i-1) steps for $1\leq i\leq n$.
Let $v_{1,j}$ be the n vertices on the base where $1\leq j\leq n$.
Let $v_{2,j}$ be the n vertices on the second stage above the base for $1\leq j\leq n$.
Let $v_{3,j}$ be the n-1 vertices on the third step for $1\leq j\leq n-1$, proceeding like this we have vertices for n-(i-1) steps. Now the vertices of $S(T_n)$ is denoted by $v_{i,j}$, where i denote the row from bottom to top and j denote the column from left to right and $1\leq i\leq n$, $1\leq j\leq n$.Now the graph $S(T_n)$ has $\frac{n^2+3n-2}{2}$ vertices and n(n+1)-2 edges with $\deg(v_{1,1}) = \deg(v_{1,n}) = \deg(v_{2,n}) = \deg(v_{n,1}) = 2$, $\deg(v_{i,n-i+2})= 2$ for $3\leq i\leq n$, $\deg(v_{i,1}) = 3$ for $2\leq i\leq n-1$, $\deg(v_{1,j}) =3$ for $2\leq j\leq n-1$ and $\deg(v_{i,j}) =4$ for $1\leq i\leq n-1$, $1\leq j\leq n-1$ and $j \neq n-i+2$.
Define f: $V(G) \rightarrow\{1,2,\ldots, \frac{n^2+3n-2}{2}\}$ as follows:
$f(v_{1,1}) =1$,
$f(v_{1,j}) = f(v_{1,j-1}) +j$ for $2\leq j\leq n$ $f(v_{i,1}) = f(v_{i-1,1}) +i-1$ for $2\leq i\leq n$
$f(v_{i,j}) = f(v_{i-1,j}) +i +j-2$ for $2\leq j\leq n-1$, $2\leq i\leq n-1$ $f(v_{i,j}) = f(v_{i-1,j}) +i +j-3$ for $2\leq j\leq n$, $2\leq i\leq n$ and $(i+j) = n+2$.
For the above vertex labeling $f^*(E(G)) = \{1, 2, 3\}$. Therefore the maximum value of $f^*(E(G))$ is equal to 3. Then $q_l(f^*) =3$. But in G, $\delta(G) =2$ and $\Delta(G) =4$.
Therefore $q_l(f^*)$ can take the value 3 or 4 or 5.Here $q_l(f^*) =3$ and it is minimum.
Hence $Q_L(S(T_n))=3$.

Example: 3.13 The quotient labeling of the step ladder $S(T_6)$ is shown below.

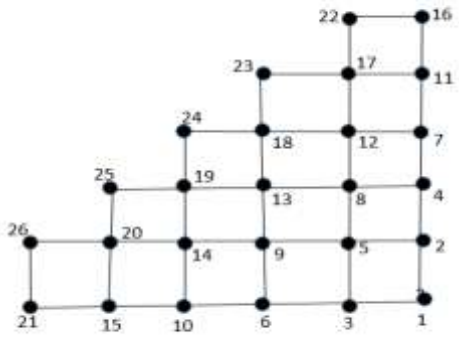

Fig.5. Quotient labeling of $S(T_6)$

Theorem: 3.14 The quotient labeling number of an open diagonal ladder graph $O(DL_n)$ is 3.
Proof: Let G =$O(DL_n)$ with
$V(G) = \{u_i, v_i : 1\leq i\leq n\}$ and $E(G) = \{(u_iu_{i+1}), (v_iv_{i+1}), (u_iv_{i+1}), (v_iu_{i+1}) : 1\leq i\leq n-1\} \cup \{(u_iv_i) : 2\leq i\leq n-1\}$
Define f: $V(G) \rightarrow \{ 1, 2, \ldots, 2n\}$ by
$f(u_i) = 2i-1$ for $1\leq i\leq n$
$f(v_1) = 4$, $f(v_2) =2$,
$f(v_i) = 2i$ for $3\leq i\leq n$.
For the above vertex labeling we get $f^*(E(G)) = \{1, 2, 3\}$
Therefore the maximum value of $f^*(E(G))$ is equal to 3.
Then $q_l(f^*) = 3$. Since minimum degree $\delta(G) = 2$ and maximum degree $\Delta(G) = 5$. Therefore $q_l(f^*)$ can take the value 3 or 4 or 5 or 6. Here $q_l(f^*) = 3$ and it is minimum. Hence $Q_L(G) = 3$.

Example: 3.15 The quotient labeling of the composition graph $O(DL_{10})$ is shown below.

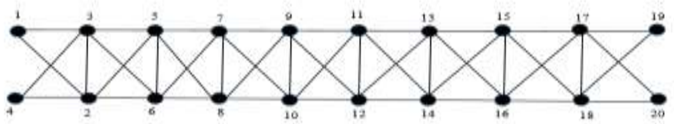

Fig.6. Quotient labeling of $O(DL_{10})$

Theorem: 3.16 The Quotient labeling number of a Mobius ladder graph M_n is 4.**Proof:** Let G = M_n with V(G) = $\{v_i, u_i : 1\leq i\leq n\}$ and E(G) = $\{\{(u_iu_{i+1}), (v_iv_{i+1}) : 1\leq i\leq n-1\} \cup \{(u_iv_i), (u_1v_n), (u_nv_1) : 1\leq i\leq n\}$.
We prove this theorem on two different cases on n.
Case (i): n is odd. Define f: V(G) → { 1, 2, ….., 2n} by
$f(v_i) = 2i-1$ for i = 1, 2.
$f(v_i) = 4(i-1) -1$ for $3\leq i\leq \left\lceil\frac{n}{2}\right\rceil$
$f(u_1) = 2$ $f(u_i) = 4i-3$ for $2\leq i\leq \left\lceil\frac{n}{2}\right\rceil$ $f(v_{n-i}) = 4(i+1)+2$ for $0\leq i\leq \left\lceil\frac{n}{2}\right\rceil - 2$
$f(u_{n-i}) = 4(i+1)$ for $0\leq i\leq \left\lceil\frac{n}{2}\right\rceil - 2$ For the above vertex labeling we get f*(E(G)) = {1, 2, 3, 4}
Therefore in this case the maximum value of f*(E(G)) is equal to 4.
Case (ii): n is even. Define f: V(G) → { 1, 2, ….., 2n} by
$f(v_i) = 2i-1$ for i = 1, 2.
$f(v_i) = 4(i-1) -1$ for $3\leq i\leq \frac{n}{2}+1$
$f(u_1) = 2f(u_i) = 4i-3$ for $2\leq i\leq \frac{n}{2}$ $f(v_{n-i}) = 4(i+1)+2$ for $0\leq i\leq \frac{n}{2} - 2$
$f(u_{n-i}) = 4(i+1)$ for $0\leq i\leq \frac{n}{2} - 1$ For the above vertex labeling we get f*(E(G)) = {1, 2, 3, 4}
Therefore in this case the maximum value of f*(E(G)) is equal to 4. By cases (i) and (ii) the maximum value of the quotient labeling is 4.
Then $q_l(f^*) = 4$. Since in G, $\delta(G) = \Delta(G) = 3$.
Therefore $q_l(f^*)$ can take the only value 4.
Here $q_l(f^*) = 4$ and it is minimum. Hence $Q_L(G) = 4$.

Example: 3.17 The quotient labeling of a Mobius ladder graph M_{12} is shown below.

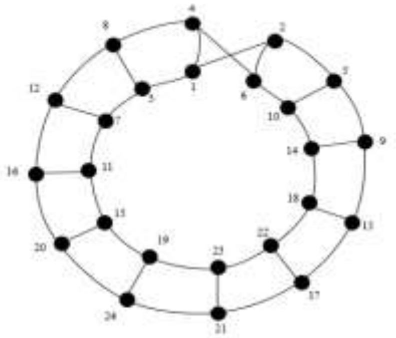

Fig.7. Quotient labeling of M_{12}

IV. CONCLUSION

Quotient labeling number for some ladder graphs are calculated in this paper Calculating quotient labeling number for other family of graphs is our future work.

REFERENCES

[1]. Gallian J.A., Dynamic Survey of Graph Labeling, Electronic Journal of Combinatorics 18, DS6 (2015).
[2]. Harary F., Graph Theory, Narosa Publishing House Reading, New Delhi (1988).
[3]. M. I. Moussa, and E. M. Badr, Ladder and subdivision of ladder graphs with pendant edges are odd graceful, International Journal on Applications of Graph Theory in Wireless Ad hoc Networks and Sensor Networks, vol.8, No.1, 1-8 (2016).
[4]. M. Joice Punitha, and S. Rajakumari, Skew chromatic index of comb, ladder and Mobius ladder graphs, International Journal of Pure and Applied Mathematics, vol. 101, no. 6, 1003-1011 (2015).
[5]. P. Sugirtha, R. Vasuki and J. Venkateswari, Some new super mean graphs, International Journal of Mathematics Trends and Technology, vol. 19, no.1, 62-73 (2015).
[6]. P. Sumathi, and A. Rathi, Some results on quotient labeling of special graphs, Global Journal of Pure and Applied Mathematics, vol. 13, no. 1, 176-178 (2017).
[7]. P. Jeyanthi, A. Maheswari, and P. Pandiaraj, One modulo three mean labeling of cycle related graphs, International Journal of Pure and Applied Mathematics, vol. 103, no. 4, 625-633 (2015).
[8]. P. Sumathi, A. Rathi, and A. Mahalakshmi, Quotient labeling of corona of ladder graphs, International Journal of Innovative Research in Applied Sciences and Engineering (IJIRASE), vol. 1, no. 3, 80-85 (2017).
[9]. S. K. Vaidya, and LekhaBijukumar, Mean labeling for some new families of graphs, PRAJNA –Journal of Pure and Applied Sciences, vol. 18, 115-116 (2010).

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