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Effect of Rotatory Inertial And Damping Coefficient on The Transverse Motion of Uniform Rayleigh Beam Under Moving Loads of Constant Magnitude

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ABSTRACT: *In this work, the effect of rotatory inertial and damping co-efficient on the transverse motion of Rayleigh beam subjected to moving loads of constant magnitude is investigated. The solution techniques employed is based on the Fourier Sine Transform, Laplace Integral Transformation and Convolution Theorem. It is observed that, the amplitude of deflection of the beam under the influence of moving load of constant magnitude decreases with an increase in the value of the rotatory inertial and damping co-efficients. Also as the values of the other structural parameters such as shear modulus, foundation modulus and axial force increases, the amplitude of vibration of the beam decreases. Finally, it is observed the effect of rotatory inertial is significant compared with damping co-efficient effect.*

Keywords: *Constant magnitude, Moving loads, Damping coefficient, Rotatory inertial, Shear modulus, Foundation modulus, Axial force.*

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I. INTRODUCTION

The behaviour of elastic structures such as beams under various forms of moving loads is of great theoretical and practical importance in the field of sciences and engineering. Among the earliest researchers on the dynamical analysis of an elastic beam was Ayre et al. [1], who studied the effect of the ratio of the weight of the load to the weight of the simply supported beam for a constantly moving mass load. They obtained the exact solution for the resulting partial differential equation by using the infinite series method. Kenny [2], investigated the dynamic response of infinite beams on an elastic foundations under the action of moving load of constant speed. The response of a finitely supported Bernoulli-Euler beam to a unit force moving at a uniform velocity was investigated by Steel [3]. The effects of this moving force on beams with and without an elastic foundation were analysed. Later, Hamada [4] applied the double Laplace transformation with respect to both time and the length coordinate along the beam to obtain solution of simply supported Euler-Bernoulli beam of finite length transversed by a constant force moving at a uniform speed.Severally other researchers have made tremendous feat in the study of dynamic of structure under moving loads in the recent years including Oni and Jimoh [5], Hsu [6], Civalek et al [7], Oni and Jimoh [8], Oni and Jimoh [9], Liu and Chang [10], Misra [11], Oni and Ogunbamike [12]. In the aforementioned, comparative effect of rotatory inertia and damping coefficients are not taken into consideration. To this end, Oni and Awodola [13] considered the dynamic response to moving concentrated masses of uniform Rayleigh beams resting on variable wrinklier elastic foundation neglecting damping coefficient. Oni and Omolofe [14], also considered the dynamic analysis of uniform Rayleigh beam subjected to concentrated moving loads moving with variable velocity with damping coefficient neglected.
Thus, this work investigated the dynamic response to moving concentrated load of uniform simply supported Rayleigh beam subjected to constant magnitude moving load.

II. MATHEMATICAL MODEL

The governing partial differential equation of uniform Rayleigh beam of length L and transverse by a moving load of constant magnitude $P(x, y)$ of mass Emoving with velocity v is given [15] as:

$$EI\frac{\partial^4 Z(x,t)}{\partial x^4} + \bar{m}\frac{\partial^2 Z(x,t)}{\partial x^2} - N\frac{\partial^2 Z(x,\ t)}{\partial x^2} - \bar{m}R_0^2\frac{\partial^4 Z(x,t)}{\partial x^2 \partial t^2} + \epsilon\frac{\partial Z(x,t)}{\partial t} + K\,Z(x,t) + \mathrm{P}f\frac{\partial^2 Z(x,t)}{\partial x^2} = P(x,t) \quad (1)$$

Where:
EI $=$ flexural rigidity of the beam,
$Z(x,t)$ $=$ transverse deflection,
$N =$ Axial force,
K $=$ foundation modulus,
$\mathrm{P}f$ $=$shear modulus,
R_0^2 $=$ rotatory inertia,
$\bar{m}$ $=$ mass per unit length of the beam,
ϵ $=$ damping coefficient,
$\mathrm{P}(x,t)$ $=$ Applied force defined as

$$\mathrm{P}(x,t) = \begin{cases} Pf(x - vt) & \\ 0, & \text{Otherwise} \end{cases} \quad (2)$$

Such that $f\ (x - vt) =$ Dirac delta function which is defined as the unit impulse function of point $x = vt$.
v $=$ constant velocity of load motion,
p $=$ moving force of constant magnitude.
The associated boundary conditions are given as:

$$Z(0,t) = 0 = Z(L,t)$$

$$\frac{\partial^2(0,t)}{\partial x^2} = 0 = \frac{\partial^2 Z(L,t)}{\partial x^2}$$

And the initial conditions are

$$Z(x,0) = 0 = \frac{\partial Z(x,0)}{\partial t} \quad (3)$$

Solution of the Mathematical Problem (4)

The effective applicable method of handling Eq. (1), Eq. (3), and Eq. (4) is integral transforma techniques specifically, the Fourier transformation for the length coordinate and Laplace transformation for the time coordinate with the given boundary and initial conditions. The Finite Fourier Sine Integral transformation for the length coordinate is defined as

$$Z(n,t) = \int_0^L Z(x,t)\sin\frac{n\pi x}{L}\,dx$$

With the inverse transformation defined as (5)

$$Z(x,t) = \frac{2}{L}\sum_{n=1}^{\infty} Z(n,t)\sin\frac{n\pi x}{L}$$

Invoking Eq. (5), with the consideration of the boundary conditions in Eq. (3) on Eq. (1); we have (6)

$$\left(1 + R_0^2\frac{n^2\pi^2}{L^2}\right)Z_{tt}(n,t) + \frac{\epsilon}{\bar{m}}Z_t(n,t) + \frac{EI}{\bar{m}}\left(\frac{n\pi}{L}\right)^4 + \frac{N}{\bar{m}}\left(\frac{n\pi}{L}\right)^2 - \frac{Pf}{\bar{m}}\left(\frac{n\pi}{L}\right)^2 + \frac{K}{\bar{m}}Z(n,t) = \frac{\mathrm{P}}{\bar{m}}\sin\frac{n\pi vt}{L}$$

Eq. (7) can conveniently be written as

$$Z_{tt}(n,t) + b_{11}Z_t(n,t) + b_{12}Z(n,t) = b_{13}\sin b_0 t \quad (8)$$

where

$$a_0 = 1 + R_0^2\frac{n^2\pi^2}{L^2},$$

$$a_1 = \frac{\epsilon}{\bar{m}},$$

$$a_3 = \frac{EIn^4\pi^4}{\bar{m}L^2}, a_4 = \frac{n^2\pi^2}{\bar{m}L^2}(N - Pf),$$

$$a_5 = \frac{K}{\bar{m}},$$

$$a^6 = \frac{\mathrm{P}}{\bar{m}}, \quad (9)$$

$$P = mg,$$
$$b_{11} = \frac{a_1}{a_0}, b_{12} = \frac{a_3 + a_4 + a_5}{a_0},$$
$$b_{13} = \frac{a_6}{a_0},$$
$$b_0 = \frac{n\pi v}{L}$$

Eq. (8) represent the finite Fourier transformed governing equation of the Rayleigh beam subjected to a constant magnitude moving load with constant.

Laplace Transformed Solution

To solve Eq. (8) we apply the method of Laplace Integral transformation for the time coordinate between zero and infinity. The operation of Laplace transform is indicated by the notation

$$L(f(t)) = \int_0^\infty f(t)e^{-st}dt$$

Where 'L' and 'S' are the Laplace transformed operator and Laplace transformed variable respectively.
In particular, we use

$$L(Z(n,t)) = Z(n,s) = \int_0^\infty Z(n,t)e^{-st}dt \tag{10}$$

(11)

Using the transformation (11) on Eq. (8) in conjunction with the set of initial conditions given in Eq. (upon simplification, we obtained

$$Z(n,t) = \left(\frac{b_{13}}{(s-c_1)(s-c_2)}\right)\left(\frac{b_0}{s^2 + b_0^2}\right)$$

where

(12)

$$c_1 = \frac{-b_{11}}{2} + \frac{\sqrt{b_{11}^2} - 4b_{12}}{2}$$
$$c_2 = \frac{-b_{11}}{2} - \frac{\sqrt{b_{11}^2} - 4b_{12}}{2} \tag{13}$$

Eq. (12) can be simplified to obtain

$$Z(n,s) = \frac{b_{13}}{c_1 - c_2}\left[\left(\frac{1}{s-c_1}\right)\left(\frac{b_0}{s^2 + b_0^2}\right) - \left(\frac{1}{s-c_2}\right)\left(\frac{b_0}{s^2 + b_0^2}\right)\right]$$

in order to obtain the Laplace inversion of Eq. (14), we shall adopt the following representations (14)

$$F_1(s) = \frac{1}{s-c_1},$$
$$F_2(s) = \frac{1}{s-c_2}$$
$$G_2(s) = \frac{b_0}{s^2 - b_0^2}$$

So that the Laplace inversion of Eq. (14) is the convolution of f_1 and g defined as (15)

$$f_i * g = \int_0^t f_i(t-u)g(u)du, \qquad i = 1,2$$

Thus, the Laplace inversion of q. (14) is given by

$$Z(n,t) = Q(e^{c_1 t}N_1 - e^{c_2 t}N_2)$$

Where (17)

$$N_1 = \int_0^t e^{-c_1 u}\sin b_0\, u du,$$
$$N_2 = \int_0^t e^{-c_2 u}\sin b_0\, u du;$$
$$Q = \frac{b_{13}}{c_1 - c_2} \tag{18}$$

Evaluating the integral of the first two equations of Eq. (18)

$$N_1 = \frac{b_0}{b_0^2 + c_1^2}(1 - \cos b_0 t e^{-c_1 t}) - \frac{c_1}{b_0^2 + c_1^2}\sin b_0 t e^{-c_1 t}$$
$$N_2 = \frac{b_0}{b_0^2 + c_2^2}(1 - \cos b_0 t e^{-c_2 t}) - \frac{c_2}{b_0^2 + c_2^2}\sin b_0 t e^{-c_2 t} \tag{19}$$

Using Eq. (19) in Eq. (17), and upon evaluation: we have

$$Z(n,t) = \frac{Qb_0}{b_0^2+c_1^2}(e^{c_1 t} - \cos b_0\, t) - \frac{Qc_1}{b_0^2+c_1^2}\sin b_0\, t - \frac{Qb_0}{b_0^2+c_2^2}(e^{c_2 t} - \cos b_0\, t) - \frac{Qc_2}{b_2^2+c_2^2}\sin b_0\, t \tag{20}$$

Putting Eq. (20) into Eq. (6), we have

$$Z(n,t) = \frac{2}{L}\sum_{n=1}^{\infty}\{\frac{Qb_0}{b_0^2+c_1^2}\left(e^{c_1 t} - \cos b_0 t\right) - \frac{Qc_1}{b_0^2+c_1^2}\sin b_0\, t - \frac{Qb_0}{b_0^2+c_2^2}(e^{c_2 t} - \cos b_0\, t) - \frac{Qc_2}{b_2^2+c_2^2}\sin b_0\, t\Big\}\frac{\sin n\pi x}{L} \tag{21}$$

Eq. (21) represents the dynamic response of a uniform elastic Rayleigh beam subjected to moving load of constant magnitude.

III. Numerical Analysis And Discussion Of Results

In order to illustrate the analytical results, the uniform elastic Rayleigh beam is taken to be of length 12.192*m*, velocity taken to be 8.12*m/s*, the flexural Rigidity EI is 6068242m^3/s^2. The values of the foundation modulus, shear modulus, damping coefficient, rotatory inertia; and the axial force are varied respectively. Important phenomena are observed in all computations: the effect of damping coefficient on the dynamic response of uniform Rayleigh beam under moving loads of constant magnitude is shown in Figure 1. Increasing the damping coefficient, increased significantly the magnitude of the deflection of the beam while other parameters were kept constant under moving loads of constant magnitude. When the value of the damping coefficient decreases from 5.00 to 0.00, the magnitude of the deflected peak decrease from 211.00 to 210.00 for moving loads of constant magnitude. Figure 2 reflects the effect of the rotatory inertia on the dynamic response of uniform Rayleigh beam under moving loads of constant magnitude. In this case, increase in the rotatory inertia, increase significantly the magnitude of the deflection of the beam.at mid-point of the beam there is peak deflection of the beam. At mid-point of the beam, there is peak deflection for the varied rotatory inertia while other parameters were kept constant.Figure 3, and Figure 4 show the effect of foundation modulus and the shear modulus on the dynamic response of uniform Rayleigh beam under moving

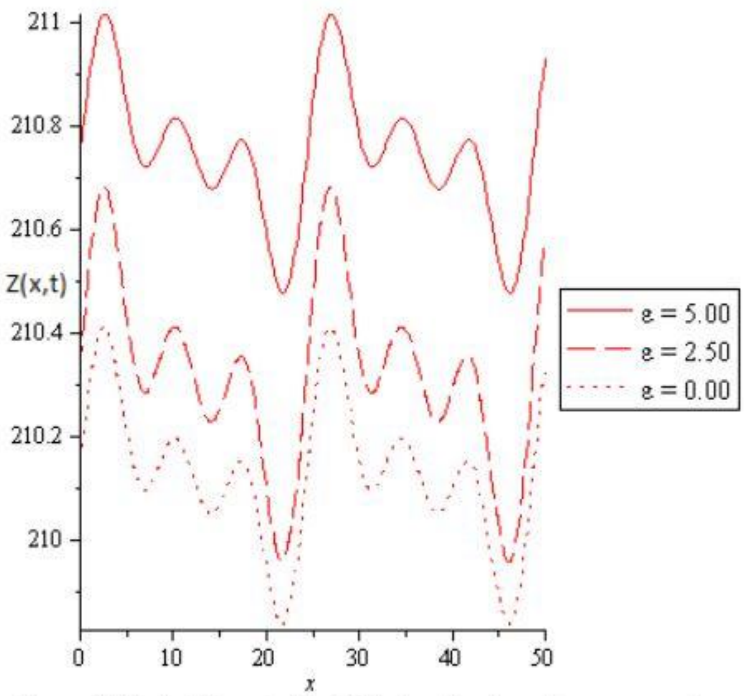

Figure 1: Effect of Damping Coefficient on the dynamic response of a uniform Rayleigh beam subjected to moving load of constant magnitude

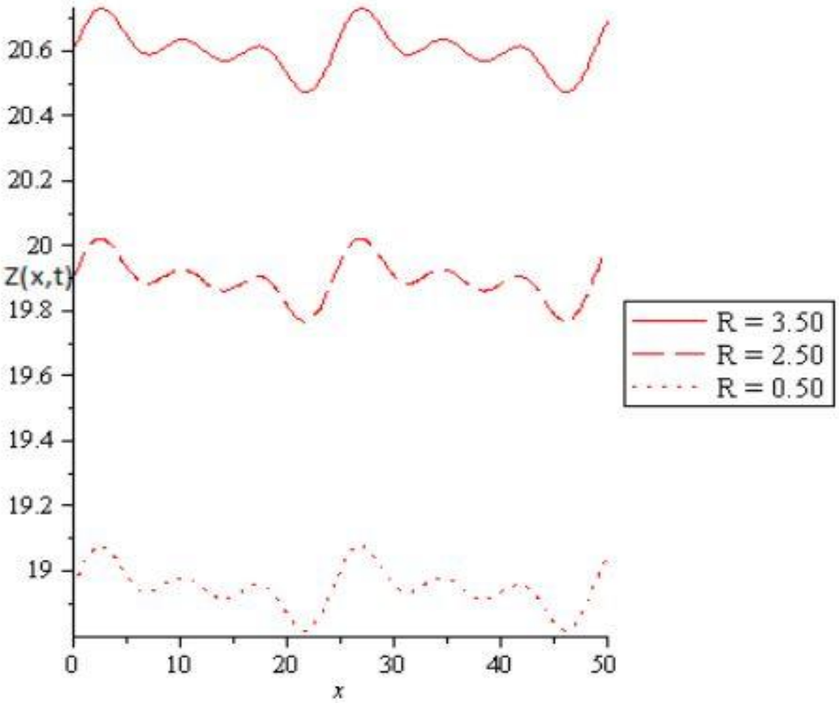

Figure 2: Effect of Rotating Inertia on the dynamic response of a uniform Rayleigh beam subjected to moving load of constant magnitude

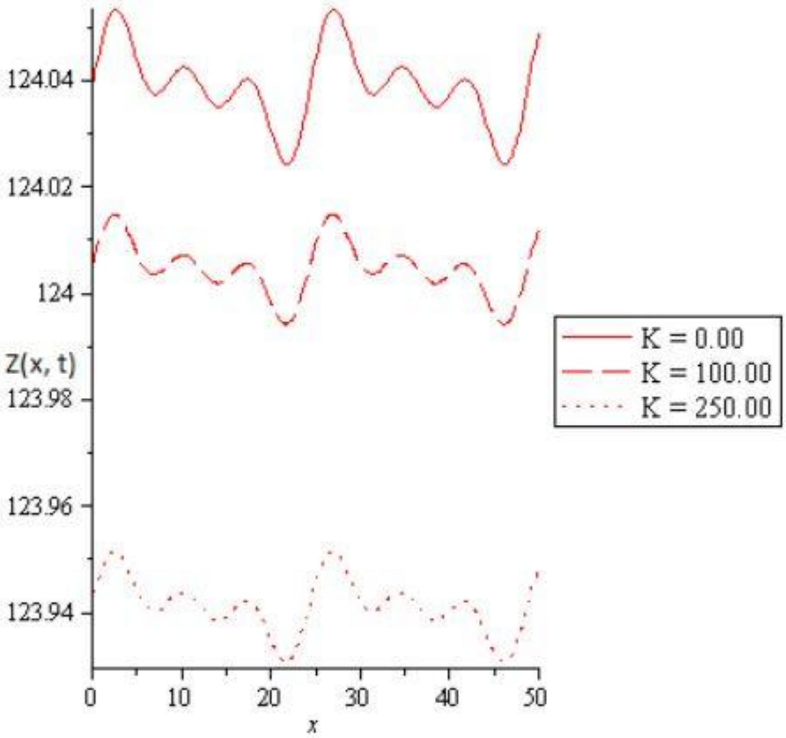

Figure 3: Effect of Foundation Modulus on the dynamic response of a uniform Rayleigh beam subjected to moving load of constant magnitude

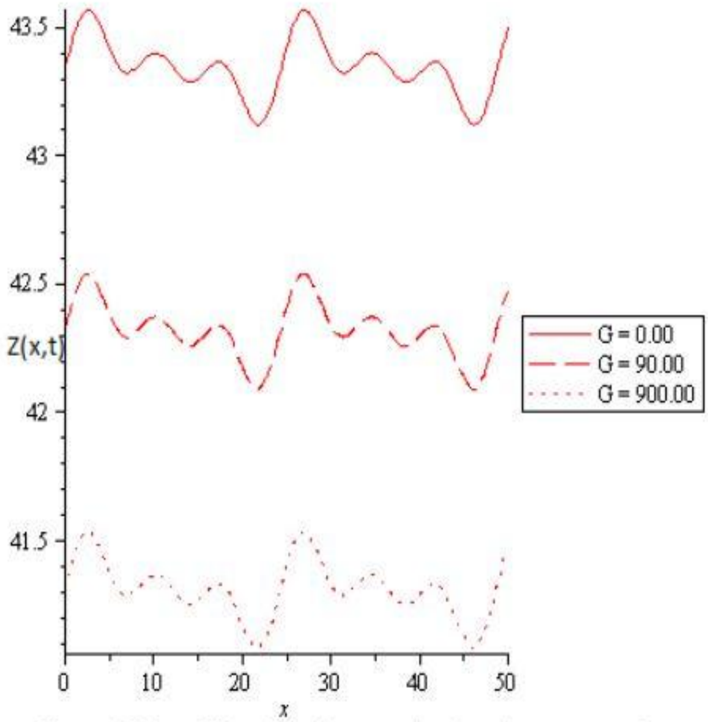

Figure 4: Effect of Shear Modulous on the dynamic response of a uniform Rayleigh beam subjected to moving load of constant magnitude

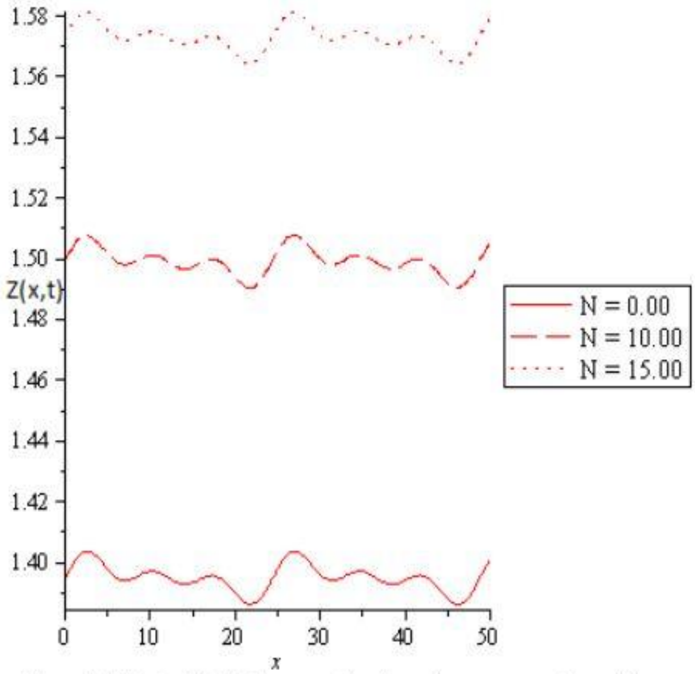

Figure 5: Effect of Axial Force on the dynamic response of a uniform Rayleigh beam subjected to moving load of constant magnitude

loads of constant magnitude respectively. In both cases, it is shown that increase in foundation modulus and shear modulus decreases the magnitude of the peak of deflection of the material. The rate of decrease in agnitude of deflection is proportional to the respective varied parameters. For the Foundation Modulus, the magnitude decrease from 124.07 to 123.95 at varies interval of 0,100.000 and 150.00; while the decrease from 124.07 to 123.95 at varies interval of 0.00, 90.00 and 900.00.Figure 5 depicts the effect of the Axial Force on the dynamic response of the uniform Rayleigh beam. Increasing the Axial Force causes significant increase in the magnitude of deflection of the beam.

IV. Conclusion

This work investigates the effect of rotatory inertial and damping coefficient on the transverse motion of Rayleigh beam subjected to moving loads of constant magnitude. Fourier Sine Transform Laplace Integral Transformation and Convolution Theorem are the solution techniques employed in obtaining the closed form solution of the transverse motion. It is observed that, the amplitude of deflection of the beam under the influence of moving load of constant magnitude decreases with an increase in the value of the rotatory inertial and damping coefficients. Also as the values of the other structural parameter such as shear modulus, foundation modulus and axial force increases, the amplitude of vibration of the beam decreases. Finally, it is observed the effect of rotatory inertial is significant compared with damping coefficient effect.Detailed analyses are also performed to investigate the effect of various parameters such as foundation stiffness and shear modulus on dynamic deflection.

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