

Influence of Electric Field Flow on MHD Nano-Fluid Over a Stretching Sheet

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ABSTRACT: This study delves into the dynamics of electric field flow in a steady magnetohydrodynamic (MHD) fluid over a stretching sheet. Magnetohydrodynamics holds significant importance in comprehending the behavior of conductive fluids amidst magnetic fields, offering applications across diverse engineering and scientific domains. The intriguing phenomena arising from the interaction between electric fields and fluid flow on a stretching surface carry practical implication. Through rigorous mathematical modeling and numerical analysis, this research endeavors to clarify the intricate interplay between electric field effects and fluid flow characteristics, providing insights into the underlying mechanisms governing the system's behavior. Understanding these dynamics is imperative for optimizing processes across industries such as aerospace, energy, and materials manufacturing.

KEYWORDS: Electric field, Magnetohydrodynamics, Stretching sheet, steady, flow.

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I. INTRODUCTION

The dynamic interaction between fluid flow and external forces has captivated the attention of researchers and engineers for decades, driven by its relevance to numerous scientific and engineering applications. Among the myriad configurations studied, the flow of a magnetohydrodynamic (MHD) fluid over a stretching surface holds particular significance due to its prevalence in industrial processes, environmental phenomena, and technological advancements. In recent years, the exploration of how electric fields influence this flow regime has emerged as a focal point of investigation, offering new insights into the complex interplay between electromagnetic forces and fluid dynamics. The pioneering work of [1] introduced the concept of boundary layer flow over a stretching sheet. [1] analysis laid the foundation for subsequent investigations into similar flow configurations. Also, the integration of magnetic fields into the analysis of fluid flow over stretching surfaces marked a significant advancement. [2] examined the influence of a transverse magnetic field on the flow over a stretching sheet. Their study demonstrated the profound alterations induced in the boundary layer characteristics due to the presence of magnetic fields. In the same approach, the introduction of electric fields adds another layer of complexity to the fluid flow phenomenon. [3] investigated the impact of an electric field on the flow and heat transfer characteristics over a stretching sheet. Their findings highlighted the significant alterations in velocity and temperature profiles induced by the electric field. Then, building upon the individual investigations of magnetic fields and electric fields, studies have explored the combined effects of these fields on fluid flow. [4] conducted a comprehensive analysis of MHD flow induced by a stretching surface in the presence of both magnetic and electric fields. Their study elucidated the intricate interplay between these fields and their influence on boundary layer characteristics. In recent times, [5] Investigate the Effects of Magnetic Fields on MHD Flows in Industrial Applications. [6] studied Numerical Simulation of MHD Flow and Heat Transfer in a Rotating Channel with Porous Walls. [7] Analyze MHD Mixed Convective Flow Over a Stretching Sheet with Chemical Reaction and Thermal Radiation. [8] studied MHD Flow and Heat Transfer Characteristics in a Curved Channel with Porous Medium and Heat Source. [9] did Analytical Study of MHD

Casson Fluid Flow over a Stretching/Shrinking Sheet with Thermal Radiation and Heat Source/Sink. [10] Did a Numerical Study of Unsteady MHD Flow and Heat Transfer Over a Stretching Sheet in Presence of Heat Source/Sink and Chemical Reaction. [11] investigate Effect of Magnetic Field on MHD Blood Flow with Radiative Heat Transfer and Joule Heating. [12] investigate MHD Flow and Heat Transfer of Nanofluid over a Stretching Sheet with Heat Source/Sink and Thermal Radiation. [13] Did Numerical Study on the Influence of Magnetic Field on MHD Flow and Heat Transfer in a T-shaped Cavity. These reviews demonstrate the evolving understanding of electric field flow on a steady MHD fluid over stretching sheets. The integration of electric and magnetic fields presents a rich area for further exploration, promising insights into fundamental fluid dynamics and potential applications in various engineering disciplines.

This study aims to analyze the interactions between the applied electric field and the fluid dynamics, focusing on the influence of parameters such as the electric field strength, fluid conductivity, and stretching sheet characteristics.

Nomenclature

Parameter	Definition	Parameter	Definition
u, v	Velocity components along x – and y – axis, respectively	τ	Skin friction
C	Concentration of the fluid	ρ	density of the fluid
T	Fluid temperature	γ_0	Viscosity Parameter
g	Acceleration due to gravity	Dimensionless Group	
B_i	Convective heat transfer parameter	M	Hartmann number
C_i	Concentration slip	Ec	Eckert Number
S	Suction/Injection Parameter	Gr	Thermal buoyancy
B_0	Magnetic Field Strength	R	Radiation Parameter
t	Time	Pr	Prandtl number
Greek Symbols		Sc	Schmidt number
λ_0	Unsteadiness Parameter	Sr	Soret Number
β_τ	Coefficient of thermal expansion	Sh	Sherwood number
β_c	Coefficient of mass expansion	Nu	Nusselt number
V	Kinematic viscosity of the fluid		

II. MATHEMATICAL FORMULATION

Consider a steady electrical magnetohydrodynamic (EMHD) fluid over a stretching sheet. The velocity of the stretching sheet is denoted as $U_w = ax$. The boundary layer equations of the fluid flow are composition of the continuity equation, the momentum equation, energy equation and concentration equation, which are formulated based on Maxwell’s equation and Ohm’s law. The incompressible flow of viscous fluid in the presence of an applied magnetic field $B(x)$ taken into consideration. The flow is due to stretching of sheet from a slot through two equal and opposite force and thermally radiative. T, C is the fluid temperature and concentration, the ambient values of temperature and nanoparticle fraction attained to constant value of T_∞ and C_∞ . Magnetic field is applied normal to the flow, such that the Reynolds number is selected small. The induced magnetic field is smaller to the applied magnetic field. Hence the induced magnetic field is absence for small magnetic Reynolds number. We choose the Cartesian coordinate system such that x is chosen along the stretching sheet and y axis denotes the normal to the stretching sheet, u and v are the velocity components of the fluid in the x and y direction. The boundary layer flow equation of an incompressible fluid is given as:

Continuity Equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

Momentum Equation:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\partial u_e}{\partial t} + u_e \frac{\partial u_e}{\partial x} + \frac{1}{\rho} \left(1 + \frac{1}{\beta} \right) \frac{\partial}{\partial y} \left(\mu_\tau \frac{\partial u}{\partial y} \right) - \frac{\sigma \mu_\tau}{\rho} B^2 (u - u_e) + \frac{g}{\rho} (\beta_\tau T - T_1) + \beta_c (C - C_1) \tag{2}$$

The energy field for temperature can be expressed due to magnetic field as:

$$\rho C_p \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \frac{\partial}{\partial y} \left(K_T \left(\frac{\partial T}{\partial y} \right) \right) + \mu_T \left(1 + \frac{1}{\beta} \right) \left(\frac{\partial u}{\partial y} \right)^2 - B_1 \left(\frac{\partial u}{\partial y} \frac{\partial}{\partial y} \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) \right) - \frac{16\sigma T_\infty^2}{3k_s} \frac{\partial^2 T}{\partial y^2} \tag{3}$$

Concentration equation:

$$\rho \left(\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} \right) = D \frac{\partial^2 C}{\partial y^2} + D_T \frac{\partial^2 T}{\partial y^2} \tag{4}$$

The magnetic field factor $B(x) = B_0$, and ν, ρ are the kinematic viscosity of the fluid and the fluid density. ρ is the density. We also have B_0, D , and D_T which represents magnetic field, the Brownian diffusion coefficient and the thermophoresis diffusion coefficient, with the boundary conditions:

$$T(t, x, y) = T_1 + (T_w - T_1)\theta(\eta)$$

$$C(t, x, y) = C_1 + (C_w - C_1)\phi(\eta)$$

$$u = U_w(x), \quad v = v_w, \quad T = T_\infty, \quad C = C_w$$

$$y \rightarrow \infty; \quad u \rightarrow U, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty$$

The wall shearing stress τ_w , on the surface of the stretching sheet and the local heat flux q_w can be expressed as

$$\tau_w = \left[v \frac{\partial u}{\partial y} - k_0 \left(u \frac{\partial^2 u}{\partial x \partial y} - \frac{\partial u}{\partial x} \frac{\partial u}{\partial y} \right) \right]_{y=0}, \quad q_w = \left[-k_T \frac{\partial T}{\partial y} - \left(\frac{-16\sigma T_\infty^3}{3k_s} \right) \left(\frac{\partial T}{\partial y} \right) \right]_{y=0}$$

with the local mass flux as

$$h_w = \left[-D \frac{\partial C}{\partial y} \right]_{y=0}$$

Hence, the friction drags, heat and mass transfer at the wall are defined by

$$c_f = \frac{\tau_w}{\frac{1}{2} \rho_n f u_w^2}, \quad Nu = \frac{q_w}{k(T - T_\infty)}, \quad Sh = \frac{h_w}{D(C - C_\infty)}$$

To obtain similarity solution of equations (1) – (4) the nondimensionalize variables are presented as:

$$\psi(x, y, t) = \left(\frac{av}{1 - \lambda t} \right)^{\frac{1}{2}} x f(\eta), \quad \eta(y, t) = \left(\frac{a}{v(1 - \lambda t)} \right)^{\frac{1}{2}} y, \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}$$

where η is similarity variable and ψ is stream function defined as:

$$u = \frac{\partial \psi}{\partial y} \text{ and } v = -\frac{\partial \psi}{\partial x}$$

The set of equations are transformed using the Similarity variables as:

Momentum Equation:

$$\lambda_0 \left(\frac{d}{d\eta} f(\eta) - \frac{\eta}{2} \frac{d^2}{d\eta^2} f(\eta) \right) + \left(\frac{d}{d\eta} f(\eta) \right)^2 - f(\eta) \frac{d^2}{d\eta^2} f(\eta) = \left(1 + \frac{1}{\beta} \right) \frac{d}{d\eta} \left(\frac{1}{1 + \gamma_0 \theta(\eta)} \frac{d^2}{d\eta^2} f(\eta) \right) - M \left(\frac{f'(\eta) - 1}{(1 + \gamma_0 \theta(\eta))} \right) + Gr(\theta(\eta) + N\phi(\eta)) \tag{5}$$

The Energy Equation:

$$Pr \left(\lambda_0 \frac{\eta}{2} - f(\eta) \right) \frac{d}{d\eta} \theta(\eta) = \frac{d}{d\eta} \left(1 + \epsilon(\theta(\eta)) \frac{d}{d\eta} \theta(\eta) \right) + Ec \left(1 + \frac{1}{\beta} \right) \left(\frac{f''(\eta)^2}{1 + \gamma_0(\eta)} \right) - B_0(f''(\eta))(f'(\eta)f'''(\eta)) - (f(\eta)f'''(\eta)) \frac{4R}{3} (\theta''(\eta)) \tag{6}$$

And the Concentration Equation:

$$Sc \left(\lambda_0 \frac{\eta}{2} \phi'(\eta) - f(\eta) \phi'(\eta) \right) = \phi''(\eta) + Sr \theta''(\eta) \quad (7)$$

With the Initial and Boundary Conditions as:

$$f'(0) = \delta, \quad f(0) = S, \quad Bi(\theta'(0)) = \theta(0), \quad \theta(0) = 1, \quad Ci(\phi'(0)) = \phi(0), \quad \phi(0) = 1$$

Also, the engineering parameters of curiosity are the Friction drags, heat and mass flux at the wall respectively given by

$$c_{fx} = \left[\frac{1}{(1 + \epsilon \theta(\eta))} f''(\eta) \right]_{\eta=0}$$

$$Nu_x = - \left(1 + \frac{4}{3} R + \epsilon \theta(\eta) \right) (\theta'(\eta)) \Big|_{\eta=0}$$

$$Sh_x = -\phi'(\eta) \Big|_{\eta=0}$$

The flow governing parameters are:

$$\frac{1}{Pr} = \frac{k}{\mu C_p}, Ec = \frac{\mu a^2 x}{k b_1}, u_w = \frac{ax}{-\lambda t + 1}, A = A^* \left(\frac{a}{b(-\lambda t + 1)} \right), R = \frac{4\sigma T_\infty^2}{k k_s}, M = \frac{\sigma \mu B_0^2}{\rho a},$$

$$Gr = \frac{g \beta_t b_2}{\rho a^2}, B = \frac{B_1 a^3 x}{k(-\lambda t + 1)}, N = \frac{\beta_\tau b_1}{\beta_c b_2}, \lambda_0 = \frac{\lambda}{a}, \delta = \frac{c}{a}, \gamma_0 = \gamma(T_w - T_1), Sc = \frac{\mu}{D},$$

$$Sr = \frac{D_T(T_w - T_1)}{DC_w - C_1}, \Omega = \frac{-\lambda t + 1}{a\rho} k_4, \beta_1 = \frac{ak_0}{a\rho}$$

III. RESULT AND DISCUSSION

We hope to clarify the complex behaviours and occurrences seen in the system under investigation by carefully analysing and interpreting the data collected, providing insight into the relevance and ramifications of our conclusions. The results were obtained by the numerical solution method using the fourth order Runge–Kutta method. The results are displayed with the help of graphical illustrations.

Table 1 indicate the numerical values of engineering parameters, that is friction drags, Nusselt number and Sherwood number, and how various flow parameters change in response to changes in factors such as magnetic field strength, fluid properties, geometry, and boundary conditions. Tables 2 and 3 display the parameter values delineating the impact of Slip and Suction conditions on wall rate transfer and the Influence of Buoyancy on wall rate transfer, illustrating numerical variations relative to friction drags, Nusselt number, and Sherwood number. These tables offer concise information regarding the heat and mass transfer attributes of the flow, facilitating both research and engineering computations.

Figures 1 and 2 illustrate how the variation in the Unsteadiness Parameter influences the temperature and species profiles. Additionally, Figures 3 and 4 depict how changes in the magnetic parameter affect the temperature profile and species distribution. Figure 3 and 4 analyzes the effect of the magnetic field term B_0 on the temperature profile. B_0 causes the temperature distribution to rise. This tendency is caused by the Lorentz force, which is a drag-like force that is produced when a transverse magnetic field is applied to a micropolar nanofluid that conducts electricity. This Lorentz force produces a frictional heating effect that improves the temperature profile by creating a barrier to the micropolar nanofluid's free flow. Furthermore, Figures 5 and 6 demonstrate the impact of the Prandtl number on the dimensionless temperature and species profiles. This is due to the fact the thermal boundary layer thickness increases with Prandtl number. These figures provide valuable insights into the interplay between different parameters and the resulting thermal and species characteristics within the system. It also shows that the presence of the magnetic parameter reduces the dimensionless velocity. This is due to the fact that, when a transverse magnetic field is applied to an electrically conducting fluid, it gives rise to a resistive force known as the Lorentz force. This force causes the fluid to experience resistance by

increasing the friction between its layers, which decreases velocity. Thus, the increasing values of the magnetic parameter decrease the velocity of the nanofluid. It can be noted that the temperature decreases with the increasing values of the Prandtl number. This is due to the fact that the thermal boundary-layer thickness decreases with an increase in the Prandtl number. Hartman number decreases the flow velocity, this is because it represents magnetic force which induce Lorentz force into the flow, this Effects on Velocity, Temperature and Species Profile are shown in Figure 7, 8 and 9. The Concentration slip affects mass transport within the fluid and alters the diffusion layer thickness around electrodes, due to the different velocities of charged and neutral species, the diffusion layer thickness for each species vary, affecting the overall transport of species in the fluid as shown in Figure 11. Effect of Convective heat transfer parameter on Species Profile indicates variations in species concentrations within the fluid, impacting overall system behavior and performance as shown in Figure 12. Herein $S < 0$ (suction) and $S > 0$ (blowing). In case of suction, it provides impending motion to flow thereby resulting in drop of flow velocity as reported in Figure 13 and 14. The reverse is the case for blowing.

Table 1: Wall rate data for various Parameters

Parameters	c_f	Nu	Sh	Parameters	c_f	Nu	Sh
$\lambda_0 = 0.5$	-0.67129	-0.40286	0.13911	$\gamma_0 = 0.0$	-0.68688	-0.42096	0.14507
$\lambda_0 = 1.0$	-0.74559	-0.47560	0.15122	$\gamma_0 = 0.5$	-0.68998	-0.41173	0.14212
$\lambda_0 = 1.5$	-0.80832	-0.54608	0.26173	$\gamma_0 = 1.0$	-0.69323	-0.40265	0.13922
$\lambda_0 = 2.0$	-0.86149	-0.61342	0.23155	$\gamma_0 = 1.5$	-0.69654	-0.39364	0.13634
$B_0 = 0.5$	-0.69292	-0.35194	0.12165	$Sr = 0.5$	-0.68767	-0.41910	0.18060
$B_0 = 1.0$	-0.70090	-0.25321	0.08810	$Sr = 1.0$	-0.68862	-0.41911	0.36128
$B_0 = 1.5$	-0.70913	-0.15104	0.05339	$Sr = 1.5$	-0.68956	-0.41913	0.54205
$B_0 = 2.0$	-0.71763	-0.04527	0.01746	$Sr = 2.0$	-0.69054	-0.41933	0.72327

Table 2: Effect of Slip and suction conditions on wall rate transfer

Parameters	c_f	Nu	Sh	Parameters	c_f	Nu	Sh
$V = 0.5$	-0.44448	-0.11040	0.03920	$Bi = 0.5$	-0.66381	-0.30489	0.10832
$V = 1.0$	-0.31039	0.00915	-0.00213	$Bi = 1.0$	-0.64789	-0.22806	0.08456
$V = 1.5$	-0.23893	0.05704	-0.01889	$Bi = 1.5$	-0.63842	-0.18235	0.07065
$V = 2.0$	-0.19437	0.08100	-0.02735	$Bi = 2.0$	-0.63212	-0.15197	0.06149
$S = 0.5$	-0.76389	-0.45717	0.15381	$Ci = 0.5$	-0.68842	-0.42027	0.13282
$S = 1.0$	-0.90110	-0.50051	0.16116	$Ci = 1.0$	-0.68970	-0.42186	0.11712
$S = -0.5$	0.76389	0.45717	-0.15381	$Ci = 1.5$	-0.69071	-0.42312	0.10475
$S = -1.0$	0.90110	0.50051	-0.16116	$Ci = 2.0$	-0.69152	-0.42414	0.09477

Table 3: Effect of Buoyancy on wall rate transfer

Parameters	c_f	Nu	Sh	Parameters	c_f	Nu	Sh
$M = 0.5$	-0.62678	-0.26116	0.09310	$Ec = 0.5$	-0.72628	0.10667	-0.03478
$M = 1.0$	-0.55228	-0.11441	0.04568	$Ec = 1.0$	-0.72036	0.02564	-0.00718
$M = 3$	-0.39283	0.08435	-0.01708	$Ec = 1.5$	-0.71458	-0.05327	0.01971
$M = 10$	-0.21981	0.17017	-0.03748	$Ec = 2.0$	-0.70892	-0.13017	0.04592

The influence of electric fields on fluids is found to be significant and multifaceted. When the electric fields are applied to the conducting fluids in the presence of magnetic fields, they interact through the Lorentz force, generating complex flow patterns. This interaction leads to the generation of electric currents, excitation of MHD waves, and triggering of instabilities and turbulence. This research offers a means of controlling fluid flows, with applications ranging from propulsion systems to energy generation. Understanding this influence is crucial for various scientific, engineering, and technological advancements in fields such as plasma physics, aerospace engineering, and magnetic confinement fusion

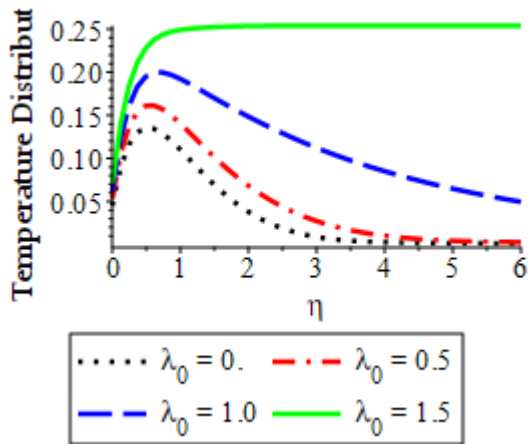


Figure 1. Effect of Unsteadiness Parameter on Temperature Profile

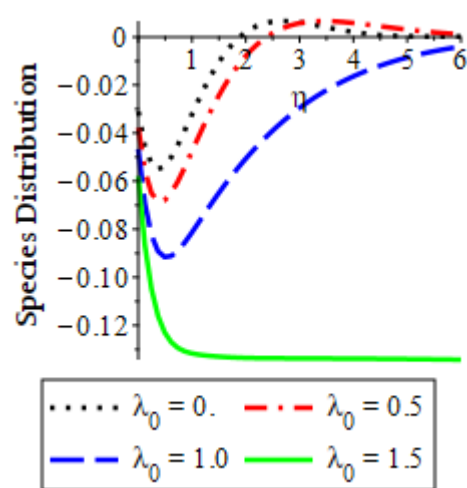


Figure 2. Effect of Unsteadiness Parameter on Species Profile

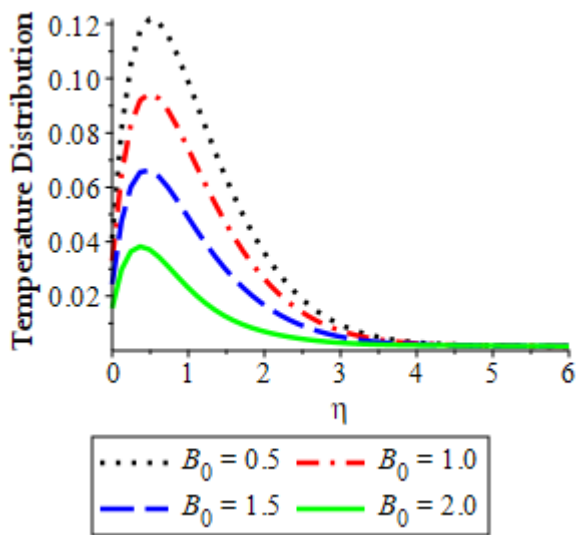


Figure 3. Effect of Magnetic Field Strength on Temperature Profile

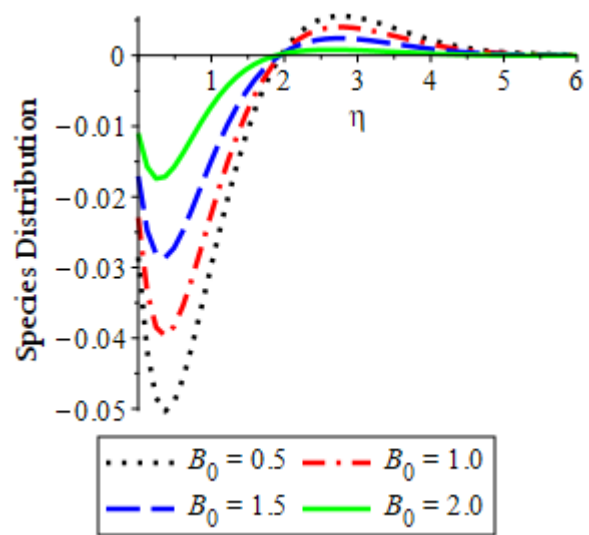


Figure 4. Effect of Magnetic Field Strength on Species Profile

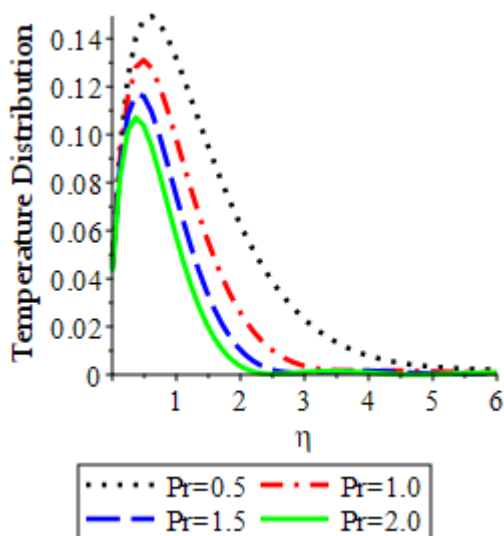


Figure 5. Effect of Prandtl number on Temperature Profile

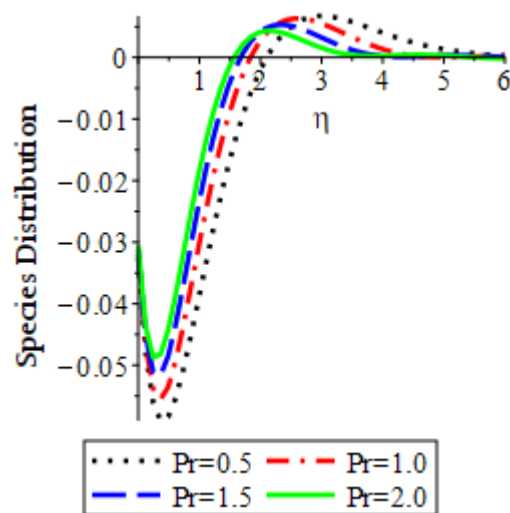


Figure 6. Effect of Prandtl number on Species Profile

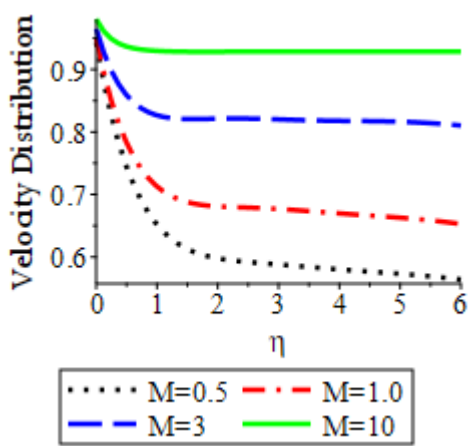


Figure 7. Effect of Hartmann number on Velocity Profile

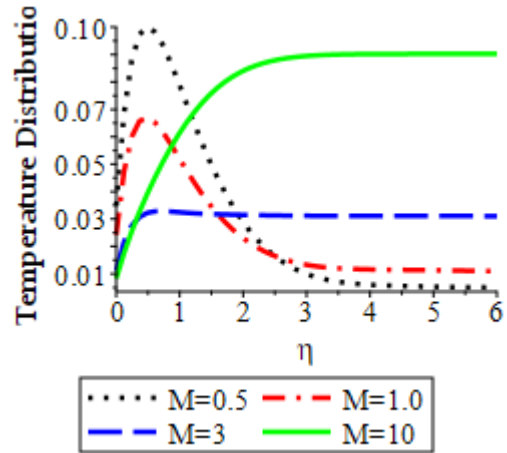


Figure 8. Effect of Hartmann number on Temperature Profile

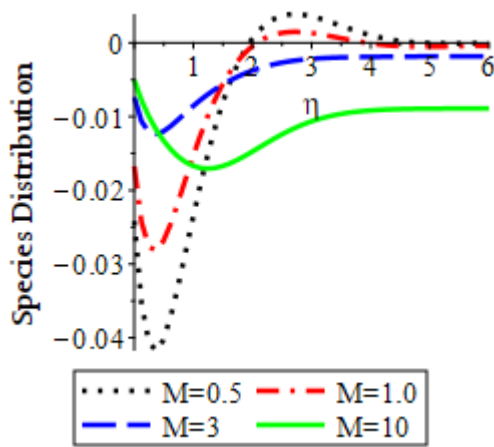


Figure 9. Effect of Hartmann number on Species Profile

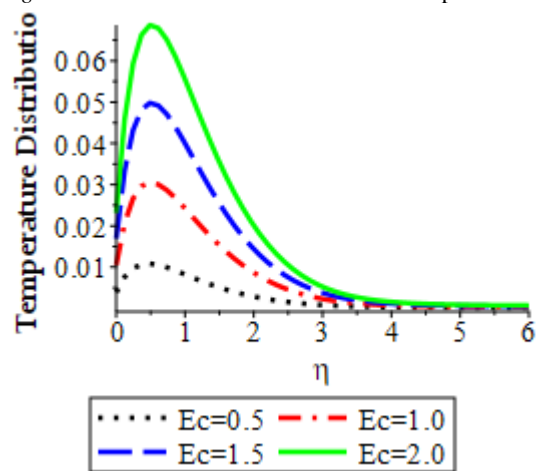


Figure 10. Effect of Eckert Number on Temperature Profile

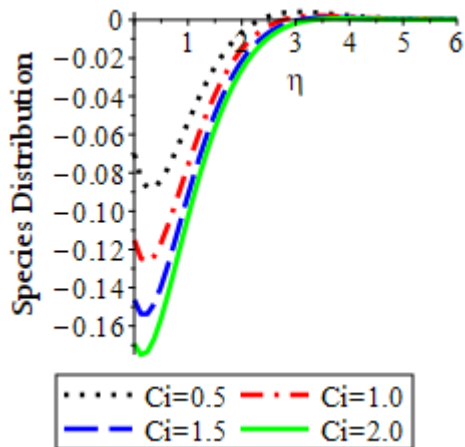


Figure 11. Effect of Concentration slip on Species Profile

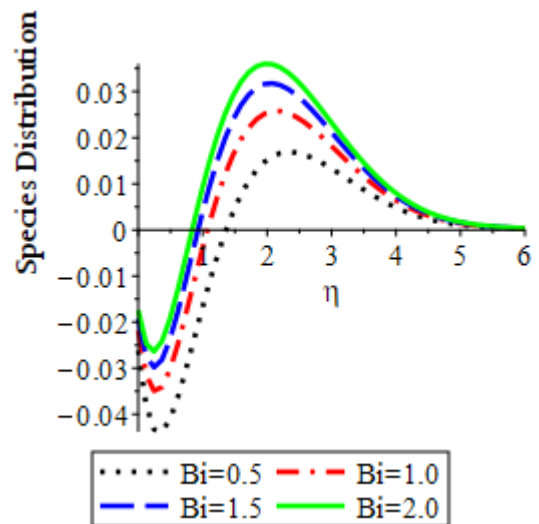


Figure 12. Effect of Convective heat transfer parameter on Species Profile

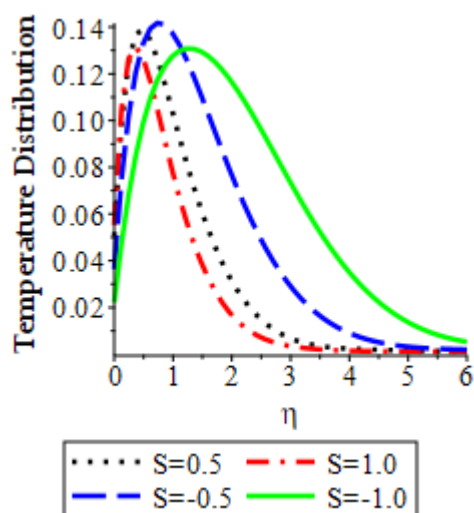


Figure 13. Effect of Effect of Suction/Injection on Temperature Profile

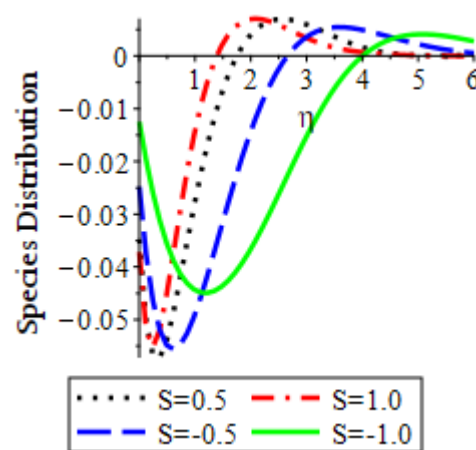


Figure 14. Effect of Effect of Suction/Injection on Species Profile

IV. CONCLUSION

This research delves into the influence of electric fields on boundary layer flow in magnetohydrodynamic (MHD) nanofluids over linearly stretching sheets. Utilizing numerical methods, we investigated the steady-state, two-dimensional boundary layer flow, focusing on heat and mass transfer within nanofluids possessing electrical conductivity due to linear stretching sheets. Our study meticulously examines the effects of various governing parameters on the intricate dynamics of heat and mass transfer. The Velocity profile increase distance along η for higher values of Electric/Magnetic field parameter. Whereas, temperature and concentration fields decreased by increasing the values of electric field.

Electric fields exert a significant influence on magnetohydrodynamic (MHD) fluids, shaping their behavior in complex ways. Through the interaction with magnetic fields, electric fields generate currents, induce waves, trigger instabilities, and facilitate control over fluid flows. Understanding and harnessing this influence have broad implications across scientific, engineering, and technological domains, from propulsion systems to energy generation and beyond. Therefore, this study is crucial for advancing various applications in plasma physics, aerospace engineering, and magnetic confinement fusion, among others.

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