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The Deductibility of the Categorical Syllogisms *AII*-1 from the Perspective of Knowledge Reasoning

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Abstract
This paper aims to realize the reduction between/among different valid categorical syllogisms and establish a concise formal axiomatic system for categorical syllogistic. Making full use of the tripartite structure of categorical propositions, the symmetry of no and some, the definable relationship between the quantifier all and the other three Aristotelian quantifiers, and some reasoning rules and facts in first-order logic, this paper takes the syllogism AII-1 as a basic axiom and derives the remaining 23 valid syllogisms. It is hoped that this research will not only promote the development of modern logic, but also provide assistance for machine reasoning in artificial intelligence.

Keywords: *categorical syllogisms; knowledge reasoning; symmetry; deductibility*

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I. Introduction

The fact that syllogistic reasoning has been a widespread and significant form of reasoning in human thinking beyond all doubt. This paper is devoted to studying categorical syllogisms which has been widely discussed from various point of views Łukasiewicz (1957), Moss (2008), Endrullis and Moss (2018), Xiaojun (2018), Kulicki (2020) , etc.

As is well known, only 24 are valid out of 256 categorical syllogisms. When deducing the remaining valid syllogisms, at least two valid syllogisms are used as basic axioms, for example by Cai (1984), Xiaojun and Sheng (2016), Mengyao and Xiaojun (2020). Different from the previous studies, this paper fully utilizes set theory, first-order logic and generalized quantifier theory to infer the remaining 23 valid syllogisms by means of the syllogism *AII-1*, so as to establish a concise formal axiomatic system for categorical syllogistic.

II. Preliminary Knowledge

In the following, let *Q* be any of the four Aristotelian quantifiers (namely, *all*, *some*, *no*, *not all*). $\neg Q$ and $Q\neg$ be respectively its outer and inner quantifier (Westerståhl, 1989). And *b*, *g* and *x* indicate lexical variables, *D* their domain. The sets composed of *g*, *b* and *x* are respectively *G*, *B*, and *X*. Let α, β, γ and δ be well-formed formulas (abbreviated as wff). '$\gamma =_{def} \delta$' stands for γ is defined by δ, and '$\vdash\alpha$' for a provable proposition α.

A categorical syllogism contains three categorical propositions which have the following four kinds of propositions: *all(b, x)*, *some(b, x)*, *no(b, x)*, and *not all(b, x)*, and mean respectively that 'all *b*s are *x*s', 'some *b*s are *x*s', 'no *b*s are *x*s' and 'not all *b*s are *x*s'. They can be respectively abbreviated as Proposition *A*, *E*, *I* and *O*. Then, the syllogism *AII-1* can be denoted by $all(b, x)\wedge some(g, b)\rightarrow some(g, x)$. The formal representations of other syllogisms are similar.

III. Formal System of Categorical Syllogistic

This formal axiomatic system of categorical syllogistic includes four parts: initial symbols, formation rules of wffs, basic axioms and deductive rules.

3.1 Initial Symbols

(1) lexical variables: *g*, *b*, *x*

(2) unary negative operator: ¬ □

(3) binary implication operator: $\rightarrow$□
(4) quantifier: *all*
(5) brackets: (,)

3.2 Formation Rules

(1) If *Q* is a quantifier, *b* and *x* are lexical variables, then *Q(b, x)* is a wff.
(2) If α and β are wffs, then so are $\neg\alpha$ and $\alpha\rightarrow\beta$.
(3) The formulas obtained just in terms of (1) and (2) are wffs.

For example, *all(b, x)*, *¬some(b, x)*, and *some(b, x)→all(x, g)* are wffs that represent 'all *b*s are *x*s', 'not some *b*s are *x*s', 'if some *b*s are *x*s, then all *x*s are *g*s', respectively. The other formulas are similar.

3.3 Related Definitions

Definition 1: $(\alpha\leftrightarrow\beta)=_{def}(\alpha\rightarrow\beta)\wedge(\beta\rightarrow\alpha)$.
Definition 2: $(\alpha\wedge\beta)=_{def}\neg(\alpha\rightarrow\neg\beta)$.
Definition 3: $Q\neg(b, x)=_{def}Q(b, D-x)$.
Definition 4: $(\neg Q)(b, x)=_{def}$It is not that $Q(b, x)$.
Definition 5: $all(b, x)=_{def} B\subseteq X$.
Definition 6: $some(b, x)=_{def} B\cap X\neq\varnothing$.
Definition 7: $no(b, x)=_{def} B\cap X=\varnothing$.
Definition 8: $not\ all(b, x)=_{def} B\nsubseteq X$.

3.4 Basic Axioms

A1: If α is a valid formula in propositional logic, then $\vdash\alpha$.
A2: $\vdash all(b, x)\wedge some(g, b)\rightarrow some(g, x)$ (i.e. the syllogism *AII-1*).

3.5 Reasoning Rules

Rule 1 (Subsequent weakening): If $\vdash(\alpha\wedge\beta\rightarrow\gamma)$ and $\vdash(\gamma\rightarrow\delta)$, then $\vdash(\alpha\wedge\beta\rightarrow\delta)$ can be inferred.
Rule 2 (anti-syllogism): If $\vdash(\alpha\wedge\beta\rightarrow\gamma)$, then $\vdash(\neg\gamma\wedge\alpha\rightarrow\neg\beta)$ can be inferred.

3.6 Related Facts

Fact 1 (inner negation):
(1) $all(b, x)\leftrightarrow no\neg(b, x)$; (2) $no(b, x)\leftrightarrow all\neg(b, x)$;
(3) $some(b, x)\leftrightarrow not\ all\neg(b, x)$; (4) $not\ all(b, x)\leftrightarrow some\neg(b, x)$.
Fact 2 (outer negation):
(1) $\neg all(b, x)\leftrightarrow not\ all(b, x)$; (2) $\neg not\ all(b, x)\leftrightarrow all(b, x)$;
(3) $\neg no(b, x)\leftrightarrow some(b, x)$; (4) $\neg some(b, x)\leftrightarrow no(b, x)$.
Fact 3 (symmetry of *some* and *no*):
(1) $\vdash some(b, x)\leftrightarrow some(x, b)$; (2) $\vdash no(b, x)\leftrightarrow no(x, b)$.
Fact 4 (assertoric subalternations):
(1) $\vdash all(b, x)\rightarrow some(b, x)$; (2) $\vdash no(b, x)\rightarrow not\ all(b, x)$.

The above four facts can be proven by the above axioms, definitions and reasoning rules (Zhang & Wu, 2021). So their proofs are omitted here.

IV. Knowledge Reasoning Based on the validity of the Syllogism *AII-1*

The following Theorem 1 shows the syllogism *AII-1* is valid. '(1) ⊢*AII-1*→*AII-3*' in Theorem 2 indicates the validity of syllogism *AII-3* can be deduced from that of the syllogism *AII-1*. In other words, there is a deducible relationship between these two syllogisms. Other cases are similar. The deductibility between/among different syllogisms is key to build logical proof systems for categorical syllogisms.

Theorem 1(*AII-1*)**:** $all(b, x)\wedge some(g, b)\rightarrow some(g, x)$ is valid.

Proof: Suppose that *all(b, x)* and *some(g, b)* are true, then $B\subseteq X$ and $G\cap B\neq\varnothing$ are true according to Definition 5 and 6, respectively. Now it follows that $G\cap X\neq\varnothing$ is true. Hence *some(g, x)* is true according to Definition 6. This proves that the syllogism $all(b, x)\wedge some(g, b)\rightarrow some(g, x)$ is valid, just as desired.

Theorem 2: The validity of the following 23 syllogisms can be inferred from that of the syllogism *AII-1*:

(1) ⊢*AII-1*→*AII-3*
(2) ⊢*AII-1*→*AII-3*→*IAI-3*
(3) ⊢*AII-1*→*IAI-4*
(4) ⊢*AII-1*→*AEE-2*
(5) ⊢*AII-1*→*AEE-2*→*AEE-4*
(6) ⊢*AII-1*→*AEE-2*→*AEE-4*→*EAE-1*
(7) ⊢*AII-1*→*AEE-2*→*EAE-2*
(8) ⊢*AII-1*→*AEE-2*→*AEO-2*
(9) ⊢*AII-1*→*AEE-2*→*AEO-2*→*AEO-4*

(10) $\vdash AII\text{-}1\rightarrow AEE\text{-}2\rightarrow AEE\text{-}4\rightarrow EAE\text{-}1\rightarrow EAO\text{-}1$
(11) $\vdash AII\text{-}1\rightarrow AEE\text{-}2\rightarrow AEE\text{-}4\rightarrow EAE\text{-}1\rightarrow EAO\text{-}1\rightarrow AAI\text{-}3$
(12) $\vdash AII\text{-}1\rightarrow AEE\text{-}2\rightarrow AEE\text{-}4\rightarrow EAE\text{-}1\rightarrow EAO\text{-}1\rightarrow EAO\text{-}2$
(13) $\vdash AII\text{-}1\rightarrow AEE\text{-}2\rightarrow AEE\text{-}4\rightarrow EAE\text{-}1\rightarrow EAO\text{-}1\rightarrow AAI\text{-}1$
(14) $\vdash AII\text{-}1\rightarrow AEE\text{-}2\rightarrow AEE\text{-}4\rightarrow EAE\text{-}1\rightarrow EAO\text{-}1\rightarrow AAI\text{-}1\rightarrow AAI\text{-}4$
(15) $\vdash AII\text{-}1\rightarrow AEE\text{-}2\rightarrow AEE\text{-}4\rightarrow EAE\text{-}1\rightarrow EAO\text{-}1\rightarrow AAI\text{-}1\rightarrow AAI\text{-}4\rightarrow EAO\text{-}4$
(16) $\vdash AII\text{-}1\rightarrow AEE\text{-}2\rightarrow AEE\text{-}4\rightarrow EAE\text{-}1\rightarrow EAO\text{-}1\rightarrow AAI\text{-}1\rightarrow AAI\text{-}4\rightarrow EAO\text{-}4\rightarrow EAO\text{-}3$
(17) $\vdash AII\text{-}1\rightarrow EIO\text{-}1$
(18) $\vdash AII\text{-}1\rightarrow EIO\text{-}1\rightarrow EIO\text{-}2$
(19) $\vdash AII\text{-}1\rightarrow EIO\text{-}1\rightarrow EIO\text{-}2\rightarrow EIO\text{-}4$
(20) $\vdash AII\text{-}1\rightarrow EIO\text{-}1\rightarrow EIO\text{-}3$
(21) $\vdash AII\text{-}1\rightarrow AEE\text{-}2\rightarrow AEE\text{-}4\rightarrow EAE\text{-}1\rightarrow AAA\text{-}1$
(22) $\vdash AII\text{-}1\rightarrow AII\text{-}3\rightarrow IAI\text{-}3\rightarrow OAO\text{-}3$
(23) $\vdash AII\text{-}1\rightarrow AEE\text{-}2\rightarrow AEE\text{-}4\rightarrow EAE\text{-}1\rightarrow AAA\text{-}1\rightarrow AOO\text{-}2$

Proof:

[1] $\vdash all(b, x)\wedge some(g, b)\rightarrow some(g, x)$ (i.e. *AII-1*, basic axiom A2)
[2] $\vdash all(b, x)\wedge some(b, g)\rightarrow some(g, x)$ (i.e. *AII-3*, by [1] and Fact 3)
[3] $\vdash all(b, x)\wedge some(b, g)\rightarrow some(x, g)$ (i.e. *IAI-3*, by [2] and Fact 3)
[4] $\vdash all(b, x)\wedge some(g, b)\rightarrow some(x, g)$ (i.e. *IAI-4*, by [1] and Fact 3)
[5] $\vdash \neg some(g, x)\wedge all(b, x)\rightarrow \neg some(g, b)$ (by [1] and Rule 2)
[6] $\vdash no(g, x)\wedge all(b, x)\rightarrow no(g, b)$ (i.e. *AEE-2*, by [5] and Fact 2)
[7] $\vdash no(x, g)\wedge all(b, x)\rightarrow no(g, b)$ (i.e. *AEE-4*, by [6] and Fact 3)
[8] $\vdash no(x, g)\wedge all(b, x)\rightarrow no(b, g)$ (i.e. *EAE-1*, by [7] and Fact 3)
[9] $\vdash no(g, x)\wedge all(b, x)\rightarrow no(b, g)$ (i.e. *EAE-2*, by [6] and Fact 3)
[10] $\vdash no(g, b)\rightarrow not\ all(g, b)$ (by Fact 4)
[11] $\vdash no(g, x)\wedge all(b, x)\rightarrow not\ all(g, b)$ (i.e. *AEO-2*, by [6], [10] and Rule 1)
[12] $\vdash no(x, g)\wedge all(b, x)\rightarrow not\ all(g, b)$ (i.e. *AEO-4*, by [11] and Fact 3)
[13] $\vdash no(x, g)\wedge all(b, x)\rightarrow not\ all(b, g)$ (i.e. *EAO-1*, by [8] and Fact 4)
[14] $\vdash \neg not\ all(b, g)\wedge all(b, x)\rightarrow \neg no(x, g)$ (by [13] and Rule 2)
[15] $\vdash all(b, g)\wedge all(b, x)\rightarrow some(x, g)$ (i.e. *AAI-3*, by [14] and Fact 1)
[16] $\vdash no(g, x)\wedge all(b, x)\rightarrow not\ all(b, g)$ (i.e. *EAO-2*, by [13] and Fact 3)
[17] $\vdash all\neg(x, g)\wedge all(b, x)\rightarrow some\neg(b, g)$ (by [13] and Fact 2)
[18] $\vdash all(x, D{-}g)\wedge all(b, x)\rightarrow some(b, D{-}g)$ (i.e. *AAI-1*, by [17] and Definition 3)
[19] $\vdash all(x, D{-}g)\wedge all(b, x)\rightarrow some(D{-}g, b)$ (i.e. *AAI-4*, by [18] and Fact 3)
[20] $\vdash \neg some(D{-}g, b)\wedge all(b, x)\rightarrow \neg all(x, D{-}g)$ (by [19] and Rule 2)
[21] $\vdash no(D{-}g, b)\wedge all(b, x)\rightarrow not\ all(x, D{-}g)$ (i.e. *EAO-4*, by [20] and Fact 3)
[22] $\vdash no(b, D{-}g)\wedge all(b, x)\rightarrow not\ all(x, D{-}g)$ (i.e. *EAO-3*, by [21] and Fact 3)
[23] $\vdash no\neg(b, x)\wedge some(g, b)\rightarrow not\ all\neg(g, x)$ (by [1] and Fact 2)
[24] $\vdash no(b, D{-}x)\wedge some(g, b)\rightarrow not\ all(g, D{-}x)$ (i.e. *EIO-1*, by [23] and Definition 3)
[25] $\vdash no(D{-}x, b)\wedge some(g, b)\rightarrow not\ all(g, D{-}x)$ (i.e. *EIO-2*, by [24] and Fact 3)
[26] $\vdash no(D{-}x, b)\wedge some(b, g)\rightarrow not\ all(g, D{-}x)$ (i.e. *EIO-4*, by [25] and Fact 3)
[27] $\vdash no(b, D{-}x)\wedge some(b, g)\rightarrow not\ all(g, D{-}x)$ (i.e. *EIO-3*, by [24] and Fact 3)
[28] $\vdash all\neg(x, g)\wedge all(b, x)\rightarrow all\neg(b, g)$ (by [8] and Fact 2)
[29] $\vdash all(x, D{-}g)\wedge all(b, x)\rightarrow all(b, D{-}g)$ (i.e. *AAA-1*, by [28] and Definition 3)
[30] $\vdash all(b, x)\wedge not\ all\neg(b, g)\rightarrow not\ all\neg(x, g)$ (by [3] and Fact 2)
[31] $\vdash all(b, x)\wedge not\ all(b, D{-}g)\rightarrow not\ all(x, D{-}g)$ (i.e. *OAO-3*, by [30] and Definition 3)
[32] $\vdash \neg all(b, D{-}g)\wedge all(x, D{-}g)\rightarrow \neg all(b, x)$ (by [29] and Rule 2)
[33] $\vdash not\ all(b, D{-}g)\wedge all(x, D{-}g)\rightarrow not\ all(b, x)$ (i.e. *AOO-2*, by [32] and Fact 2)

So far, on the basis of the above rules, definitions and theorems, Theorem 2 deduces the other 23 valid categorical syllogisms just from the valid syllogism *AII-1*.

V. Conclusion

In order to realize the reduction between/among different valid categorical syllogisms and establish a concise formal axiomatic system for categorical syllogistic. Making full use of the tripartite structure of categorical propositions, the symmetry of *no* and *some*, the definable relationship between the quantifier *all* and the other three Aristotelian quantifiers, and some reasoning rules and facts in first-order logic, this paper takes the syllogism *AII-1* as a basic axiom and derives the remaining 23 valid syllogisms.

The formal processing of natural language in artificial intelligence technology has developed rapidly and has occupied an important position. Therefore, how to take advantage of this method to benefit natural language information processing?

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