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Doppler frequency shift formula derived solely based on path difference

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Abstract—*When the wave source or observer has relative motion, the velocity in the observation system is regarded as a linear superposition of the instantaneous velocities of various components. On this basis, calculate the distance difference and directly obtain the Doppler frequency shift from the path difference by observing the time period of propagation of a wavelength.*

Keywords—*Doppler effect, Doppler frequency shift, path difference*

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I. INTRODUCTION

Doppler effect is a phenomenon of frequency shift caused by the relative motion between the light source and the observer. The Doppler frequency shift formula is a mathematical formula that describes the frequency change of an object during relative motion. Its physical meaning is that there is relative motion between a signal source and receiver, and the frequency of the signal will change [1-5].

There are many methods for deriving the Doppler frequency shift formula [6-17], one of which is based on geometry and derives the formula by utilizing the distance difference [16-17]. This article proposes a new derivation method for Doppler frequency shift formula based on the path difference. The main difference from existing methods is that when there is relative motion between the wave source and observer, the velocity in the observation system is regarded as a linear superposition of the instantaneous velocities of various components. And by observing the time period of propagation of a wavelength, the Doppler frequency shift is directly obtained from the path difference.

II. ANALYSIS

The Doppler effect studies the relationship between the frequency of waves received by observers and the natural frequency of the wave source, which has the characteristic of instantaneity.

Firstly, analyze the situation where the observer is stationary and only the wave source is moving. And the distance from the wave source is much greater than the wavelength, which is the far-field situation. The geometric modeling is shown in Fig. 1. The wave source moves at the speed of v_s. At a certain moment, the distance between the two is r_1. Among them, points S_1 and S_2 represent the positions of the wave source when $t = 0$ and $t = \Delta t_1$, respectively.

Because the motion speed of the wave source itself is v_s, there is:

$$\overline{S_1S_2} = v_s \Delta t_1 \quad (1)$$

In the case of relative motion of the wave source or observer, the propagation speed of waves in the entire observation space is regarded as a linear superposition of the motion velocities of each part. Assuming $t = \Delta t_2$ and $\Delta t_2 > \Delta t_1$, the propagation distances of the wave source signals emitted from positions s1 and s2 are:

$$\overline{OS_1} = (v_c + v_s \cos\beta_s)\Delta t_2 \quad (2)$$

$$\overline{MS_2} = (v_c + v_s \cos\beta_s)(\Delta t_2 - \Delta t_1) \quad (3)$$

Where: v_c is the propagation speed of the wave source signal. β_s is the angle between the propagation direction of the wave source signal and the radial distance.

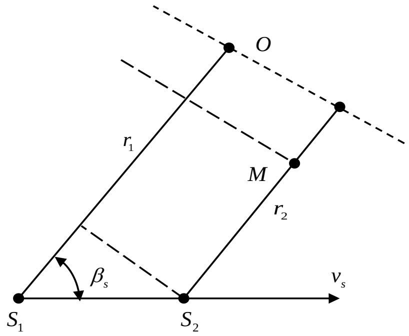

Fig.1. Geometric modeling

The difference in wave path between the two is:

$$\Delta r=\overline{OS_1}-\overline{MS_2}=(v_c+v_s\cos\beta_s)\Delta t_1 \quad (4)$$

Within the classical range, the length of the same object observed in different inertial reference frames remains unchanged. Because both the source and the observer move at low speeds, for the same wave, the wavelength measured at the source end and the wavelength measured at the observer end are both equal to the wavelength of this wave measured in the medium reference frame. Based on this, assuming the wave path difference $\Delta r\to\lambda$, there should be $\Delta t_1\to T_s$. T_s is the time period of signal propagation during the movement of the wave source. So there are:

$$\lambda=(v_c+v_s\cos\beta_s)T_s \quad (5)$$

At this point, the equivalent frequency of the wave source received by the observer is $f_{es}=1/T_s$, so the equation can be rewritten as:

$$\lambda f_{es}=\lambda f_0+v_s\cos\beta_s \quad (6)$$

Among them, $v_c=\lambda f_0$ and f_0 are the natural frequencies of the wave source in the observation system without any relative motion.

From this, it can be proven that:

$$\lambda f_{ds}=\lambda\left(f_{es}-f_0\right)=v_s\cos\beta_s \quad (7)$$

In the case where only observer motion exists, it is assumed that observer O is moving at velocity v_o. By doing a completely similar derivation, it can be proven that:

$$\lambda f_{do}=\lambda\left(f_{eo}-f_0\right)=v_0\cos\beta_o \quad (8)$$

Finally, with the wave source and observer moving simultaneously, a linear superposition is performed, which includes:

$$\lambda f_d=v_s\cos\beta_s+v_0\cos\beta_o \quad (9)$$

III. CONCLUSION

In references [16-17], the distance difference is obtained using geometric projection. However, this article directly calculates the propagation distance of the wave source in the direction relative to the observer using the equivalent wave source propagation speed within the specified propagation time through the linear superposition of various motion velocities. Then calculate the distance difference of the wave.

Another slight difference is that in this article, the signal rays emitted by the wave sources at various positions are directly regarded as parallel to each other. In this case, the angle between the velocity direction and the propagation direction of the wave at each wave source position will be equal.

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