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Non-Garmonic Oscillations of a System of Four Electric Charges

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Annotation. *In this paper, we consider the problem of fluctuations in electric charges, brought out of equilibrium as a result of burning one of the silk threads connecting them. Initially, these charges form an equilibrium configuration in the form of a regular polygon. In the future, the configuration changes, but the position of the center of mass of the system remains unchanged. The following problem is considered: four identical small balls, connected by silk threads of the same length, have the same electric charge and form a regular quadrangle. Gravity is absent. One of the filaments burns out, and as a result of the Coulomb repulsion, oscillations arise between the charges. Determine the period of arising oscillations. The problem of calculating the oscillation period of four electric charges is solved. Differential equations are obtained that describe the motion of charges. These equations are solved numerically by the improved Euler method. MS Excel spreadsheets are used as a programming environment, the program is written in VBA (Visual Basic for Application).*

Key words: *Coulomb repulsion of electric charges, conservation of the position of the center of mass. differential equations of motion, Euler's method*

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I. Introduction

Nowadays, more than one serious scientific problem, in which it is necessary to obtain a specific numerical result, cannot be solved without the use of a computer. Computer modeling of physical processes and setting up computer experiments have become an integral part of the study of physics. At present, along with experimental and theoretical physics, a new name has even appeared "computer physics" [1-2].

The solution of many physical problems is reduced to the solution of differential equations describing the considered physical process. Analytical solutions to these equations are often difficult to find. In many cases, it is easier to find numerical solutions to these equations using a computer. As our teaching experience shows, simulation of physical processes develops a deeper understanding of physics.

Previously, we considered the problem of oscillations of three charges forming an initially regular triangle [3]. Here we solve a similar problem for four charges. Here is her condition.

Four identical balls, having mass and electric charge, are connected by silk threads of the dyne, forming a square. There is no gravity. One of the threads is burned out. Determine the period of oscillations that occur.

Since only internal forces act in this system, the position of the center of mass remains unchanged and, in addition, the law of conservation of energy is fulfilled. These two laws make it possible to obtain a differential equation describing these oscillations.

The analytical solution of these equations leads to elliptic integrals. This equation is solved numerically by the improved Euler method. MS Excel spreadsheets are used as a programming environment. The program is written in VBA (Visual Basic for Application).

II. Solution

In the considered system of charges, only internal forces act. These forces cannot change the position of the center of mass of the system, so the position of the center of mass is preserved. The center of mass of the square is at the point where the diagonals intersect. We choose the origin of the Cartesian coordinate system $x0y$ at the center of mass, pointing the axis y vertically upwards, and the axis x horizontally, as shown in Fig. 1.

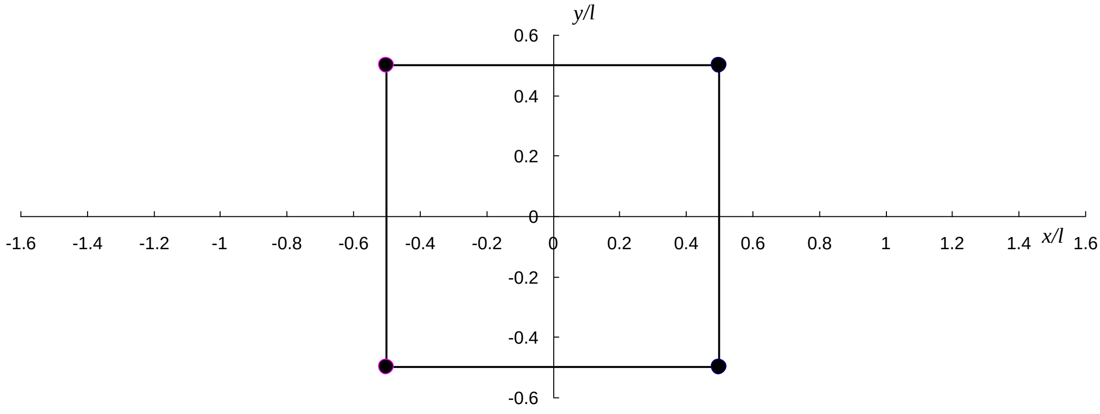

Fig. 1 – Four identical charges connected by threads of length l form a square

After burning out the lower horizontal thread, the forces acting between the nearest charges do not change, and oscillations arise due to the presence of Coulomb repulsive forces between other charges. Due to symmetry, the two central charges will move along the axis y. Thus, this mechanical system has one degree of freedom, which will be described by the coordinate y of two central charges. Since this system of charges is closed, its total energy is preserved E, which after burning the thread is equal to:

$$E = \frac{q^2}{4\pi\varepsilon_0 l}\left(\sqrt{2}+1\right). \tag{1}$$

Taking into account the conservation of momentum, we write the law of conservation of energy in the form:

$$2m\dot{y}^2 + m\dot{x}_1^2 + U(y) = E. \tag{2}$$

Here, x_1 is the *x*-coordinate of the rightmost charge and it is taken into account that in the center-of-mass system the *y*-coordinate of the edge charges is equal in absolute value to the coordinate y of the central charges; dots denote derivatives with respect to time t; $U(y)$ is the potential energy of repulsion of charges:

$$U(y) = \frac{q^2}{4\pi\varepsilon_0 l}\left(\frac{\sqrt{2}}{\sqrt{1+\sqrt{1-4\left(\frac{y}{l}\right)^2}}} + \frac{1}{2\sqrt{1-4\left(\frac{y}{l}\right)^2}+1}\right). \tag{3}$$

Since the position of the center of mass of the system does not change during the movement, the relation must be satisfied:

$$x_1 = l\left[\frac{1}{2} + \sqrt{1-4\left(\frac{y}{l}\right)^2}\right]. \tag{4}$$

Differentiating (4) with respect to time t, we obtain:

$$\dot{x}_1 = \frac{-4\dfrac{y}{l}\dot{y}}{\sqrt{1-4\left(\dfrac{y}{l}\right)^2}}. \qquad (5)$$

Taking into account (5), we write the law of conservation of energy (2) in the form:

$$\dot{y}^2\left[\frac{1+4\left(\dfrac{y}{l}\right)^2}{1-4\left(\dfrac{y}{l}\right)^2}\right] = \frac{q^2}{8\pi\varepsilon_0 l\, m}\left(\frac{\sqrt{2}}{\sqrt{1+\sqrt{1-4\left(\dfrac{y}{l}\right)^2}}} + \frac{1}{2\sqrt{1-4\left(\dfrac{y}{l}\right)^2}+1}\right). \qquad (6)$$

We will measure the coordinates of charges y and x_1 in units l and introduce dimensionless time τ:

$$\tau = \frac{t}{\tau_0}, \qquad (7)$$

where

$$\tau_0 = \frac{l^{\frac{3}{2}}}{q}\sqrt{8\pi\varepsilon_0 m}. \qquad (8)$$

Let us write relation (6) in dimensionless variables:

$$\dot{y}^2\frac{1+4y^2}{1-4y^2} = \frac{\sqrt{2}}{\sqrt{1+\sqrt{1-4y^2}}} + \frac{1}{2\sqrt{1-4y^2}+1}. \qquad (9)$$

Here, the dots now denote the derivatives with respect to the dimensionless time τ. From relation (9) we find:

$$\dot{y}^2 = \frac{1-4y^2}{1+4y^2}\left(\frac{\sqrt{2}}{\sqrt{1+\sqrt{1-4y^2}}} + \frac{1}{2\sqrt{1-4y^2}+1}\right). \qquad (10)$$

At the initial moment $y = y_0 = \frac{1}{2}$ and $\dot{y} = y_0 = 0$, and at $y = 0$, we get $\dot{y} = \mp\sqrt{1+\frac{1}{3}} = \mp\frac{2}{\sqrt{3}}$. The resulting relation (10) allows one to construct the phase diagram of oscillations, which is shown in Fig. 2.

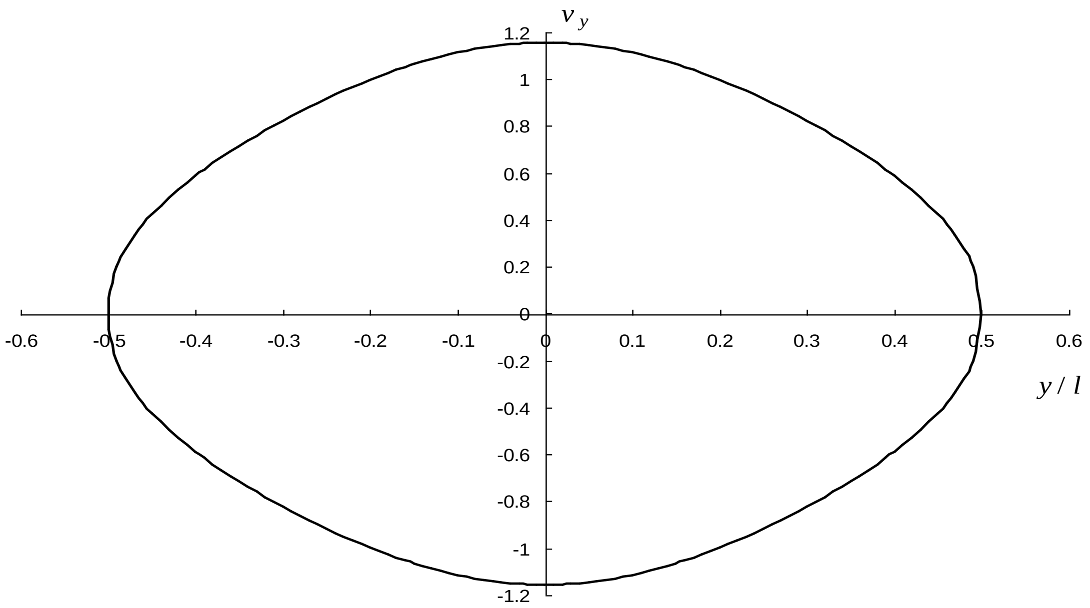

Fig. 2 – Phase diagram of oscillations

Here, the dimensionless velocity of the central charge is given by:

$$v_y = \frac{dy}{d\tau} = \dot{y}. \tag{11}$$

Equation (10) is not integrated in elementary functions. Its solution can be written in terms of elliptic integrals [4-5]. Differentiating expression (10) with respect to dimensionless time τ , we find the acceleration with which the central charge moves.

$$\begin{aligned} 2\dot{y}\,\ddot{y} = {} & \left(\frac{1-4y^2}{1+4y^2}\right)' \dot{y} \left(\frac{\sqrt{2}}{\sqrt{1+\sqrt{1-4y^2}}} + \frac{1}{2\sqrt{1-4y^2}+1}\right) + \\ & + \left(\frac{1-4y^2}{1+4y^2}\right)\left(\frac{\sqrt{2}}{\sqrt{1+\sqrt{1-4y^2}}} + \frac{1}{2\sqrt{1-4y^2}+1}\right)' \dot{y} \end{aligned} \tag{12}$$

From whence

$$\begin{aligned} \ddot{y} = {} & \frac{-8y}{\left(1+4y^2\right)^2}\left(\frac{\sqrt{2}}{\sqrt{1+\sqrt{1-4y^2}}} + \frac{1}{2\sqrt{1-4y^2}+1}\right) + \\ & + \frac{y\sqrt{1-4y^2}}{1+4y^2}\left(\frac{\sqrt{2}}{\left(1+\sqrt{1-4y^2}\right)^{\frac{3}{2}}} + \frac{4}{\left(2\sqrt{1-4y^2}+1\right)^2}\right) \end{aligned} \tag{13}$$

Equation (13) can be integrated numerically, for example, by the improved Euler method [6-8]. The dependence of the dimensionless coordinate obtained as a result of numerical integration is $y(\tau)$ shown in Fig. . 3. From the obtained dependence, the period of arising oscillations can be determined.

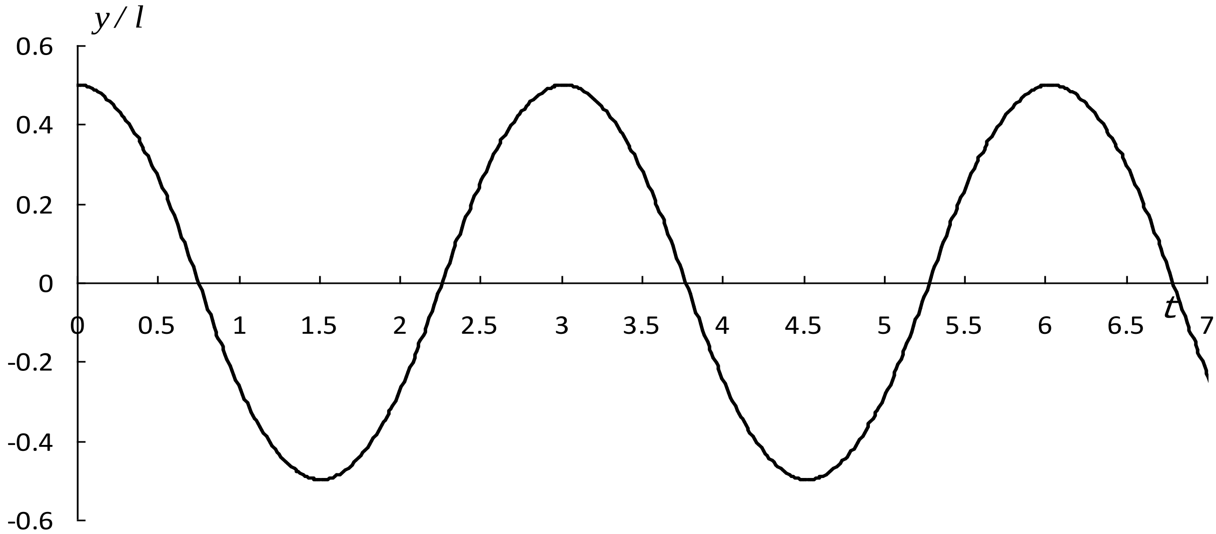

Fig. 3 – Dependence of the y-coordinate of the central charge on the dimensionless time τ

A program has been created that visualizes the resulting oscillations. On fig. 4 shows one of the configurations of the vibrational system.

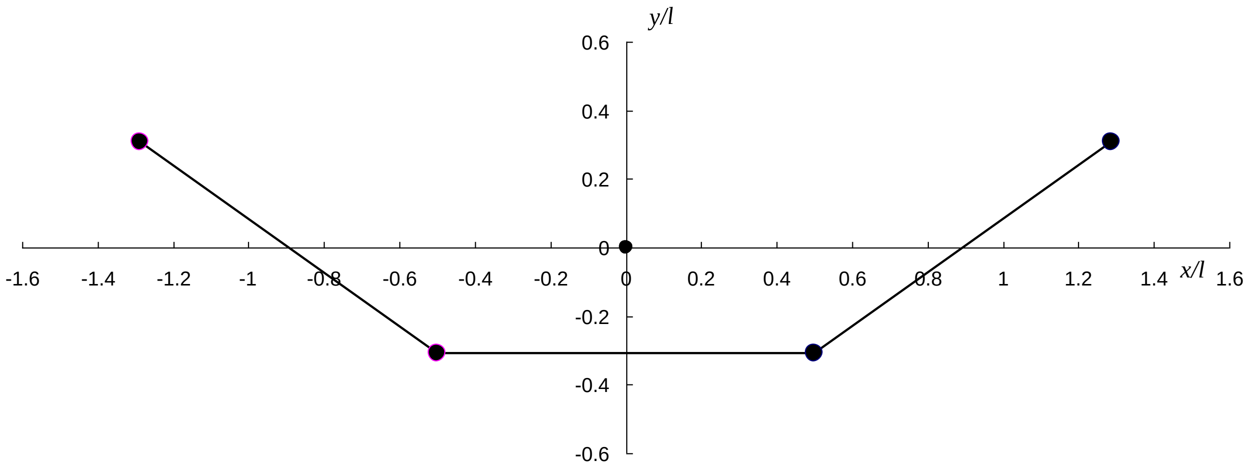

Fig. 4 – One of the configurations of the oscillating system

Using expression (11), we can also write the following formula for the oscillation period:

$$T = 4\tau_0 \int_0^{0.5} \frac{dy}{|\dot{y}|}. \tag{14}$$

To calculate the integral, one can use the Monte Carlo method and the mean value theorem [3]. Numerically calculating the integral (14), we obtain:

$$\int_0^{0.5} \frac{dy}{|\dot{y}|} = \int_0^{0.5} \frac{\sqrt{1+4y^2}\,dy}{\sqrt{\left(1-4y^2\right)\left(\frac{\sqrt{2}}{\sqrt{1+\left(1-4y^2\right)}} + \frac{1}{2\left(1-4y^2\right)+1}\right)}} \cong 0.7545\,. \tag{15}$$

Substituting the calculated value of the integral into (14), we find that in units τ, the period is approximately 3.018. Thus, the following expression can be written for the oscillation period:

$$T \cong 3.018\,\frac{l^{\frac{3}{2}}}{q}\sqrt{8\pi\varepsilon_0 m}\,. \tag{16}$$

From expression (13) we find the acceleration with which the central charges move. So, at the initial moment of time, immediately after the thread is burned out: $y = \frac{1}{2}$, $\dot{y} = 0$, we get $\ddot{y} = \ddot{y}_0 = -\left(\sqrt{2}+1\right)$. In dimensional variables, the initial acceleration is determined by the expression:

$$a_0 = \frac{l}{\tau_0^2}\ddot{y}_0 = -\frac{q^2\left(\sqrt{2}+1\right)}{8\pi\varepsilon_0 l^2 m}\,. \tag{17}$$

III. Conclusion

In this article, we have shown the advantages of introducing dimensionless variables. They simplify numerical calculations and make the results more universal.

It follows from expression (16) that the square of the period of the emerging oscillations is proportional to the cube of the linear dimensions of the system; such a dependence is typical for all fields of the Coulomb type. In such force fields, the interaction force of point bodies is inversely proportional to the square of the distance between the bodies, and the potential energy is inversely proportional to the first power of the distance between the bodies. In particular, such dependence of period on distance leads to Kepler's third law for the motion of planets in the gravitational field of the Sun.

An animation program has been created that visualizes the arising non-harmonic oscillations.

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