

# Applied Statistics For The Positioning Of Control Points In The Horizontal Deformation Monitoring Network. Case Study of "Gura Apelor" Dam, Romania

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**ABSTRACT:** For monitoring the horizontal deformations of dams in Romania, the geodetic monitoring networks were usually designed as micro-triangulation networks, which are constrained on a series of points considered fixed, installed onsite by concrete pillars. In relation to these points, through repeated measurements, the horizontal position of the new points located on the dam body and in the adjacent areas is determined at different time intervals in order to extract the deformation vectors and to establish the construction's trend over time. Before starting a new cycle of measurements it is always necessary to test the stability of the reference points in the fixed network or to check the validity of the previous position coordinates in case of a change in the measurement technology. Often, this procedure is avoided a priori by selecting two or three pillars whose coordinates will not change in the current measurement processing cycle, followed by re-determining the coordinates of the remaining pillars by least-squares adjustment. The paper presents the method of testing the fixed pillars network using the global congruence test, after which, depending on the results obtained, procedures for selective application of statistical tests may be carried out in order to strictly identify only those pillars that require a redetermination of position. This ensures continuity and coherence of interpretations resulting from the periodic preparation of deformation graphs of the mobile landmarks on the monitored construction.

**KEYWORDS:** deformation, monitoring, dam, network, stability.

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## I. INTRODUCTION

The monitoring of large dams in Romania began in the last century and was based on geodetic methods combined in some cases with physical methods to highlight the horizontal and vertical deformations of the monitored structure at millimeter precision level [1]. In the case of horizontal deformations, micro-triangulation networks were chosen, whose geometric configurations are currently determined by measurements with accurate total stations. Although there are studies on the implementation of GNSS technology in the structure of monitoring networks [2], the technical regulations impose the classical measurements of angles and eventually distances, requirements that are also found in other European countries, e.g. Portugal [3]. Unlike the GNSS networks, the horizontal networks determined by observations of angles and distances, require a good geometric configuration, which also adapts to the particular situation in which certain directions along which the deformations will be maximum are anticipated [4]. The precision of determining the new points, both in the network of fixed pillars and in the entire network that includes mobile landmarks on the body of the construction, is analyzed by means of the error ellipses resulting from the rigorous processing of the measurements applying the least squares method. In this case, the orientation of the error ellipse will be of such a nature that its flattening in the direction of the minor axis corresponds as much as possible to the direction in which the maximum deformations are expected. At the end of the adjustment of the current measurement cycle, the differences in plane coordinates in relation to the previous cycle or the reference cycle will indicate the displacements that occurred over time, the most significant ones corresponding to the axis that was chosen in the local system in the direction of water flow.

As a rule, in each new cycle of measurements, two points of known coordinates are considered in the fixed network of pillars, compared to which the other pillars are determined; in this way the adjustment of the network is carried out under the condition of a minimally constrained network. Practically, in this way, the remaining pillars are considered new points each time and are determined anew at each cycle of measurements. This a priori procedure eliminates the study of the stability of the pillars in the control network, because in the case of a displacement, its influence is removed by determining a new set of coordinates that will be used later to determine the coordinates of the mobile landmarks in the fully constrained network.

However, this way of working introduces to the calculation of the current cycle specific measurement errors that relativize the results from the deformation graphs, reducing the accuracy of the interpretation. Things are completely different if at each measurement cycle a statistical test for the stability of the pillars in the control network is carried out, so that based on a certain level of confidence, the coordinates of the fixed pillars are accepted and only those pillars that are not fit in the initial hypothesis are to be determined anew and used later in the adjustment of the entire network.

For this purpose, the workflow for the application of the global congruence test will be presented, aiming to establish the concordance between the configurations of the networks from two successive cycles and then, to identify those pillars that require a redetermination for the current cycle.

## II. EQUIPMENT AND GEODETIC METHODS FOR THE DAM MONITORING

The “Gura Apelor” Dam is the largest rock dam with a central clay core in Europe, being located in the Southern Carpathians of Romania in the “Râul Mare” valley (Figure 1). The dam was built between 1975 and 1986, having the following characteristics [5]:

- dam height: 168 m;
- crest length: 460 m;
- width of the base dam: 574 m.
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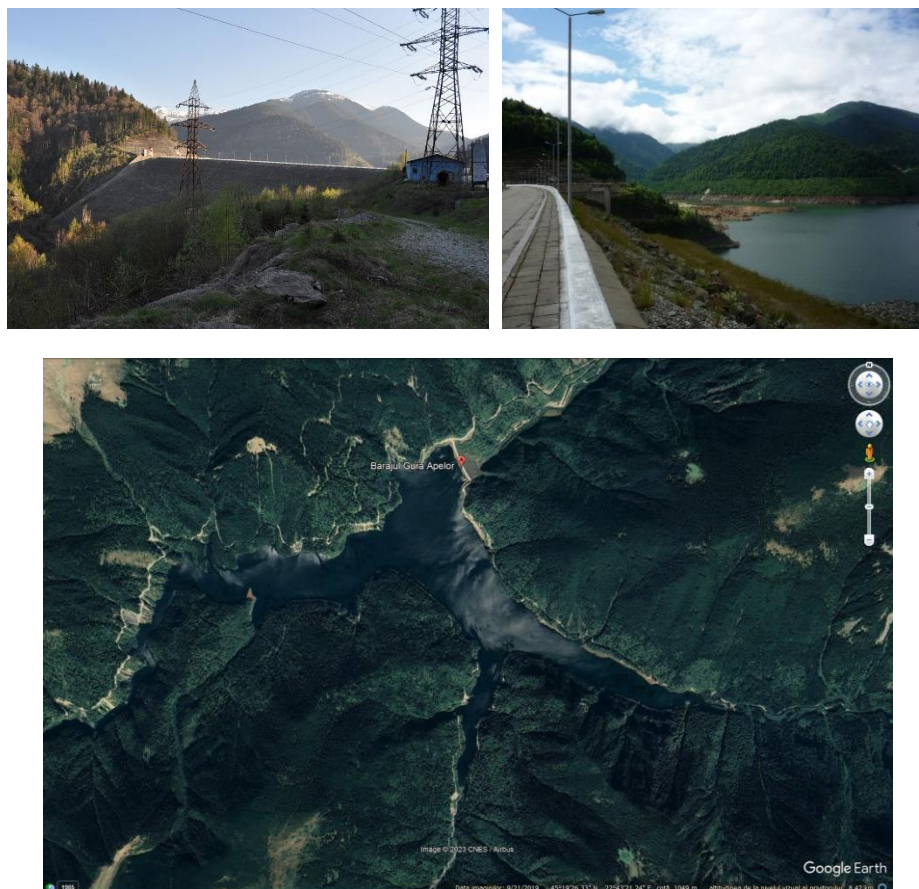


Fig.1. “Gura Apelor” dam accumulation (Romania)

The reservoir has an area of 373 ha and totals 210 million cubic meters of water. The hydropower development serves to produce electricity, to regulate the river and mitigate flood waves.

The coordinate reference system used for the dam monitoring is based on an engineering datum "Local RMR" with the local origin in the P1 pillar of false coordinates (1000m; 0m), with the OY axis oriented in the direction of water pressure (downstream-upstream) and the OX axis in the direction of the crest axis (right bank-left bank). The orientation of the network is done by fixing the coordinates of a second pillar: P6 (847.0704m; 375.9402m).

The geodetic equipment for the dam monitoring includes the following categories of points for the horizontal network:

- **The fixed network** was initially made up of nine cylindrical concrete pillars (P1÷P9), located on the two slopes of the valley downstream of the dam. Three pillars (P1, P3 and P6) were arranged on the right slope, and the six pillars (P2, P4, P5, P7, P8 and P9) on the left slope. As vertical positioning, P5, P6, P7, P8 and P9 pillars were placed at the level of the crest, and the rest at lower levels (Figure 2). The P2, P4 and P8 pillars, located on the left slope in an area where the forest was cleared when the construction of the dam began, are currently surrounded by forest vegetation, which no longer allows visibility for doing observations towards these network points, which led to their removal from the pillar network scheme.

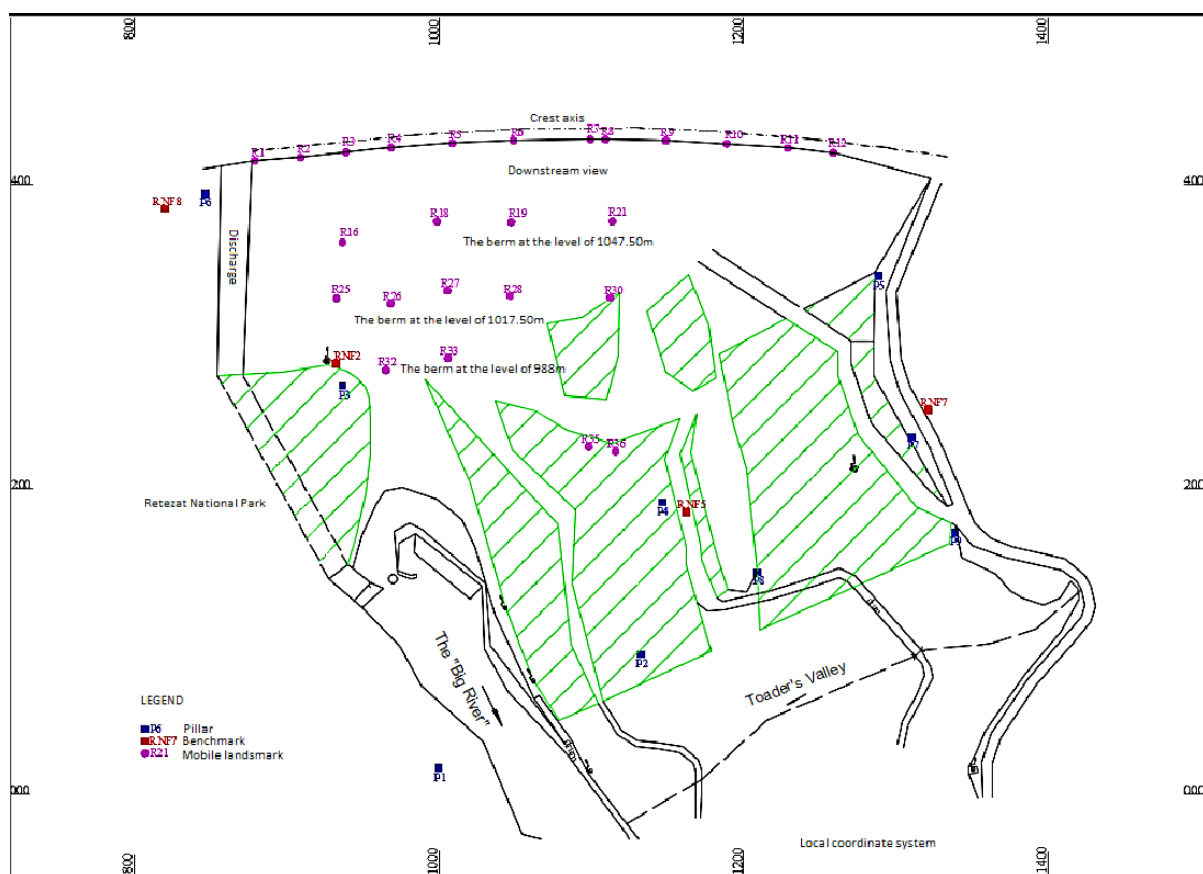


Fig. 2. Positioning of geodetic equipment for the horizontal monitoring network

- **The mobile network** consists of 24 mobile landmarks (R1÷R36), which were placed at the level of the crest (R1÷R12) and on the downstream face of the dam, arranged on three levels of height. A total of 19 mobile landmarks were selected for sketching the micro-triangulation network visas (Figure 3).

The observations made in the micro-triangulation horizontal network for two successive measurement cycles (2012, 2013) were selected for the present study. For this, the Leica TDM 5005 total station was used, which ensures an angular accuracy of 0.5". After in-station processing of the horizontal angles, standard deviations of the adjusted measurements resulted in the range of 0.4" – 5" (2012) and 0.3" – 6.7" (2013).

The presence of a blunder in the observations file can affect the geometry of the monitoring network and distort the accuracy of the results obtained after the adjustment process. That is why it is necessary to filter the initial data by testing geodetic observations using confidence intervals, with the following steps [6]:

- performing a least squares adjustment of the minimally constrained network;
- identifying residual errors that fail to pass the rejection criterion;
- removal of the largest detected error;
- resuming of adjustment.

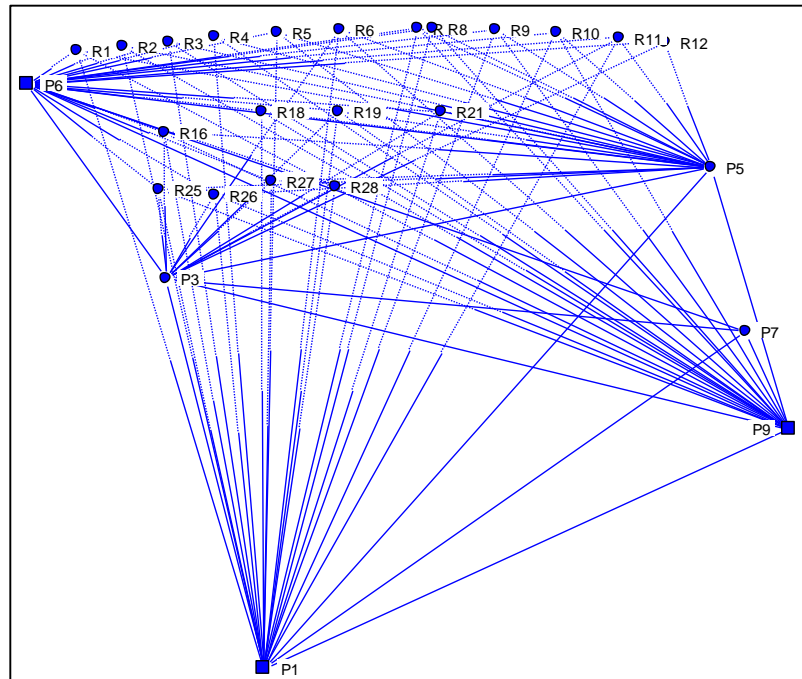


Fig. 3. Sketch of visas in the geodetic micro-triangulation network

A rejection level of  $3.29 s_0$  was accepted for the detection of blunders in the post adjustment check of the observations, where  $s_0$  is the reference standard deviation. In the case of the two series of measurements (2012, 2013), values of 1.4 and 1.1 were obtained for  $s_0$ , respectively. From the analysis of the adjustment reports, it follows that both the 2012 and 2013 series observations carried out in the horizontal monitoring network of the "Gura Apelor" dam have standard residuals lower than the calculated limits (4.487; 3.460) and therefore the measurements do not contain blunders.

### III. TESTING THE STABILITY OF FIXED PILLARS IN THE HORIZONTAL MONITORING NETWORK

The application of the global congruence test was performed for the horizontal monitoring network, taking into account the preservation of the geometric configuration of the network between the two cycles [7]. For both series of measurements, an adjustment was performed as a free network by means of the "S transformation". Since no point is fixed within the free network, the actual geodetic measurements cannot fit this network into a specific coordinate system. The rank defect in the case of the micro-triangulation network has a value of 4 and is represented by the number of degrees of freedom consisting of two translations, a rotation angle and a scale factor. The "S" transformation involves transforming shifted, non-unique solutions into estimated approximate values using an S-transformation matrix, which has the same rank defect as the matrix of the normal equations system [8].

The processing algorithm involves the following steps:

- the processing of the observations in the free network, from which the elements of the cofactor matrix  $Q_D$  result from replacing the four columns and four lines with the value 0 in the matrix of coefficients of the normal equations system and from solving the inverse of the matrix;
- the calculation of displaced parameters:

$$x_D = -Q_D A^T W L,$$

- where A is the configuration matrix, W is the weight matrix, L is the vector of observations;
- the design of the reference data matrix (D), which for some point "i" will have the form:
 
$$\begin{pmatrix} 1 & 0 & -y_i & x_i \\ 0 & 1 & x_i & y_i \end{pmatrix};$$
- computing the elements of the transformation matrix:
 
$$S = I - DD^+,$$
 where  $D^+$  is the pseudo-inverse matrix:
 
$$D^+ = (D^T D)^{-1} D^T,$$
 whence it follows:
 
$$S = I - D(D^T D)^{-1} D^T;$$
- the calculation of unshifted solutions:
 
$$x = S x_D;$$
- calculating the elements of the cofactor matrix of the unshifted solutions:
 
$$Q_x = S Q_D S^T;$$
- the calculation of the reference standard deviation:
 
$$s_0 = \pm \sqrt{\frac{v^T W v}{r - n + d}},$$
 where V is the residual matrix, W is the weight matrix, r is the number of observations, n is the number of unknowns and d is the rank defect;
- the calculation of the standard deviations of the unknowns (the adjusted coordinates of the new point):
 
$$s_x = \pm s_0 \sqrt{Q_{xx}} ; s_y = \pm s_0 \sqrt{Q_{yy}},$$
 where  $Q_{xx}$ ,  $Q_{yy}$  are the diagonal elements of the cofactor matrix ( $Q_x$ ) for the unknowns (dx, dy).
 The results of the accuracy evaluation in the case of least squares adjustment of the free network, in the two measurement cycles, are presented in Table 1.

**Table 1. Accuracy evaluation of free network adjustment**

Point	Measurement series	Reference standard deviation $\pm s_0$	Standard deviation		The Helmert's error $s_t$ [mm]	Coordinate differences d(mm)
			$\pm s_x$ (mm)	$\pm s_y$ (mm)		
P1	2012	1.3639	1.35	1.40	1.95	-0.7
	2013	1.0517	1.26	0.80	1.49	-1.9
P6	2012	1.3639	1.33	1.95	2.36	1.1
	2013	1.0517	1.08	0.87	1.39	-2.2
P9	2012	1.3639	0.80	0.92	1.22	-1.6
	2013	1.0517	0.40	0.97	1.04	1.5
P3	2012	1.3639	1.23	1.51	1.95	-0.7
	2013	1.0517	0.69	0.65	0.95	2.6
P5	2012	1.3639	1.03	1.11	1.52	2.2
	2013	1.0517	0.54	0.65	1.00	0.5
P7	2012	1.3639	0.61	0.72	0.94	-0.3
	2013	1.0517	0.50	1.03	1.15	-0.5

After performing the micro-triangulation geodetic network adjustment as a free network through the "S" transformation, the global congruence test is carried out in the case of networks with the same configuration, performing the following operations [7]:

- determining the vector of the coordinate differences of the network pillars between the two measurement cycles:
 
$$d = x_2 - x_1 \text{ (Table 1);}$$
- calculation of the cofactor matrix:
 
$$Q_d = Q_{x_1} + Q_{x_2};$$
- determination of the normally distributed quadratic form:
 
$$R = d^T Q_d^{-1} d = 23,1055;$$
- the calculation of the h factor under the condition that the network has the same configuration at the two measurement epochs:
 
$$h = \text{rank}(Q_{x_1}) = \text{rank}(Q_{x_2}) = n - d = 12 - 4 = 8,$$



- the estimate  $s_0^2$  for the weight unit variance  $\sigma_0^2$  is obtained from a separate adjustment of the measurements from the two cycles:

$$s_0^2 = \frac{v_1 s_{01}^2 + v_2 s_{02}^2}{v_1 + v_2} = \frac{12 \cdot 1.3639^2 + 14 \cdot 1.0517^2}{12 + 14} = 1,4541.$$

where  $v_1, v_2$  are the degrees of freedom for the 2012 and 2013 measurement epoch.

The basic hypothesis is expressed only after the condition that the same a priori factor was used or, more precisely, the estimates have the same expectation and were checked with F-test [9]:

$$F_{v_1, v_2, 1-\alpha/2} \leq s_{01}^2 / s_{02}^2 \leq F_{v_1, v_2, \alpha/2}$$

or

$$0.312 \leq 1.297 \leq 3.050$$

Then, the size of the congruence test is calculated with the following relationship:

$$F = \frac{d^T Q_d^{-1} d}{s_0^2 h} = \frac{R}{s_0^2 h} = \frac{23,1055}{1,4541 \cdot 8} = 1,99.$$

The null hypothesis is tested

$$P(F > F_{h, v_1 + v_2, 1-\alpha} | H_0) = \alpha,$$

where  $F_{h, v_1 + v_2, 1-\alpha} = F_{8, 26, 0.05} = 2,32$  for a 95% level of confidence.

The null hypothesis is rejected due to inequality ( $1.99 < 2.32$ ), which leads to the validation of the alternative hypothesis, which indicates that there are displacements of some pillars in the network as a whole, relative to the two measurement epochs.

#### IV. IDENTIFYING AND HIGHLIGHTING THE SHIFTED PILLARS OF THE MONITORING NETWORK

Following the global congruence test performed, it can be argued that there are displacements of fixed pillars between the two epochs. To detect these points, the values of the vector of the normally distributed quadratic form corresponding to each point were analyzed (table 2):

$$R_i = d_i^T Q_{d_i}^+ d_i.$$

It is observed that the maximum value of the vector  $R_i$  belongs to the point P5, which can thus be initially considered the first unstable point.

**Table 2. The values of the vector of the normally distributed quadratic form**

Point \ Value	P1	P6	P9	P3	P5	P7
$R_i$	2.3235	1.7158	5.5695	6.9049	8.1387	0.8520

This fact can also be highlighted by further constrained adjustments for the 2013 measurement series, considering combinations of 5 fixed points. Using the reference standard deviations resulting from the adjustment as the minimally constrained ( $s_1$ ) and additionally constrained network ( $s_2$ ), the null hypothesis  $s_1^2 = s_2^2$  should be rejected when the following statement is satisfied by applying the Fisher test:

$$F = \frac{s_2^2}{s_1^2} > F_{v_2, v_1, \alpha/2}$$

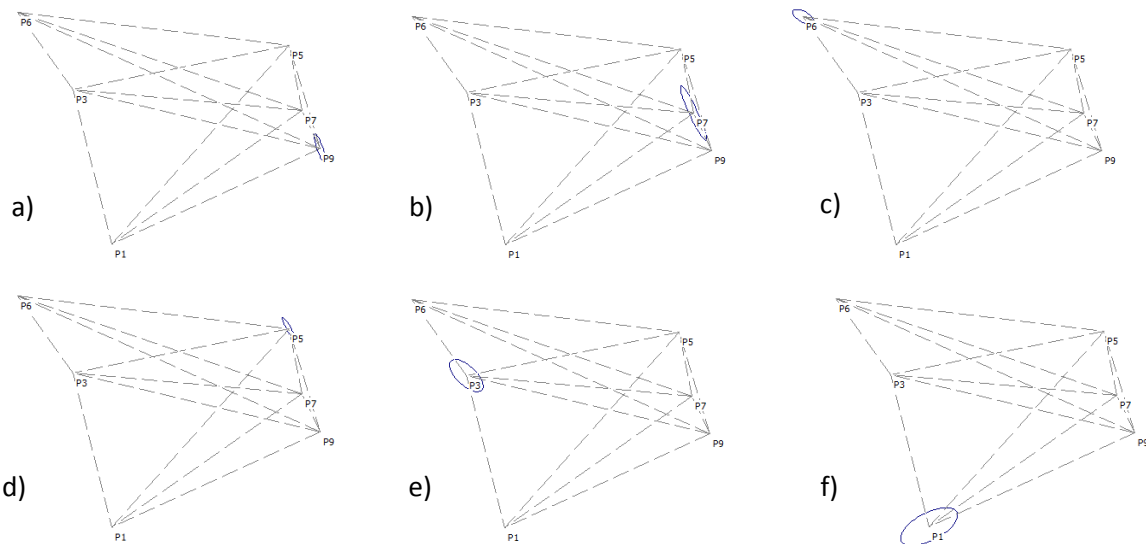
for a 95% ( $\alpha = 0.05$ ) confidence level for the variances obtained from the adjustment as the additionally constrained and the minimally constrained network (Table 3). The alternative hypothesis is stated by  $s_1^2 \neq s_2^2$  and it is validated in 3 cases where point P5 appears each time. For each case, it is observed that the error ellipses in the new point have much larger areas (Figure 4.b,e,f) compared to those in which the null hypothesis was validated (Figure 4.a,c,d). In addition, the P6-P5-P9 angle appears flagged with a possible blunder in the observations, but this is precisely due to the incorrect constraint coordinates of the P5 point.

Significantly, in the case of the combination P1–P6–P9–P3–P7, point P5 is determined within the accepted accuracy limits (Figure 4.d). The conclusion that emerges is that the P5 pillar has undergone significant shifts in the 2013 cycle compared to the 2012 cycle and needs to be re-determined for the current cycle. For this, the coordinates of point P5 from the validated adjustment P1–P6–P9–P3–P7 are accepted, having the standard deviations,  $s_x = 0.0011$  m,  $s_y = 0.0018$  m.

In order to verify the correctness of the chosen solution and the eventual displacement of other points, the calculation in table 3 is repeated for the cases in which the null hypothesis ( $s_1^2 = s_2^2$ ) was rejected, by applying the F test for a 95% confidence level for the variances obtained from the adjustment as an additional and minimally constrained network (Table 4). All three cases of point combinations are now validated for coordinate consistency, without indicating any possible blunders in the observations. Also, the error ellipses plots show the positioning errors of the new points in the same trend as those of the previously validated points. (Figure 5 a,b,c). In the other two previously validated cases, in which P5 pillar is included (P<sub>1</sub>-P<sub>3</sub>-P<sub>5</sub>-P<sub>6</sub>-P<sub>7</sub>, P<sub>1</sub>-P<sub>3</sub>-P<sub>5</sub>-P<sub>7</sub>-P<sub>9</sub>), the reference standard deviations were reduced to the range [1.7-1.8].

**Table 3. Testing the null hypothesis by applying the F-test to the control network**

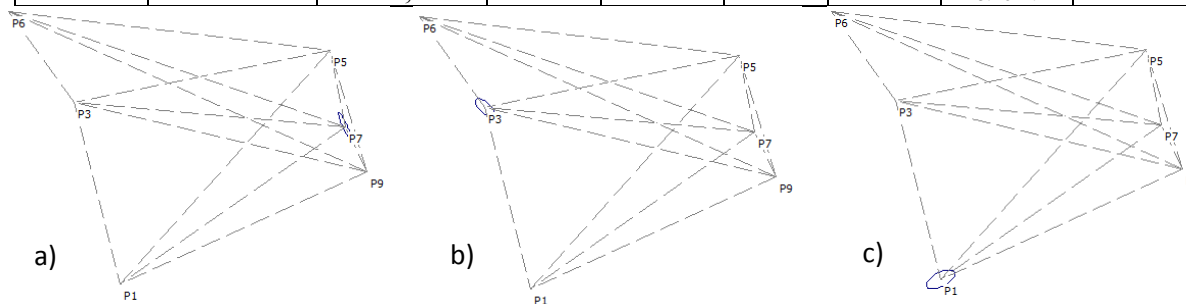
Measurement series	Network adjustment	Point Combinations	Reference standard deviation $s_0$	No. degrees of freedom $\nu$	Variance $s^2$	Statistical test $F = s_2^2/s_1^2$	Rejection criterion $F > F_{\alpha/2}$	Null hypothesis validation
2013	minimally constrained	P <sub>1</sub> - P <sub>6</sub>	1.1	14	1.106	-	-	-
	additionally constrained	P <sub>1</sub> - P <sub>3</sub> - P <sub>5</sub> - P <sub>6</sub> - P <sub>7</sub>	2.8	20	7.935	7.175	7.175 > 8.202 ?	YES
		P <sub>1</sub> - P <sub>3</sub> - P <sub>5</sub> - P <sub>6</sub> - P <sub>9</sub>	3.9	20	15.144	13.693	13.693 > 8.202 ?	No
		P <sub>1</sub> - P <sub>3</sub> - P <sub>5</sub> - P <sub>7</sub> - P <sub>9</sub>	1.7	20	2.969	2.684	2.684 > 8.202 ?	YES
		P <sub>1</sub> - P <sub>3</sub> - P <sub>6</sub> - P <sub>7</sub> - P <sub>9</sub>	1.9	20	3.672	3.320	3.320 > 8.202 ?	YES
		P <sub>1</sub> - P <sub>5</sub> - P <sub>6</sub> - P <sub>7</sub> - P <sub>9</sub>	3.8	20	14.276	12.908	12.908 > 8.202 ?	No
		P <sub>3</sub> - P <sub>5</sub> - P <sub>6</sub> - P <sub>7</sub> - P <sub>9</sub>	3.9	20	14.830	13.409	13.409 > 8.202 ?	No



**Fig. 4. Errors ellipse of the new point for the additionally constrained adjustment of the control network**

**Table 4 – Testing the null hypothesis by applying the F-test to the newly determined control network**

Measurement series	Network adjustment	Point Combinations	Reference standard deviation $s_0$	No. degrees of freedom $\nu$	Variance $s^2$	Statistical test $F = s_2^2/s_1^2$	Rejection criterion $F > F_{\alpha/2}$	Null hypothesis validation
2013	minimally constrained	$P_1 - P_6$	1.1	14	1.106	-	-	-
	additionally constrained	$P_1 - P_3 - P_5 - P_6 - P_9$	1.7	20	2.909	2.630	$2.630 > 8.202 ?$	Yes
		$P_1 - P_5 - P_6 - P_7 - P_9$	1.8	20	3.116	2.817	$2.817 > 8.202 ?$	Yes
		$P_3 - P_5 - P_6 - P_7 - P_9$	1.6	20	2.664	2.409	$2.409 > 8.202 ?$	Yes



**Fig. 5. Errors ellipse of the new point after resuming the additionally constrained network adjustment**

**IV. CONCLUSION**

The objective of this work was to study the application of statistical tests for tracking the stability of the control points of a horizontal monitoring network. The case study was chosen for the "Gura Apelor" dam for two consecutive measurement cycles (2012 and 2013) within the micro-triangulation control network containing 6 pillars.

Before performing the statistical tests to verify the stability over time of the control pillars, it is necessary to filter the measurement for blunder detection. Next, for the processing of the geodetic measurements in the two selected cycles, the network adjustment strategy by the least squares method was used under the minimally constrained and additionally constrained hypothesis.

Although network adjustment is currently performed using 2 or 3 pillars as constraints for the rest of the network pillars, it is useful to perform an analysis on the entire network of the 6 pillars, and for this the global congruence test was applied to the networks with the same configuration at each measurement cycle.

The invalidation of the null hypothesis led to the conclusion that there are pillars that have displacements in the last selected cycle, which indicated the need to identify them by applying statistical tests and comparing the minimally constrained and the additionally constrained network. In this way, the point P5 which introduces large errors in the network when included in the group of fixed pillars was identified and re-determined by least square adjustment, using the rest of the pillars as constraints.

To verify the correctness of this decision, the statistical comparison between the minimally constrained and the additionally constrained network containing the newly determined P5 pillar was repeated, and this time the null hypothesis of equality of variances was accepted for all cases.

Regarding the case study it is recommended for greater consistency in the construction of the graphs of the horizontal deformations of the mobile landmarks on the dam body and in the adjacent areas, to keep the same coordinates of the fixed pillars in the control network, except for the P5 pillar which was the only re-determined in the final analysis cycle.

By complying with these requirements for control of the positioning precision of the fixed pillars, the correspondence and continuity of the data between the different measurement cycles is preserved, so that the deformation vectors in the analyzed points reflect the real situation in the field to a great extent.

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