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Exponential Ultimate Boundedness of Impulsive Switched Systems with Time Delays

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ABSTRACT : *In this paper, by establishing differential delay inequalities, we discuss the exponential ultimate boundedness of a class of impulsive switched systems with time delays, the sufficient condition of exponential ultimate boundedness of the system are derived. Finally, an example is given to verify the effectiveness of the results.*

KEYWORDS *impulsive switched system,time delay,exponential ultimate boundedness.*

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I. INTRODUCTION

Hybrid Dynamic System(HDS) which is characterized by continuous changes over time and driven by discrete emergencies, also called Hybrid System(HS). In 1966, H.S. Witsenhausen proposed the earliest literature on hybrid dynamic system[1]. Since then, scholars at home and abroad have devoted themselves to relevant research. Switched system can be regarded as a typical hybrid system, consisting of continuous or discrete subsystems and a logical rule which coordinates switched machines among subsystems.Switched system has many applications in industry, such as network control, power control, traffic controland process control[2,3]. In addition to switched system, impulsive system[4,5], also known as impulsive signal, is another kind of important hybrid system, which includes the jump or reset of instantaneous states. And the concept of impulse can be understood as a sudden change of some variables at a certain moment during the operation of the system. The force of such a short, sudden change is considerable. Under the influence of impulse, condition of the system will change greatly. Moreover, time delay may occur during the course of the establishment of the switched system. Time delay means that the rate of state change of the system is related to the state of the past moment. Time-delay phenomenon is very common in practical application, especially actual industrial production process, such as chemical system, hydraulic system[6,7], and other fields. The universality of time-delay phenomenon is the reason why timedelay can become the favored research object of many scholars.

In this paper, both time-delay effect and impulsive effect are considered in switched system. So far, the academic circle studied the switched system has covered a wide range of areas, such as controllability problem, observability problem, stability problem, robustness problem, stabilization problem, etc.[8-10]. However, there are few researches on the boundedness of switched system at present. In such cases, it is meaningful to study the boundedness of switched system and the sufficient conditions for the exponential ultimate boundedness of impulsive switched system with time delays are derived by using two differential delay inequalities, which are vital tools in the study of boundedness of delay systems.

II. MODEL AND PRELIMINARIES

For the convenience of proof and derivation, the following symbols are given:

Let $\square[\Phi_{k-1},\square]$ represent a family of continuous functions from Φ_{k-1} to $\square$, I_n represents the n-dimensional identity matrix, $\square_+ = [0,\infty)$, $\mathrm{K} = \{1,2,\ldots,\}$, $\square_n = \{1,2,\ldots,n\}$, $\square = \{1,2,\ldots,N\}$. $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ are the maximum and minimum eigenvalue of a real symmetric matrix $P \in \square^{n\times n}$, respectively. The symbol $PC[[-\tau,0],\square^n]$ represents the space of piecewise right continuous $\square^n$-valued functions ϕ, which are defined on $[-\tau,0]$ with a norm $\|\phi\| = \sup_{-\tau\le\theta\le 0}|\phi(\theta)|$, in which $|\cdot|$ stands for Euclidean norm.

Consider the following impulsive switched system with time delays:

$$\begin{cases} x'(t) = P_{i_k} x(t) + f_{i_k}(t, x(t-\tau)),\ t \ge t_0,\ i_k \in \mathbb{N}, \\ \Delta x = x(t_k^+) - x(t_k^-) = I_k x(t_k^-),\ k \in \mathrm{K}, \\ x(t_0 + \theta) = \psi(\theta),\ \theta \in [-\tau, 0]. \end{cases} \quad (1)$$

where the constant $\tau > 0$ denotes the time delay, and $\psi(\theta) \in PC[[-\tau, 0], \mathbb{R}^n]$ represents the initial function . $I_k, P_{i_k} \in \mathbb{R}^{n\times n}$, $\{i_k\}$ denotes the switching signal $\sigma : \mathbb{R}_+ \to \mathbb{N}$ on the basis upon $\Phi_{k-1} = [t_{k-1}, t_k) \to i_k \in \mathbb{N}$, is a piecewise constant function, represents that the system has switched from the i_{k-1} -th subsystem to i_k -th subsystem at the time t_k .A piecewise continuous vector-valued function $f : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}^n$ satisfies $f_{i_k}(t, 0) = 0,\ t \in \mathbb{R}_+$, and ensuresthat system (1)exists and it is unique as well. Meanwhile, theimpulsive moments $t_k (k \in \mathrm{K})$ that are fixed should satisfy $0 \le t_0 < t_1 < t_2 < ... < t_k < ...,\ \lim_{k\to\infty} t_k = \infty$.

Obviously, there are N different subsystems in the system (1) showing as follows:

$$x'(t) = P_i x(t) + f_i(t, x(t-\tau)),\ i \in \mathbb{N}. \quad (2)$$

For every switched function $\sigma, t \in \mathbb{R}_+, t > t_0$, m_i denotes the total amount of activations of i-th subsystem (2) in the period of $[t_0, t]$, the symbols $\ell_{ij}(t_0, t)(j \in \{1, 2, ..., m_i\})$ denote the j-th continuous duration of work of the i-th subsystem, the symbol $\ell_i(t_0, t)$ denotes the total activation time of the i-th subsystem (2) in the period of $[t_0, t]$, each Lebesgue measure of the sets $\ell_i(t_0, t)$ and $\ell_{ij}(t_0, t)$ is represented by $\omega(\ell_i(t_0, t))$ and $\omega(\ell_{ij}(t_0, t))$. The continuous part of system (1) can be represented as $x'(s) = P_i x(s) + f_i(s, x(s-\tau)),$ $s \in \bigcup_{j=1}^{m_i} \ell_{ij}(t_0, t), t \ge t_0$, where $i \in \mathbb{N}$, $\bigcup_{i=1}^{N} \bigcup_{j=1}^{m_i} \ell_{ij}(t_0, t) = \bigcup_{i=1}^{N} \ell_i(t_0, t) = [t_0, t].$

The definition of exponential ultimate boundedness which is a slightly modified version of Definition 2.1 in [11] is given below.

Definition 2.1.*(Exponential Ultimate boundedness)*

System (1) is said to be exponentially ultimately bounded if there are positive constants λ *, K and* M_i *such that for any initial value* x_0 *,*

$$|x_i(t; t_0, x_0)| \le K e^{-\lambda(t-t_0)} \|x_0\| + M_i,\ i \in \mathbb{N}.$$

For the discussion of boundedness of delay systems, differential delay inequalities play an important role. In order to prove the exponentialultimate boundedness for the impulsive switched systems with time delays, the following two lemmas are introduced.

Lemma 2.2.*Let* $y(t) \in \mathbb{C}[\Phi_{k-1}, \mathbb{R}],\ k \in \mathrm{K}$ *, if there exists constants* $P_i > 0,\ Q_i > 0,\ i \in \mathbb{N}$ *such that*

$$y'(t) \le P_i y(t) + Q_i y(t-\tau),$$

then

$$y(t) \le \rho_k e^{\beta_{i,k}(t-t_{k-1})},\ t \in \Phi_{k-1},$$

where $\rho_k \ge \max_{t_{k-1}-\tau \le r \le t_{k-1}} y(r),$ *and* $\beta_{i,k} > 0$ *is a root of the equation*

$$Q_i e^{-\beta_{i,k}\tau} + P_i - \beta_{i,k} = 0.$$

Proof. It can be proved by Lemma 1 in literature [12] when $\alpha = 1$ with some minor modifications.

Lemma 2.3.*Let* $y(t) \in \mathbb{C}[\Phi_{k-1}, \mathbb{R}],\ k \in \mathrm{K}$ *, if there exists constants* $P_i < 0,\ Q_i > 0,\ i \in \mathbb{N}$ *and* $-P_i > Q_i$ *such that*

$$y'(t) \le P_i y(t) + Q_i y(t-\tau) + J,$$

then

$$y(t) \le \rho_k e^{-\gamma_{i,k}(t-t_{k-1})} + \frac{J}{-P_i - Q_i},\ t \in \Phi_{k-1},$$

where $\rho_k \ge \max_{t_{k-1}-\tau \le r \le t_{k-1}} y(r)$ *,* $\gamma_{i,k} > 0$ *is a root of the equation*

$$Q_i e^{\gamma_{i,k}\tau} + P_i + \gamma_{i,k} = 0.$$

Proof. It can be proved by Lemma 3 in literature [13] when $\alpha = 1$ with some minor modifications.

III. BOUNDEDNESS ANALYSIS

In order to proof the exponential ultimate boundedness of system (1), we begin to introduce the following symbols:

$\hat{P}_i = \lambda_{\max}(\Lambda_i^{-1}(P_i^T\Lambda_i + P_i\Lambda_i)) + \tilde{\xi}_i < 0$ for $i \in U_s \triangleq \{1,2,...,\kappa\}$.

$\hat{P}_i = \lambda_{\max}(\Lambda_i^{-1}(P_i^T\Lambda_i + P_i\Lambda_i)) + \tilde{\xi}_i \geq 0$ for $i \in U_u \triangleq \{\kappa+1,\kappa+2,...,N\}$.

$U_u = \varnothing$ if $\kappa = N$, $\ell_s(t_0,t) = \sum_{i=1}^{\kappa}\omega(\ell_i(t_0,t))$, $\ell_u(t_0,t) = \sum_{i=\kappa+1}^{N}\omega(\ell_i(t_0,t))$.

Then, we introduce the following assumptions:

(H_1) For any $\mu,\ \nu \in \mathbb{R}^n$, and Λ_i is a symmetric positive definite matrix, there exists nonnegative constants ξ_i and $\tilde{\xi}_i$ such that

$$2f_i^T(t,\mu)\Lambda_i\nu \leq \xi_i\mu^T\Lambda_i\mu + \tilde{\xi}_i\nu^T\Lambda_i\nu,\ i \in \mathbb{N},\ t \geq t_0.$$

(H_2) There exists a constant $\hat{\lambda}_i \geq 0$ such that

$$e^{-\gamma_i(\ell_{ij}(t_0,t)-\tau)} \leq e^{-\hat{\lambda}_i\ell_{ij}(t_0,t)},$$

where $\gamma_i = \inf_{k\in\mathrm{K}}\{\gamma_{i,k}\}$, and $\gamma_{i,k} > 0$ satisfies

$$\tilde{\xi}_i e^{\gamma_i\tau} + \hat{P}_i + \gamma_i = 0,\ i \in U_s.$$

(H_3) For any $t > t_0$, there exists a constant $\vartheta > 0$ such that

$$\sup_{t>t_0}\frac{\lambda^*\ell_u(t_0,t) - \lambda_*\ell_s(t_0,t)}{t-t_0} = -\vartheta < 0, \quad (3)$$

where $\lambda^* = \max_{i\in U_u}\{\beta_i\}$, $\lambda_* = \min_{i\in U_s}\{\gamma_i\}$, $\breve{\lambda}_i = \beta_i > 0$ satisfies $\tilde{\xi}_i e^{-\beta_i\tau} + \hat{P}_i - \beta_i = 0,\ i \in U_u$.

(H_4) There exists a constant $0 < \eta < \vartheta$ such that

$$\eta_k \leq e^{\eta(t_k - t_{k-1})},\ k \in \mathrm{K}, \quad (4)$$

where

$$\eta_k = \max\{1, \lambda_{\max}[\Lambda_{i_k}^{-1}(I_n + I_k)^T\Lambda_{i_k}(I_n + I_k)]\},\ k \in \mathrm{K}.$$

(H_5) There exists a constant $\delta > 1$ such that

$$(\prod_{i=1}^{k-1}\eta_i)[\prod_{i=l+1}^{N}\prod_{j=1}^{m_i}e^{\beta_i\ell_{ij}(t_0,t)}] < \delta,$$

$$(\prod_{i=1}^{k-1}\eta_i)[\prod_{i=l+1}^{N}\prod_{j=1}^{m_i}e^{-\gamma_i\ell_{ij}(t_0,t)}] + (\prod_{i=2}^{k-1}\eta_i) + (\prod_{i=3}^{k-1}\eta_i) + \cdots + \eta_{k-1} + 1 < \delta.$$

Theorem 3.1. *Assume that (H_1) to (H_5) hold, and $\tau < t_k - t_{k-1}$ for all $k \in \mathrm{K}$. Then the system (1) is exponentially ultimately bounded.*

Proof. Define $G(x(t)) = x^T\Lambda_{i_k}x$. By($H_1$), we have

$$\begin{aligned}
G'(x(t)) &\leq 2x^T(t)\Lambda_{i_k}x'(t) \\
&\leq 2x^T(t)\Lambda_{i_k}[P_{i_k}x(t) + f_{i_k}(t,x(t-\tau))] \\
&= x^T(P_{i_k}^T\Lambda_{i_k} + \Lambda_{i_k}P_{i_k})x + 2f_{i_k}^T(t,x(t-\tau))\Lambda_{i_k}x \\
&\leq \lambda_{\max}(\Lambda_{i_k}^{-1}(P_{i_k}^T\Lambda_{i_k} + P_{i_k}\Lambda_{i_k}))x^T\Lambda_{i_k}x \\
&\quad + \xi_i x^T(t-\tau)\Lambda_{i_k}x(t-\tau) + \tilde{\xi}_{i_k}x^T\Lambda_{i_k}x \\
&= [\lambda_{\max}(\Lambda_{i_k}^{-1}(P_{i_k}^T\Lambda_{i_k} + P_{i_k}\Lambda_{i_k})) + \tilde{\xi}_{i_k}]G(t) \\
&\quad + \xi_{i_k}G(t-\tau),\ t \in \Phi_{k-1}.
\end{aligned}$$

It is feasible to suppose that the i-th subsystem is active on $[t_0,t_1)$ and $i \in U_s$, since the generality still exist there. According to Lemma 2.3, there exists a positive constant $\rho_1 \geq \bar{G}_\tau(t_0)$ such that

$$G(x(t)) \leq \rho_1 e^{-\gamma_{i,1}(t-t_0)} + \frac{J}{-P_i - Q_i},\ t \in [t_0,t_1). \quad (5)$$

On the other side,

$$\begin{aligned} G(x(t_1)) &= (x(t_1^-)+I_1x(t_1^-))^T\Lambda_{i_1}(x(t_1^-)+I_1x(t_1^-)) \\ &= x^T(t_1^-)(I_n+I_1)^T\Lambda_{i_1}(I_n+I_1)x(t_1^-) \\ &\le \lambda_{\max}[\Lambda_{i_1}^{-1}(I_n+I_1)^T\Lambda_{i_1}(I_n+I_1)][x^T(t_1^-)\Lambda_{i_1}x(t_1^-)] \\ &\le \eta_1 G(x(t_1^-)). \end{aligned}$$

Combining with (5), we have

$$G(x(t_1)) \le \eta_1[\rho_1 e^{-\gamma_{i,1}(t_1-t_0)} + \frac{J}{-P_i-Q_i}].$$

Therefore, we can obtain

$$G(x(t)) \le \eta_1[\rho_1 e^{-\gamma_{i,1}(t-t_0)} + \frac{J}{-P_i-Q_i}],\ t\in[t_1-\tau,t_1]. \quad (6)$$

Suppose that the j-th subsystem is activated on $[t_1,t_2)$, let $I=\frac{J}{-P_i-Q_i}$.

According to (6) and Lemma 2.2, Lemma 2.3, for $t\in[t_1,t_2)$, we have

$$G(x(t)) \le \begin{cases} [\eta_1[\rho_1 e^{-\gamma_{i,1}(t_1-\tau-t_0)}+I]]e^{\beta_{j,2}(t-t_1)},\ j\in U_u, \\ \eta_1[\rho_1 e^{-\gamma_{i,1}(t_1-\tau-t_0)}+I]e^{-\gamma_{j,2}(t-t_1)}+I,\ j\in U_s. \end{cases} \quad (7)$$

By(7), we obtain

$$G(x(t_2)) \le \eta_2[\eta_1[\rho_1 e^{-\gamma_{i,1}(t_1-\tau-t_0)}+I]]e^{\beta_{j,2}(t_2-t_1)},$$

or

$$G(x(t_2)) \le \eta_2[\eta_1[\rho_1 e^{-\gamma_{i,1}(t_1-\tau-t_0)}+I]e^{-\gamma_{j,2}(t_2-t_1)}+I]+I.$$

Thus, for $t\in[t_2-\tau,t_2]$, we have

$$G(x(t)) \le \eta_2[\eta_1[\rho_1 e^{-\gamma_{i,1}(t_1-\tau-t_0)}+I]]e^{\beta_{j,2}(t-t_1)},$$

or

$$G(x(t)) \le \eta_2[\eta_1[\rho_1 e^{-\gamma_{i,1}(t_1-\tau-t_0)}+I]e^{-\gamma_{j,2}(t-t_1)}+I]+I.$$

Now do a repetition ofsteps above, using a simple induction, we obtain

$$G(x(t)) \le \begin{cases} (\prod_{i=1}^{k-1}\eta_i)\rho_1[\prod_{i=1}^{l}\prod_{j=1}^{m_i}e^{-\gamma_i(\ell_{ij}(t_0,t)-\tau)}]\times[\prod_{i=l+1}^{N}\prod_{j=1}^{m_i}e^{\beta_i\ell_{ij}(t_0,t)}] \\ +(\prod_{i=1}^{k-1}\eta_i)[\prod_{i=l+1}^{N}\prod_{j=1}^{m_i}e^{\beta_i\ell_{ij}(t_0,t)}]I,\ t\in\Phi_{k-1},\ k\in\mathrm{K},\ j\in U_u, \\ (\prod_{i=1}^{k-1}\eta_i)\rho_1[\prod_{i=1}^{l}\prod_{j=1}^{m_i}e^{-\gamma_i(\ell_{ij}(t_0,t)-\tau)}]\times[\prod_{i=l+1}^{N}\prod_{j=1}^{m_i}e^{-\gamma_i\ell_{ij}(t_0,t)}] \\ +(\prod_{i=1}^{k-1}\eta_i)[\prod_{i=l+1}^{N}\prod_{j=1}^{m_i}e^{-\gamma_i\ell_{ij}(t_0,t)}]I \\ +(\prod_{i=2}^{k-1}\eta_i+\prod_{i=3}^{k-1}\eta_i+\cdots+\eta_{k-1}+1)I,\ t\in\Phi_{k-1},\ k\in\mathrm{K},\ j\in U_s. \end{cases}$$

Based on (H_5), it is easy to get

$$\begin{aligned} G(x(t)) &\le (\prod_{i=1}^{k-1}\eta_i)\rho_1[\prod_{i=1}^{l}\prod_{j=1}^{m_i}e^{-\gamma_i(\ell_{ij}(t_0,t)-\tau)}] \\ &\times[\prod_{i=l+1}^{N}\prod_{j=1}^{m_i}e^{\beta_i\ell_{ij}(t_0,t)}]+\delta I,\ t\in\Phi_{k-1},\ k\in\mathrm{K}. \end{aligned} \quad (8)$$

In the light of(H_3) and (H_2), we have

$$\begin{aligned} \prod_{i=l+1}^{N}\prod_{j=1}^{m_i}e^{\beta_i\ell_{ij}(t_0,t)} &= \prod_{i=l+1}^{N}\prod_{j=1}^{m_i}e^{\bar{\lambda}_i\ell_{ij}(t_0,t)} \\ &\le e^{\lambda^*\sum_{i=l+1}^{N}\sum_{j=1}^{m_i}\ell_{ij}(t_0,t)} = e^{\lambda^*\ell_u(t_0,t)}, \end{aligned} \quad (9)$$

and

$$\begin{aligned} \prod_{i=1}^{l}\prod_{j=1}^{m_i}e^{-\gamma_i(\ell_{ij}(t_0,t)-\tau)} &\le \prod_{i=1}^{l}\prod_{j=1}^{m_i}e^{-\hat{\lambda}_i\ell_{ij}(t_0,t)} \\ &\le e^{-\lambda_*\sum_{i=1}^{l}\sum_{j=1}^{m_i}\ell_{ij}(t_0,t)} = e^{-\lambda_*\ell_s(t_0,t)}. \end{aligned} \quad (10)$$

By (8) to (10), for $t\in\Phi_{k-1},\ k\in\mathrm{K}$, we have

$$G(x(t)) \le \eta_1...\eta_{k-1}\rho_1 e^{\lambda^*\ell_u(t_0,t)}e^{-\lambda_*\ell_s(t_0,t)}+\delta I. \quad (11)$$

On the basis of(3), we have

$$\lambda^{*}\ell_u(t_0,t)-\lambda_{*}\ell_s(t_0,t)\le-\vartheta(t-t_0),\ t>t_0. \quad (12)$$

By (4), for $t\in\Phi_{k-1},\ k\in K$, we can derive that

$$\eta_1...\eta_{k-1}\le e^{\eta(t_{k-1}-t_0)}\le e^{\eta(t-t_0)}. \quad (13)$$

Substituting (12) and (13) into (11), we can get

$$G(x(t))\le\rho_1 e^{-(\vartheta-\eta)(t-t_0)}+\delta I,\ t\in\Phi_{k-1},\ k\in K.$$

That is

$$|x(t)|\le\sqrt{\frac{\rho_1 e^{-(\vartheta-\eta)(t-t_0)}+\delta\dfrac{J}{-P_i-Q_i}}{\min_{i\in\square}\lambda_{\min}(\Lambda_i)}},t\ge t_0.$$

Thus, the system (1) is exponential ultimately bounded.

Remark. In Theorem 3.1, according to Lemma 2.2 and Lemma 2.3, we derive the boundedness conditions for a class of systems, including unstable and stable subsystems. Lemma 2.2 and Lemma 2.3 is used to handle unstable and stable subsystems, respectively. Obviously,for the systemonly containing stable subsystems, this conclusion still holds.

Corollary 3.2.*Assume that (H_1) to (H_4) hold, and $\tau<t_k-t_{k-1}$ for all $k\in K$. If $J=0$ in Lemma 2.3, then the system (1) is globally exponentially stable.*

Proof. It can be proved by Theorem 1 in literature [12] when $\alpha=1$ with some minor modifications.

IV. EXAMPLE

Example 4.1. Consider a two-dimensional case of system (1). Assume the parameters

$$P_1=\begin{pmatrix}-9 & 0\\ 5 & -9\end{pmatrix},\ P_2=\begin{pmatrix}1 & 3\\ 0 & 1\end{pmatrix},$$

$$f_i^T(t,x)=(3-i)(x_1,x_2),\ i=1,2,$$

$$I_k=\begin{pmatrix}e^{0.015}-1 & 0\\ 0 & e^{0.015}-1\end{pmatrix},$$

the switching signal $\sigma(\Phi_{k-1})=i_k=\begin{cases}1,\ if\ 3\nmid k,\\ 2,\ if\ 3\mid k,\end{cases}\ k\in K,$

the time delay τ=0.15,

the impulsive moments $t_k: t_k=0.3+t_{k-1},\ t_0=0.$

Taking $\xi_i=\tilde{\xi}_i=(3-i)$ and $\Lambda_i=I_2$, it is easy to verify that (H_1) holds.

According to

$\hat{P}_i=\lambda_{\max}(\Lambda_i^{-1}(P_i^T\Lambda_i+P_i\Lambda_i))+\tilde{\xi}_i<0$ for $i\in U_s\ \square\ \{1,2,...,\kappa\}.$

$\hat{P}_i=\lambda_{\max}(\Lambda_i^{-1}(P_i^T\Lambda_i+P_i\Lambda_i))+\tilde{\xi}_i\ge 0$ for $i\in U_u\ \square\ \{\kappa+1,\kappa+2,...,N\}.$

Then when i=1,

$$\begin{aligned}&\Lambda_1^{-1}(P_1^T\Lambda_1+P_1\Lambda_1)\\ &=\begin{pmatrix}1&0\\0&1\end{pmatrix}[\begin{pmatrix}-9&5\\0&-9\end{pmatrix}\begin{pmatrix}1&0\\0&1\end{pmatrix}+\begin{pmatrix}-9&0\\5&-9\end{pmatrix}\begin{pmatrix}1&0\\0&1\end{pmatrix}]\\ &=\begin{pmatrix}-18&5\\5&-18\end{pmatrix},\end{aligned}$$

then $|\lambda E-(\Lambda_1^{-1}(P_1^T\Lambda_1+P_1\Lambda_1))|=\begin{vmatrix}\lambda+18 & -5\\ -5 & \lambda+18\end{vmatrix}=(\lambda+18)^2-25=0,$

we obtain $\lambda_1=-23,\ \lambda_2=-13.$

Thus, $\lambda_{\max}(\Lambda_1^{-1}(P_1^T\Lambda_1+P_1\Lambda_1))+\tilde{\xi}_1=-13+(3-1)=-11<0.$

Therefore, we get $\hat{P}_1=-11.$

When i=2, in the same way, we have $\hat{P}_2=6.$

Clearly, $U_s=\{1\}$, $U_u=\{2\}$.

We can get $6<\gamma_{1,k}<6.1$, $6.2<\beta_{2,k}<6.5$, where $\gamma_{1,k}$ and $\beta_{2,k}$ are the roots of the equations $2\times e^{0.15\gamma_{1,k}}-11+\gamma_{1,k}=0$ and $e^{-0.15\beta_{2,k}}+6-\beta_{2,k}=0$, respectively.

Hence, (H_2) holdswhen taking $\hat{\lambda}_1=2$, $\breve{\lambda}_2<7$.

In fact,

$$e^{-\gamma_1(\ell_{2j}(t_0,t)-\tau)}\le e^{-6\times 0.15}=0.4066\le e^{-2\times 0.3},$$
$$e^{\beta_2\ell_{1j}(t_0,t)}\le e^{6.5\times 0.3}=7.0287\le e^{7\times 0.3}.$$

Furthermore, $\lambda^*=\breve{\lambda}_2<7$, $\lambda_*=\hat{\lambda}_1=2$,

$$\eta_k=\max\{1,\lambda_{\max}[\Lambda_{i_k}^{-1}(I_n+I_k)^T\Lambda_{i_k}(I_n+I_k)]\}$$
$$=\max\{1,\lambda_{\max}[\begin{pmatrix}1&0\\0&1\end{pmatrix}^{-1}[\begin{pmatrix}1&0\\0&1\end{pmatrix}+\begin{pmatrix}e^{0.015}-1&0\\0&e^{0.015}-1\end{pmatrix}]^T\begin{pmatrix}1&0\\0&1\end{pmatrix}[\begin{pmatrix}1&0\\0&1\end{pmatrix}+\begin{pmatrix}e^{0.015}-1&0\\0&e^{0.015}-1\end{pmatrix}]]\}$$
$$=\max\{1,\lambda_{\max}\begin{pmatrix}e^{0.03}&0\\0&e^{0.03}\end{pmatrix}\}=\max\{1,e^{0.03}\}=e^{0.03},$$
$$\ell_u(t_0,t)\le 0.15(t-t_0),\ \ell_s(t_0,t)\ge 0.85(t-t_0).$$

Based on these parameters, we can obtain

$$\sup_{k\in K}\frac{\lambda^*\ell_u(t_0,t)-\lambda_*\ell_s(t_0,t)}{t-t_0}=-\vartheta<\sup_{t>t_0}\frac{7\ell_u(t_0,t)-2\ell_s(t_0,t)}{t-t_0}=-0.65<0.$$

That is $\vartheta>0.65$.

$$\frac{\ln\eta_k}{t_k-t_{k-1}}=\frac{\ln e^{0.03}}{0.3}=0.1=\eta<\vartheta.$$

Thus, (H_3)and (H_4) hold.

Then by (H_4), we get

$$1\le\eta_1\le e^{\eta(t_1-t_0)}=e^{0.1\times 0.3}=e^{0.03},$$
$$1\le\eta_2\le e^{\eta(t_2-t_1)}=e^{0.1\times 0.3}=e^{0.03},$$
$$\vdots$$
$$1\le\eta_{k-1}\le e^{\eta(t_{k-1}-t_{k-2})}=e^{0.1\times 0.3}=e^{0.03}.$$

Now take $\eta_1=\frac{1}{k}e^{\frac{\delta_1}{(2k)^k}-1.95}$, $\eta_2=\frac{1}{k}e^{\frac{\delta_1}{(2k)^k}\times 2-1.95}$,…, $\eta_{k-1}=\frac{1}{k}e^{\frac{\delta_1}{(2k)^k}\times(k-1)-1.95}$, $k\in K$, δ_1 is a constant,

$$(\prod_{i=1}^{k-1}\eta_i)[\prod_{i=l+1}^{N}\prod_{j=1}^{m_i}e^{\beta_i\ell_{ij}(t_0,t)}]$$
$$<(e^{6.5\times 0.3})^{k-1}\times(e^{\frac{\delta_1}{(2k)^k}-1.95}\times e^{\frac{\delta_1}{(2k)^k}\times 2-1.95}\times\cdots\times e^{\frac{\delta_1}{(2k)^k}(k-1)-1.95})\times\frac{1}{k^{k-1}}$$
$$=\frac{1}{k^{k-1}}e^{1.95(k-1)+\frac{\delta_1}{(2k)^k}\frac{(1+k-1)(k-1)}{2}-1.95(k-1)}=\frac{1}{k^{k-1}}e^{\frac{\delta_1 k(k-1)}{2(2k)^k}}<e^{\frac{\delta_1(k-1)}{2^k k^{k-2}(k-1)}}<e^{\delta_1},$$
$$(\prod_{i=1}^{k-1}\eta_i)[\prod_{i=l+1}^{N}\prod_{j=1}^{m_i}e^{-\gamma_i\ell_{ij}(t_0,t)}]$$
$$<\frac{1}{k^{k-1}}e^{-0.9(k-1)+\frac{\delta_1}{(2k)^k}\frac{(1+k-1)(k-1)}{2}-1.95(k-1)}=\frac{1}{k^{k-1}}e^{\frac{\delta_1 k(k-1)}{2(2k)^k}-2.85(k-1)}<\frac{1}{k}e^{\delta_1},$$
$$(\prod_{i=2}^{k-1}\eta_i)=\frac{1}{k^{k-2}}e^{\frac{\delta_1}{(2k)^k}\frac{(2+k-1)(k-2)}{2}-1.95(k-1)}=\frac{1}{k^{k-2}}e^{\frac{\delta_1(k+1)(k-1)}{2(2k)^k}-1.95(k-1)}<\frac{1}{k}e^{\delta_1},$$
$$\vdots$$
$$\eta_{k-1}=\frac{1}{k}e^{\frac{\delta_1}{(2k)^k}\times(k-1)-1.95}<\frac{1}{k}e^{\delta_1},$$

As $1\le\eta_{k-1}$, $1<\frac{1}{k}e^{\delta_1}$.

Hence, $(\prod_{i=1}^{k-1}\eta_i)[\prod_{i=l+1}^{N}\prod_{j=1}^{m_i} e^{-\gamma_i \ell_{ij}(t_0,t)}]+(\prod_{i=2}^{k-1}\eta_i)+(\prod_{i=3}^{k-1}\eta_i)+\cdots+\eta_{k-1}+1<k\times\frac{1}{k}e^{\delta_1}=e^{\delta_1}$.

Meanwhile, we can get $1\le\frac{1}{k}e^{\frac{\delta_1}{(2k)^k}-1.95}\le e^{0.03},\cdots,\ 1\le e^{\frac{\delta_1}{(2k)^k}(k-1)-1.95}\le e^{0.03}$.

So $\delta_1>0$, that is $e^{\delta_1}>1$.

Let $\delta=e^{\delta_1}$, then $\delta>1$.

Therefore, (H_5) holds.

Hence, by Theorem 3.1, the system (1) is exponentially ultimately bounded.

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