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Research on underwater cooperative positioning error compensation method based on multi-source data correction

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Abstract :*In the process of underwater formation cooperative driving, the error of navigation and positioning is mainly caused by the time and distance of information transmission. The error caused by time is related to communication delay, clock deviation and other factors. Transmission distance error is affected by multiple factors such as water temperature, ocean current and water depth, which is difficult to quantify. In this paper, the Kalman filter algorithm is used to model the trajectory of the micro-platform, analyze the generation mechanism of navigation and positioning errors, classify the errors, and divide them into systematic errors and random errors. Combined with the multiple linear regression model and the method based on Bayesian particle filter, the positioning error characteristics of the micro-platform in the coordinated movement of underwater formation are studied. The experimental conditions are changed, and the corresponding relationship between the dependent variables and independent variables in the model is studied to realize the compensation and estimation of the positioning error.*

Keywords**:** *underwater formation cooperation, Kalman filter, multiple linear regression model, particle filter, error estimation*

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I. INTRODUCTION

Underwater multi-person / aircraft formation [1,2] cooperative work is an important way of working whether in the civil field of marine engineering underwater maintenance operations or in the military field of underwater mission execution of special combatants. In order to achieve formation maintenance and accurate formation route in a wide range of complex water environment, it is necessary to integrate multi-source data generated by different sensors, which will inevitably produce navigation and positioning errors caused by systematic errors and random errors. The systematic error refers to the error formed by the accumulated calculation of sensors such as inertial navigation and Doppler ergometer. Its influence on navigation and positioning often conforms to certain rules ; random error refers to the error caused by environmental conditions or certain factors according to certain rules, which is unpredictable.

In this paper, the motion model and positioning model of micro-platform based on Kalman filtering algorithm are established, and the mechanism of relative positioning error under the working conditions of cooperative formation is analyzed. The multiple linear regression model method and the algorithm based on Bayesian particle filter are adopted. By changing the experimental conditions, the corresponding relationship between the dependent variable and the independent variable in the model is studied, and the estimation of navigation and positioning error is completed.

II. ESTABLISHMENT OF MICRO-PLATFORM MOTION MODEL

In the cooperative navigation of underwater formation, the master and slave micro-motion platforms first synchronize their clocks. Assuming that the underwater acoustic signal of the autonomous micro-motion platform is received from the micro-motion platform at a certain time τ, the position information and the corresponding variance of the master micro-motion platform and the distance between the master and slave micro-motion platforms can be analyzed. Then, when the underwater acoustic signal packets coming from the autonomous micro motion platform are received again from the micro motion platform at $\tau + 1$, the relevant information of the main micro motion platform can be analyzed from the micro motion platform. According to

the position information obtained from these two times and the dead reckoning system from the micro motion platform, the position estimation information at the time of the micro motion platform can be solved.
Combined with the Kalman filter algorithm[3], it is assumed that the motion state of the micro platform at K time is transformed from the state at K-1 time, and the transformation formula is :

$$x_k = F_k x_{k-1} + B_k u_k + w_k \quad (1.)$$

F_k is a state transformation model acting on state x_{k-1};
B_k is the input control model acting on the controller u_k;
w_k is the process noise, its mean is zero, and the covariance matrix is Q_k, which is a multivariate standard normal distribution:

$$w_k \sim N(0, Q_k) \quad (2.)$$

At k time, The measurement model formula of the measurement variable z_k of state x_k is :

$$z_k = H_k x_k + v_k \quad (3.)$$

Among them, H_k is the observation model, is the transformation function of the state quantity to the observation quantity, v_k is the observation noise. Like the previous w_k, the covariance matrix is R_k, which obeys the multivariate standard normal distribution :

$$v_k \sim N(0, R_k) \quad (4.)$$

The initial state and the noise at each time $\{x_0, w_1, \dots, w_k, v_1 \dots v_k\}$ are considered to be independent of each other.
Combined with the Kalman filtering algorithm, the value of the motion state x of the micro platform with time can be derived. That is to predict the current state optimal estimation $x_{k-1|k-1}$ by the last state optimal estimation $x_{k|k-1}$; and the error $P_{k-1|k-1}$ between the last state value and the measured value is used to predict the error $P_{k|k-1}$ between the current state value and the measured value. The prediction update equation of the motion state is :

$$\hat{x}_{k|k-1} = F_k \hat{x}_{k-1|k-1} + B_k u_k \quad \text{(Predictive state estimation)} \quad (5.)$$

$$P_{k|k-1} = F_k P_{k-1|k-1} F_k^T + Q_k \quad \text{(Prediction estimation covariance matrix)} \quad (6.)$$

In the predictive state estimation formula, $\hat{x}_{k-1|k-1}$ denotes the estimator $\hat{x}_{k-1}$ obtained by comparing the last state value x_{k-1} with z_{k-1}. The same $\hat{x}_{k|k-1}$ denotes the predicted value of the current estimator $\hat{x}_k$, which is derived from the previous estimator $\hat{x}_{k-1}$.
By subtracting the state prediction equation and the system state equation :

$$x_k - \hat{x}_{k|k-1} = F_k \left(x_{k-1} - \hat{x}_{k-1|k-1}\right) + w_k \quad (7.)$$

$$P_{k|k-1} = E[(F_k(x_{k-1} - \hat{x}_{k-1|k-1}) + w_k) \times (F_k(x_{k-1} - \hat{x}_{k-1|k-1}) + w_k)^T] \quad (8.)$$

Since the state estimation error is not correlated with the system noise, it can be converted to :

$$\mathrm{E}\left[\left(x_{k-1} - \hat{x}_{k-1|k-1}\right) {w_k}^T\right] = \mathrm{E}\left[{w_k}^T \left(x_{k-1} - \hat{x}_{k-1|k-1}\right)^T\right] = 0 \quad (9.)$$

$$\Rightarrow P_{k|k-1} = F_k P_{k-1|k-1} F_k^T + Q_k \quad (10.)$$

The equation can also be updated to :

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k (z_k - H_k \hat{x}_{k|k-1}) \quad (11.)$$

$$P_{k|k} = P_{k|k-1} - K_k H_k P_{k|k-1} \quad (12.)$$

K_k is a Kalman gain, , which determines that the predicted value should be transformed into the updated value to meet :

$$K_k = P_{k|k-1} H_k^T (H_t P_{k|k-1} H_k^T + R_k)^{-1} \quad (13.)$$

Thus, the motion measurement model of micro-platform can be established as :

$$\begin{cases} X_{k+1} = f\,(X_k, u_k,\ w_k) \\ Z_{k+1} = g\,(X_{k+1}^S, X_{k+1}^M, X_k^M, \mathrm{D}X_{k+1}^M, \upsilon) \end{cases} \quad (14.)$$

III. ERROR CORRECTION OF NAVIGATION POSITIONING SYSTEM BASED ON MULTIPLE LINEAR REGRESSION EQUATION

System error refers to the error formed by a certain or certain factor changing according to a certain law under certain experimental conditions, and its influence on the determination results often conforms to a certain law. For the long-distance navigation and positioning of micro-platform, it is difficult to form a closed-loop motion. In order to find the corresponding relationship between navigation errors, the closed-loop path of short-range eight-character navigation (shown in Figure 1 and 2) is proposed to replace the cooperative working state of underwater formation in long distance, so as to carry out error correction analysis.

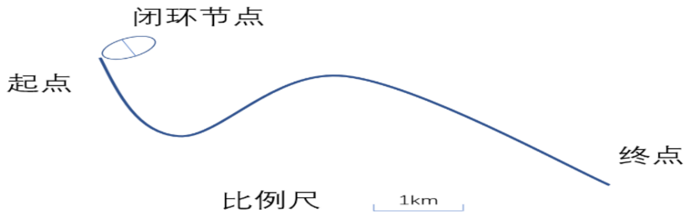


Fig 1 Closed-loop node setting diagram

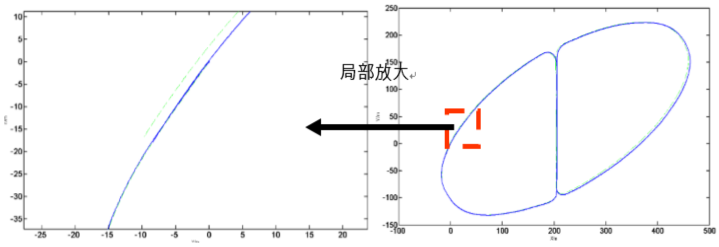
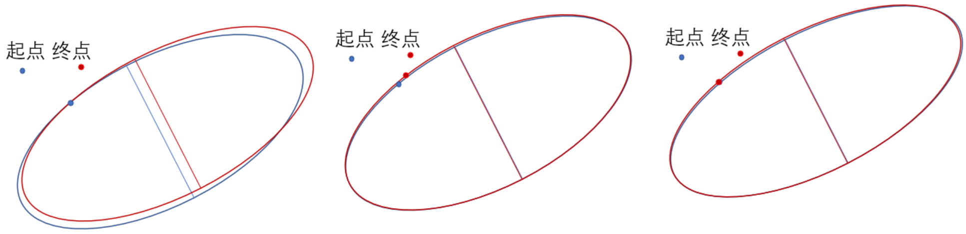


Fig. 2 Comparison method of driving trajectory at nodes

There are three cases of micro-platform running through an 8-shaped closed-loop map as shown in Figure 3 :



a) 起点终点重合、路径不重合　b)起点终点不重合、路径基本重合　c)起点不重合、路径高度重合

Fig 3. Closed-loop correction schematic of nodes

In view of the above situation, the feature extraction and matching are carried out at the starting and ending points of the closed-loop route, and the multivariate linear regression equation between the system error of the micro-platform and multiple influencing factors is established by combining with the environmental characteristics around the underwater micro-platform and the parameters of the dead reckoning information. The dead reckoning position of the micro-platform is corrected by combining the method of parameter matching and optimization, so that the calculated position value coincides with the actual position value of each point in the driving area as much as possible (Figure c), so as to complete the correction of navigation and positioning error.

The system error is expressed as follows :

$$\mu(y) = \beta_0 + \beta_1 x_1 + \ldots \beta_p x_p \quad (15.)$$

By observing group N data, it can be written as a matrix：

$$Y = X\beta + \varepsilon \quad (16.)$$

Y is the observation error，X is the observation quantity,β is the observation matrix, ,ω is the observation error. Because the cause of system error is often some deterministic variables, not random variables, so X is a full rank matrix, assuming the regression equation satisfies G-M conditions, observation error obeys normal distribution, then the multivariate linear regression equation[4] can be written as the following matrix form :

$$\begin{cases} Y = X\beta + \varepsilon \\ \varepsilon \sim N(0, \sigma_2 I_n) \end{cases} \quad (17.)$$

According to the least squares estimation and maximum likelihood estimation method, the factor parameters affecting the system error are obtained, and the regression equation is brought back into the driving process of the micro platform before the node for positioning correction comparison, so as to find the source and size of the system error, and complete the compensation and correction of the system error [5].

IV. RANDOM ERROR CORRECTION OF NAVIGATION AND POSITIONING BASED ON BAYESIAN FILTERING

Random error often comes from some accidental and unexpected in the process of measurement, which has no regularity and can not be predicted, but multiple measurements have statistical regularity. The random error sample function of the state space propagation of the micro-platform can be obtained by navigating the closed map the 8 - word path, which can approximately represent the posterior probability density of the positioning error of the micro platform. Combined with the particle filter based on Bayesian filtering[6] , the mapping relationship between small distance and large distance is established, and then the positioning error of the large-scale micro platform is corrected.

In this paper, the research object isτ_k. According to the above content,τ_k can be obtained byτ_{k-1} and inputU_k, and the process equation of the system can be established accordingly. At the same time, it represents the positioning error of the inertial navigation of the system, so the observation is the error of environmental information e_k, where $e_k$is the error between the real-time environmental detection and the predicted value. Based on this, the observation equation of the system can be established as follows :

Process equation :

$$\tau_k = g(\tau_{k-1}, U_k) \quad (18.)$$

Observation equation :

$$e_k = h(\tau_k) \quad (19.)$$

According to Bayesian and Markov hypothesis, the navigation and positioning state variable τ_k of underwater micro platform is estimated recursively :

Let $\tau_{0:k}$ represent the state variable series $\tau_{0:k} = (\tau_i i = 0, \cdots, k)$,$\tau_{1:k}$ represent the observation variables, and the Bayesian formula shows that :

$$p(\tau_{o:k} \mid \tau_{1:k}) = \frac{p(e_{1:k} \mid \tau_{0:k}) p(e_{0:k})}{p(\tau_{1:k} \mid \tau_{0:k}) d(\tau_{0:k})} \quad (20.)$$

$P(\tau_{0:k})$is the prior probability density,$P(\tau_{1:k}|e_{0:k})$ is the likelihood probability density when the observation is $e_{1:k}$, and$P(\tau_{0:k}|e_{1:k})$ is the posterior probability density. Filtering problem is to deal with the posterior probability density$P(\tau_k|e_{1:k})$, that is, the edge density of$P(e_{0:k}|\tau_{1:k})$:

$$p(\tau_k \mid e_{1:k}) = \iint \ldots \int p(\tau_{0:k} \mid e_{1:k}) d\tau 0 d\tau_1 \ldots d\tau_{k-1} \quad (21.)$$

For the above formula, whenever a new observation data comes, the posterior probability density $P(\tau_k|e_{1:k})$ will be recalculated once, which is very inconvenient. Therefore, the posterior probability density is obtained by the following recursive updating method :

Prediction : Starting from the posterior probability density $P(\tau_{k-1}|e_{1:k-1})$ obtained at $k-1$ time, the system model is used to predict the probability density of k time τ_k, and the prior probability density$P(\tau_k|e_{1:k-1})$ of k time τ_k is obtained.

Update : When the observed value e_k at k arrives, it is used to correct the above prior probability density to obtain the posterior probability density $P(\tau_k|e_{1:k})$ at k ;

Assuming that $k-1$time and$P(\tau_{k-1}|e_{1:k-1})$ is known, the system state τ_k follows the first-order Markov process and the system observation e_k is independent.

First of all, the prior probability density $P(\tau_k|e_{1:k-1})$of $K-1$time system state without Ktime observations is obtained by the prediction step .

$$p(\tau_k \mid e_{1:k-1}) = \int p(\tau_k \mid \tau_{k-1}) p(\tau_{k-1} \mid e_{1:k-1}) d\tau_{k-1} \quad (22.)$$

Where$P(\tau_k|\tau_{k-1})$ is the transition probability density of the system state.

Then, the posterior probability density$P(\tau_k|e_{1:k})$ of system state at $K-1$time points is obtained by updating step $P(\tau_k|e_{1:k-1})$ with the observation value e_k atKtime points:

$$p(\tau_k \mid e_{1:k}) = \frac{p(e_{1:k} \mid \tau_k) p(\tau_k)}{p(e_{1:k})} = \frac{p(\tau_k, \tau_{1:k-1} \mid \tau_k) p(\tau_k)}{p(e_k, e_{1:k-1})} \quad (23.)$$

Defined by conditional probabilities :

$$p(e_k, e_{1:k-1}) = p(e_k \mid e_{1:k-1}) p(e_{1:k-1}) \quad (24.)$$

From the joint probability formula：

$$p(e_k, e_{1:k-1} \mid \tau_k) = p(e_k \mid e_{1:k-1}, \tau_k) p(e_{1:k-1} \mid \tau_k) \quad (25.)$$

From Bayesian formula：

$$p(e_{1:k-1} \mid \tau_k) = \frac{p(\tau_k \mid e_{1:k-1})\, p(e_{1:k-1})}{p(\tau_k)} \quad (26.)$$

$$p(\tau_k \mid e_{1:k}) = \frac{p(e_k \mid e_{1:k-1}, \tau_k)\, p(\tau_k \mid e_{1:k-1})\, p(e_{1:k-1})}{p(e_k \mid e_{1:k-1})\, p(e_{1:k-1})} \quad (27.)$$

Assuming that each system observation e_k is independent of each other, we can get:

$$p(e_k \mid e_{1:k-1}, \tau_k) = p(e_k \mid \tau_k) \quad (28.)$$

$$p(\tau_k \mid e_{1:k}) = \frac{p(e_k \mid \tau_k)\, p(\tau_k \mid e_{1:k-1})}{p(e_k \mid e_{1:k-1})} \quad (29.)$$

Record them as：

$$bel(\tau_k) = p(\tau_k \mid e_{1:k}) \quad (30.)$$

$$\overline{bel(\tau_k)} = p(\tau_k \mid e_{1:k-1}) \quad (31.)$$

However, $\int P(e_k|e_{1:k-1}) = \int P\,(e_k|\tau_k)P(\tau_k|e_{1:k-1})d_{\tau_k}$ is usually a normalized constant, so :

$$bel(\tau_k) = \eta\, p(e_k \mid \tau_k)\overline{bel(\tau_k)} \quad (32.)$$

The Bayesian particle filter algorithm can approximate the posterior probability density of navigation and positioning in the whole motion process by weighted sum of random samples, which is expressed as follows :

$$p(\tau_{0:k} \mid e_{1:k}) \approx \sum_{i=1}^{N} w_k^i \delta(\tau_{0:k} - \tau_{0:k}^i) \quad (33.)$$

Among them, $\{\tau_k^i\}$ is a series of samples obtained by Monte Carlo random sampling at k moments, also known as ' particles ',$\{\omega_k^i\}$ is the normalized weight of particles, N is the number of particles, δ(·) is the Dirac-Delta function. Therefore, the posterior probability density $P(\tau_{0:k}|e_{1:k})$ of the system can be approximately described by using the particle set and its weight value$\{\tau_k^i, \omega_k^i\}_{i=1}^{N}$, as shown in Fig. 4.

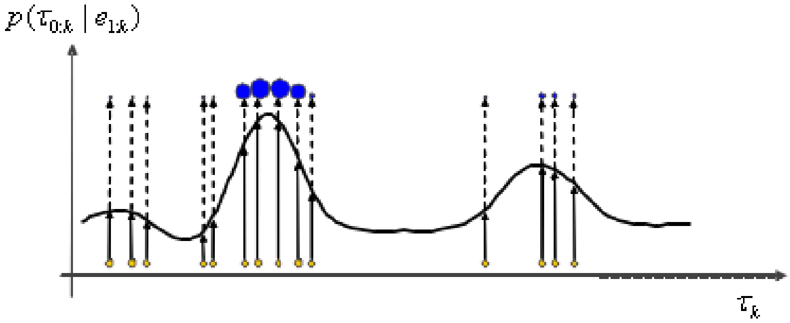


Fig. 4 posterior probability density distribution

Normalized constant $P(e_k|e_{1:k-1})$ is generally unknown, so the particle set cannot be directly sampled from posterior probability density $P(\tau_{0:k}|e_{1:k})$. The importance density function $q(\tau_{0:k}|e_{1:k})$, which is easier to sample, is usually used to sample through importance sampling. The importance density $q(\tau_{0:k}|e_{1:k})$, can be expressed as :

$$q(\tau_{0:k} \mid e_{1:k}) \approx \sum_{i=1}^{N} \delta(\tau_{0:k} - \tau_{0:k}^i) \quad (34.)$$

So the particle weight satisfies :

$$w_k^i \propto \frac{p(\tau_{0:k}^i \mid e_{1:k})}{q(\tau_{0:k}^i \mid e_{1:k})} \quad (35.)$$

If the importance density can be decomposed into :

$$q(\tau_{0:k} \mid e_{1:k}) = q(\tau_k \mid \tau_{0:k-1}, e_{1:k}) q(\tau_{0:k-1} \mid e_{1:k-1}) \quad (36.)$$

According to the recursive Bayesian estimation method, the updating formula of weight $\{\omega_k^i\}$ can be derived as follows :

$$w_k^i \propto \frac{p(e_k \mid \tau_k^i) p(\tau_k^i \mid \tau_{k-1}^i) p(\tau_{0:k-1}^i \mid e_{1:k-1})}{q(\tau_k^i \mid \tau_{0:k-1}^i, e_{1:k}) q(\tau_{0:k-1}^i \mid e_{1:k-1})} = w_{k-1}^i \frac{p(e_k \mid \tau_k^i) p(\tau_k^i \mid \tau_{k-1}^i)}{q(\tau_k^i \mid \tau_{0:k-1}^i, e_{1:k})} \quad (37.)$$

If $q(\tau_k|\tau_{0:k-1}, e_{1:k}) = q(\tau_k|\tau_{k-1}, e_k)$,the importance density function only depends onτ_{k-1} ande_k, and it is unnecessary to store the historical values of particle sets and observations. The weight updating formula of the above equation can be simplified as :

$$w_k^i \propto w_{k-1}^i \frac{p(e_k \mid \tau_k^i) p(\tau_k^i \mid \tau_{k-1}^i)}{q(\tau_k^i \mid \tau_{k-1}^i, e_k)} \quad (38.)$$

By recursively calculating the weight of the particle set obtained by sequential sampling[7] of the importance density function according to the above formula, the posterior probability density of the state can be recursively estimated, so as to obtain the statistical information of the state expectation :

$$q(\tau_k^i \mid \tau_{k-1}^i, e_k) = p(\tau_k^i \mid \tau_{k-1}^i) \quad (39.)$$

$$w_k^i \propto w_{k-1}^i p(e_k \mid \tau_k^i) \quad (40.)$$

After weight normalization :

$$\tilde{w}_k^i = w_k^i / \sum_{i=1}^{N} w_k^i \quad (41.)$$

The posterior probability density can be expressed as :

$$p(\tau_k \mid e_{1:k}) \approx \sum_{i=1}^{N} \tilde{w}_k^i \delta(\tau_k - \tau_k^i) \quad (42.)$$

When the particle number is $N \to \infty$, the large number theorem can ensure that the above equation can approximate the real posterior probability$P(\tau_k|e_{1:k})$, so as to complete the random error estimation in the navigation and positioning of micro platform.

V. CONCLUSIONS

In this paper, a method for systematic error estimation and correction in underwater cooperative formation navigation and positioning is proposed. The motion model of underwater micro-platform is established by using various sensor information carried by the micro-platform itself and combining with Kalman filter algorithm. At the same time, the error relationship of navigation and positioning is characterized by the error of the closed-loop path of the short-distance eight-character navigation before the formation of the micro-platform is coordinated. Based on multivariate linear regression equation and Bayesian particle filter method, a rapid calibration and correction method for systematic error and random error in cooperative positioning is proposed, which can improve the accuracy of underwater cooperative formation navigation.

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