American Journal of Engineering Research (AJER)
e-ISSN: 2320-0847 p-ISSN : 2320-0936
Volume-11, Issue-05, pp-65-82
www.ajer.org
Review Paper Open Access

Development of an Improved Contingency Analysis Model for Power Transmission Expansion System (Camtes) Part-1

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ABSTRACT: *The issue of deregulation of an electric power industry in no doubt will increase the present generation capacity upward by involving electrical utilities and independent power producers (IPPS) which will in turn increase the congestion on the already congested national grid. This paper presents an investigation into five bus system, which also applied to the Nigerian power transmission system with a view to expand the existing network as a solution to the congestion scenario are distributed throughout the network and bus voltages change. The new steady – state bus voltages and line currents can be predicted using the contingency analysis. This models will enables system planners and operators who must undertake hundred of studied in a short time period which are more concerned with knowing if "overloaded level" of currents and out of limit voltages exist than with finding the exact values of those quantities. Often resistance is considered neglible and the network model becomes purely reactive. Line charging and off nominal tap charging of transformers are also frequently omitted. Contingency model use Z_{bus} and Y_{bus} which become attractive from the computational new point-especially if loads can be treated as constant currents injections into the various buses of the system.*
KEYWORDS: *Contingency Analysis Model, Transmission Expansion, Triangular Decomposition, Power System, Deregulation, Impedance Matrix and Admittance Matrix, five bus system.*

Date of Submission: 04-05-2022 Date of acceptance: 18-05-2022

I. INTRODUCTION

Removing a line from service can be simulated in the system model by adding the negative of the series impedance of the line between its two end buses. When considering line additions or removal from an existing system it is not always necessary to build a new Z_{bus} or to calculate new triangular factor of Y_{bus} matrix especially if the interest is to establish the impact of the changes on the existing bus voltages and line flows. An alternative procedure is considered to injection of compensating currents into the existing system to account for the effect of the line changes [1].

The formulation of this technique is `to examine the steady – state effects of adding lines to an existing system using the developed mathematical formulation. The concept of injecting compensating currents are introduced in order to allow the existing system Z_{bus} material formation. [2].

II. MATHEMATICAL FORMULATION OF THE PROBLEM

Calculation of Z_{bus} Elements from Y_{bus}

When the full numerical form of Z_{bus} is not explicitly required for an application, then it can readily calculate the elements of Z_{bus} as needed parameter if the upper and lower triangular factors of Y_{bus} are available to analyse how this can be done while considering the post multiplying operation of Z_{bus} by a vector with only one nonzero elements Im = 1 in row m and all other elements equal to zero. When Z_{bus} is a N X N matrix.

III. MODEL FORMULATION FOR CALCULATION OF Z_{bus} ELEMENTS FROM Y_{Bus}, IN A FIVE (5) Bus SYSTEM.

When the final numerical elements of Z_{bus} is not explicitly required from an application then the calculated elements of Z_{bus} are needed for the upper and lower triangular factors of Y_{bus} which are available. To analyse how this can be done consider the post multiplying Z_{bus} by a vectorwith only one nonzero element L=1in row m and all other elements equal to zero, Where Z_{bus} is a N/Nmatrix this can be presented as:

$$\begin{matrix} 1 \\ 2 \\ \vdots \\ m \\ \vdots \\ N \end{matrix} \underbrace{\begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1m} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} & \cdots & Z_{2m} & \cdots & Z_{2N} \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ Z_{ml} & Z_{m2} & \cdots & Z_{mm} & \cdots & Z_{mV} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ Z_{NI} & Z_{N2} & \cdots & Z_{Nm} & \cdots & Z_{NN} \end{bmatrix}}_{Z_m} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1_m \\ \vdots \\ 0 \end{bmatrix} = \begin{matrix} 1 \\ 2 \\ \\ m \\ \\ N \end{matrix} \underbrace{\begin{bmatrix} Z_{1m} \\ Z_{2m} \\ \vdots \\ Z_{mm} \\ \vdots \\ Z_{Nm} \end{bmatrix}}_{Z_{bus}^{(m)}} \quad (1)$$

Thus in post multiplying Z_{bus} by the vector shown in the mth column matrix operation which are called the vector $Z_{bus}^{(m)}$ that is;

$$Z_{bus}^{(m)} \underset{=}{\Delta} \begin{bmatrix} column \\ of \\ Z_{bus} \end{bmatrix} = \begin{matrix} 1 \\ 2 \\ \\ m \\ \\ N \end{matrix} \begin{bmatrix} Z_{1m} \\ Z_{2m} \\ \vdots \\ Z_{mm} \\ \vdots \\ Z_{Nm} \end{bmatrix} \quad (2)$$

Since the product of Y_{bus} and Z_{bus} are equals to unit matrix then; it becomes

$$Y_{bus} Z_{bus} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1_m \\ \vdots \\ 0 \end{bmatrix} = Y_{bus} Z_{bus}^{(m)} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1_m \\ \vdots \\ 0 \end{bmatrix} \quad (3)$$

Similarly, if the lower-triangular matrix ‘L’ and the upper-triangular matrix ‘U’ of Y_{bus} are available, then equation – 4 can be written as;

$$LUZ_{bus}^{(m)} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1_m \\ \vdots \\ 0 \end{bmatrix} \quad (4)$$

It is important to note that the elements in the column vector $Z_{bus}^{(m)}$ can be found in equation (4) by forward elimination and back substitution operation, if only some of the elements of $Z_{bus}^{(m)}$ are required then the calculations can then be reduced accordingly. Suppose it is required to generate Z_{33} and Z_{43} of Z_{bus} for a four-bus system, while using convenient notation for the elements of L and U given as;

$$\begin{bmatrix} l_{11} & . & . & . \\ l_{21} & l_{22} & . & . \\ l_{31} & l_{32} & l_{33} & . \\ l_{41} & l_{42} & l_{43} & l_{44} \end{bmatrix} \begin{bmatrix} 1 & U_{12} & U_{13} & U_{14} \\ . & 1 & U_{23} & U_{24} \\ . & . & 1 & U_{34} \\ . & . & . & 1 \end{bmatrix} \underbrace{\begin{bmatrix} Z_{13} \\ Z_{23} \\ Z_{33} \\ Z_{43} \end{bmatrix}}_{Z_{bus}^{(3)}} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad (5)$$

Evidently, it can be solve using this equation for $Z_{bus}^{(m)}$ in two steps as follows:

$$\left.\begin{aligned} &\begin{bmatrix} l_{11} & . & . & . \\ l_{21} & l_{22} & . & . \\ l_{31} & l_{32} & l_{33} & . \\ l_{41} & l_{42} & l_{43} & l_{44} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \\ &\text{or} \\ &\text{Where; } \begin{bmatrix} 1 & u_{12} & u_{13} & u_{14} \\ . & 1 & u_{23} & u_{24} \\ . & . & 1 & u_{34} \\ . & . & . & 1 \end{bmatrix} \underbrace{\begin{bmatrix} Z_{13} \\ Z_{23} \\ Z_{33} \\ Z_{43} \end{bmatrix}}_{Z_{bus}^{(3)}} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \end{aligned}\right\} \quad (6)$$

By forward substitution equation (6) yields as;

$$x_1 = 0,\ x_2 = 0,\ x_3 = \frac{1}{l_{33}} \quad x_4 = \frac{-l_{43}}{l_{44} l_{33}} \quad (7)$$

By back substitution of this results into equation (7) in order to find the required elements of column 3 of Z_{bus}

$$\left.\begin{aligned} Z_{43} &= x_4 \\ Z_{33} &= x_1 - u_{34} Z_{43} \end{aligned}\right\} \quad (8)$$

That is, if all elements of $Z_{bus}^{(m)}$ are required, then the operations can be continued following the calculation process as;

$$\left.\begin{aligned} Z_{23} &= x_2 - u_{23} Z_{33} - u_{24} Z_{43} \\ Z_{13} &= x_1 - u_{12} Z_{23} - u_{13} Z_{33} - u_{14} Z_{43} \end{aligned}\right\} \quad (9)$$

The computational effort in generating the required elements can be reduced by judiciously choosing the numbers.

However it is necessary to evaluate the terms ($Z_{im} - Z_{in}$) involving difference between column (m) and (n) of Z_{bus}. If the elements of Z_{bus} are not available explicitly, then it is possible to calculate the required differences by solving system of equations given as;

$$LUZ_{bus}^{(m-n)} = \begin{bmatrix} 0 \\ \vdots \\ 1_m \\ \vdots \\ -1_n \\ \vdots \\ 0 \end{bmatrix} \quad (10)$$

Where; $Z_{bus}^{(m-n)} = Z_{bus}^{(m)} - Z_{bus}^{(n)}$ is the vector formed by subtracting column (n) from column (m) of Z_{bus} and I_m and1_m = 1 in row m and -1_n = -1 in row n of the vector column.

IV. ADDING AND REMOVING MULTIPLE LINES CONSIDERATION

Considering line additions or removals from an existing system it is not always necessary to build a new Z_{bus} to calculate new triangular factors for instance if the only interest is to establish the impact of the changes on the existing bus voltages and flows then an alternative procedure is to consider the injection of compensating currents into the existing system to account for the effect of the line changes. To express this mathematically the basic concepts can be considered by adding to the lines of impedances Z_a and Z_b to an existing system with known Z_{bus}. This means that we can consider three or more lines addition. [3].

Suppose the impedance Z_a and Z_b are to be added between buses m-n and p-q respectively,as referred to figure 1 then assume that the bus voltages V_1, V_2 …, V_N are produced in the original system (without Z_a and Z_b) by the current injections $I_1, I_2 \ldots I_n$ which are known and that this injection are fixed value and therefore are unaffected by the addition of Z_a and Z_b on a phase basis that the bus impedance equations for the original system are then given as:

$$V = \begin{bmatrix} V_1 \\ \vdots \\ V_m \\ V_n \\ V_p \\ V_q \\ \vdots \\ V_N \end{bmatrix} \begin{matrix} 1 \\ \\ m \\ n \\ p \\ q \\ \vdots \\ N \end{matrix} \begin{bmatrix} Z_{11} & \cdots & Z_{lm} & Z_{ln} & Z_{lq} & \cdots & Z_{lm} \\ \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ Z_{ml} & \cdots & Z_{mm} & Z_{mn} & Z_{mq} & \cdots & Z_{mN} \\ Z_{nl} & \cdots & Z_{nm} & Z_{nn} & Z_{nq} & \cdots & Z_{nN} \\ Z_{pl} & \cdots & Z_{pm} & Z_{pn} & Z_{pq} & \cdots & Z_{qN} \\ V_{ql} & \cdots & Z_{qm} & Z_{qn} & Z_{qq} & \cdots & Z_{qq} \\ \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ Z_{Nl} & \cdots & Z_{Nm} & Z_{Nn} & Z_{Nq} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} I_i \\ \vdots \\ I_m \\ I_n \\ I_p \\ I_p \\ \vdots \\ I_n \end{bmatrix} \quad (11)$$

Figure 1(a): System Independence matrix

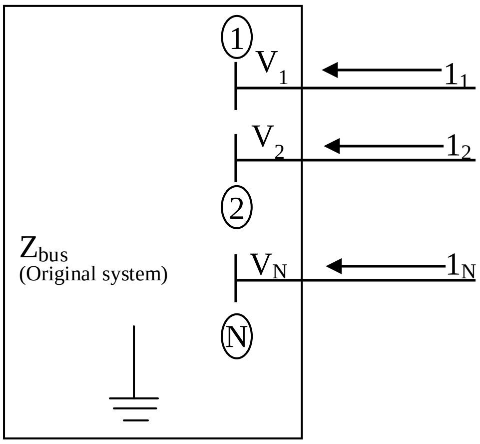

Figure 1(b): System voltages due to current injecton

Figure 1: System with voltages V_1, V_2…, V_N due to the current injections I_1, I_2,…I_N

Now in order to determine the changes in the bus voltage due to adding the two new line impedances. Let V = $(V_1, V_2, \ldots, V_N)^T$ denoted as the vector of bus voltages which apply after Z_a and Z_b are been added. The voltage change at a typical bus (K) is then given as:

$$\Delta V_k = V_k^1 - V_k \quad (12)$$

The currents I_a and I_b in the added branch impedance Z_a and Z_b are related to the new bus voltages by the equations given as;

$$Z_a I_a = V_m^1 - V_n^1 \text{, and} \qquad Z_b I_b = V_p^1 - V_q^1 \quad (13)$$

Figure 2(a): Shows the branch currents flowing into bus (m) to bus (n) from bus (p) to bus (q).This can be rewritten as equation 13 in the vector-matrix form given as:

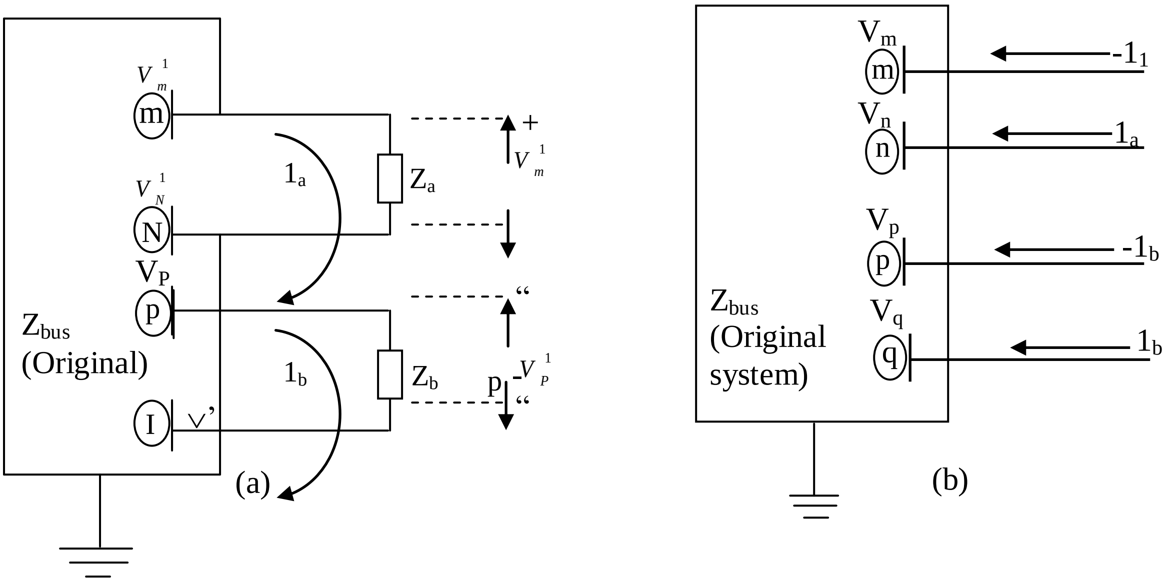

Figure 2: Shows original system with voltages V^1_i changed to V^1_i by adding in figure 2(a) impedance Z_a and Z_b while figure 2(b) equivalent compensating current injections.

From Vector Matrix Form given as:

$$\begin{bmatrix} Z & 0 \\ 0 & Z_b \end{bmatrix}\begin{bmatrix} I_a \\ I_b \end{bmatrix} = \begin{matrix} a \\ b \end{matrix}\begin{bmatrix} 0 & \cdots & 1 & -1 & 0 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & 1 & -1 & \cdots & 0 \end{bmatrix}\begin{bmatrix} V_i^1 \\ \vdots \\ V_m^1 \\ V_n^1 \\ V_q^1 \\ \vdots \\ V_N^1 \end{bmatrix} = A_c V^1 \qquad (14)$$

Where A_c is the branch to node incidence matrix, which shows that the incidence of two new branches to the nodes of the system then the new branch currents I_a and I_b have the same effect on the voltages of the original system as two sets of injected currents $-I_a$ at bus (p), I_b at bus (q), as shown in Figure 2 (b). This the equivalent current injections combine with the actual current injections into the original system to produce the bus voltages $V_1, V_2 \ldots, V_N$ the same as if the branch impedances Z_a and Z_b had been actually added to the network. In other words currents 1_a and 1_b arecompensate for not modifying Z_{bus} of the original system to include Z_a and Z_b on this account, they are called compensating currents [5].

It can be express using the compensating currents in vector-matrix form as follows:

$$I_{comp} = \begin{matrix} 1 \\ \\ m \\ n \\ p \\ q \\ \\ N \end{matrix} \begin{bmatrix} 0 \\ \vdots \\ -1_a \\ 1_a \\ -I_b \\ I_b \\ \vdots \\ 0 \end{bmatrix} = \begin{matrix} I \\ \\ m \\ n \\ p \\ q \\ \\ N \end{matrix} \begin{bmatrix} 0 & 0 \\ \vdots & \vdots \\ -I & 0 \\ I & 0 \\ 0 & -I \\ 0 & I \\ \vdots & \vdots \\ 0 & 0 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \end{bmatrix} = A_C^T \begin{bmatrix} I_a \\ I_b \end{bmatrix} \quad (15)$$

The changes in the bus voltages from $V_1, V_2, \ldots, V_N$ to $V^1_1, \ldots, V^1_N$ can be calculated by multiplying the original system Z_{bus} by the vector I_{comp} (of compensating current.) Adding $Z_{bus}\ I_{comp}$ to the vector V of existing bus voltages yields [6]

$$V^1 = V + Z_{bus}\ I_{comp} = V - Z_{bus}\ A_C^T \begin{bmatrix} I_a \\ I_b \end{bmatrix} \quad (16)$$

The equation shows that the changes at the buses of the original system due to the addition of the branch impedances Z_a and Z_b between the buses (m) – (n) and (p) – (q) respectively are given as:

$$\Delta V = V^1 - V = Z_{bus}\ A^T{}_c \begin{bmatrix} I_a \\ I_b \end{bmatrix} \quad (17)$$

Where I_a and I_b are the compensating currents, It is useful to check the dimensions of each term in equation (17) from which the voltage changes $\Delta V = V^1 - V$ can be calculated directly once the values for the currents I_a and I_b are determined. Thus this show how this can be determined. We now show how this determination can be made by; pre-multiplying Eq. (16) by A_c and then substituting for A_cV^1 from Eq. (14), to obtain as:

$$\begin{bmatrix} Z_a & 0 \\ 0 & Z_b \end{bmatrix} \begin{bmatrix} I_a \\ I_b \end{bmatrix} = A_c V - A_c Z_{bus}\ A^T{}_c \begin{bmatrix} I_a \\ I_b \end{bmatrix} \quad (18)$$

Collecting terms which involve I_a and I_b which gives as:

$$\underbrace{\left(\begin{bmatrix} Z_a & 0 \\ 0 & Z_b \end{bmatrix} + A_c Z_{bus} A_C^T \right)}_{Z} \begin{bmatrix} I_a \\ I_b \end{bmatrix} = A_c V = \begin{bmatrix} V_m & V_n \\ V_p & V_q \end{bmatrix} \quad (19)$$

When Z is a loop impedance matrix which can be formed directly from the original bus impedance matrix of the system, then solve equation (19) for I_a and I_b to find that;

$$\begin{bmatrix} Ia \\ Ib \end{bmatrix} = Z - 1AcV = Z_{-1} \begin{bmatrix} V_m - V_n \\ V_p - V_q \end{bmatrix} \quad (20)$$

That is $V_m - V_n$ and $V_p - V_q$ are open-circuit voltage drops between buses (m) – (n) and (p) – (q) in the original network, that is with branch impedances Z_a and Z_b open in Figure 1. These open-circuit voltages are either known or can be easily calculated from Eq. 11. The definition of matrix Z in Eq. 19 includes the term $A_cZ_{bus}A^T_C$ which can be determined as follows;

$$A_c Z_{bus} A_C^T = \begin{matrix} a \\ b \end{matrix} \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{matrix} m \\ n \\ p \\ q \end{matrix} \begin{bmatrix} Z_{mm} & Z_{mn} & Z_{mp} & Z_{mq} \\ Z_{nm} & Z_{nm} & Z_{np} & Z_{nq} \\ Z_{pm} & Z_{pn} & Z_{pn} & Z_{pd} \\ Z_{qm} & Z_{qn} & Z_{qp} & Z_{qq} \end{bmatrix} \begin{matrix} m \\ n \\ p \\ q \end{matrix} \begin{bmatrix} 1 & 0 \\ -1 & -0 \\ 0 & 1 \\ 0 & -1 \end{bmatrix} \quad (21)$$

This equation shown only those elements of A_c and Z_{bus} which substitute to the calculations. Since all other elements of A_c are zeros. It is now necessary to display the full Z_{bus}. The indicated multiplication yield[7, 8, 9].

$$A_c Z_{bus} A^T{}_c = \begin{matrix} a \\ b \end{matrix}\begin{bmatrix} (Z_{mm} - Z_{mn}) - (Z_{nm} - Z_{nn}) & (Z_{m} - Z_{mq}) - (Z_{np} - Z_{nq}) \\ (Z_{pm} - Z_{pn}) - (Z_{qm} - Z_{qn}) & (Z_{pp} - Z_{pq}) - (Z_{qp} - Z_{qq}) \end{bmatrix} \qquad (22)$$

The diagonal element in the equation can be recognized from fig. 1 and 2 as the thevenin impedance Z_{thmn} and Z_{thpq} when looking into the original system between buses (m) – (n) and (p) – (q) respectively. This is,

$Z_{thmn} = Z_{mn} + Z_{nn} - Z_{mn} - Z_{nm}$

$Z_{thpq} = Z_{pp} + Zqq - Z_{pq} - Z_{qp}$ 22(a) 22(a)

Substituting from Equation 22 into Equation 19 we obtain as:

$$\begin{bmatrix} \begin{matrix} a \\ b \end{matrix} \underbrace{\begin{matrix} (Z_{mm} - Z_{mn}) - (Z_{nm} - Z_{nn}) & (Z_{m} - Z_{mq}) - (Z_{np} - Z_{nq}) \\ (Z_{pm} - Z_{pn}) - (Z_{qm} - Z_{qn}) & (Z_{pp} - Z_{pq}) - (Z_{qp} - Z_{qq}) \end{matrix}}_{Z} \end{bmatrix}$$

or

$$X \begin{bmatrix} I_a \\ I_b \end{bmatrix} = \begin{bmatrix} V_m - V_n \\ V_p - V_q \end{bmatrix} \qquad (23)$$

Which shows that the compensating currents I_a and I_b can be calculated by using the known bus voltages V_m, V_n, V_p, and V_q of the original network and the elements of its Z_{bus} shown in Eq. 23. Thus, Eq. 23 and 17, constitute a two-step procedure for the closed-form solution of the voltages changes at the buses of the original system due to simultaneous addition of branch impedance Z_a and Z_b under the assumption of constant externally injected currents into the original system, first calculated the compensating currents using Eq. 23 and then substitute for these currents in Eq. 17 to find the new bus voltages which result from adding the new branches. The removal of branch impedances Z_a and Z_b from the original system can be analyzed in a similar manner simply by treating the removals as additions of the negative impedances – Z_a and - Z_b. The elements in the 2 x 2 matrix of Equation 22 can calculated by using the appropriate elements of columns m, n, p, and q of Z_{bus}, or they can be generated from the triangular factors L and U of Y_{bus}[10, 11, 12].

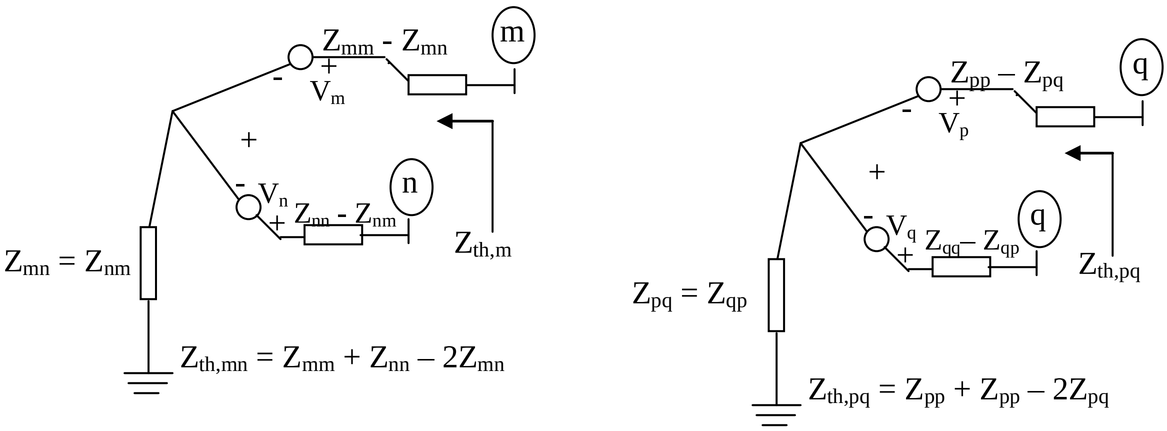

Figure 3(a): Thevenin Equivalent circuit, Zth, m **Figure3(b):** Thevenin equivalent Zth, Pq

Figure 1 and 2 are thevenin equivalent impedances looking into the system between case (a) buses (m) and (n) and case (b) buses (q).

Since the vectors $Z_{bus}^{(m-n)}$ and $Z_{bus}^{(p-q)}$ are the respective solutions of the equations.

$$LUZ_{bus}^{(m-n)} = \begin{bmatrix} 0 \\ \vdots \\ 1_m \\ \vdots \\ -1_n \\ \vdots \\ 0 \end{bmatrix}, \quad LUZ_{bus}^{(p-q)} = \begin{bmatrix} 0 \\ \vdots \\ 1_p \\ \vdots \\ -1_p \\ \vdots \\ 0 \end{bmatrix} \qquad (24)$$

This can be express as:

$(Z_{mn} - Z_{mn}) - (Z_{nm} - Z_{nn})$ = (row m – row n) of $Z_{bus}^{(m-n)}$

$(Z_{pm} - Z_{pn}) - (Z_{np} - Z_{qn})$ = (row p – row q) of $Z_{bus}^{(m-n)}$

$(Z_{mp} - Z_{mq}) - (Z_{np} - Z_{nq})$ = row m – row n) of $Z_{bus}^{(p-q)}$ (25)

$(Z_{pp} - Z_{pq}) - (Z_{qp} - Z_{qq})$ = row p – row q) of $Z_{bus}^{(p-q)}$

It can also follows from Eq. 24 that $Z_{bus} A^T{}_c$ in Eq. 17 is a Nx2 matrix given as:

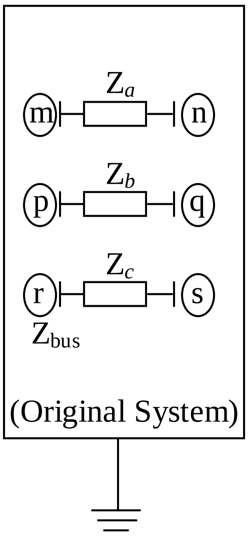

Figure 4(a): Inpedence matrix

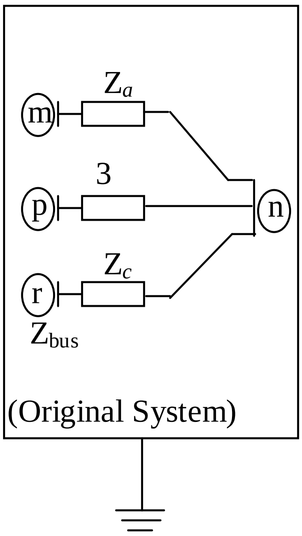

Figure 4(b): Inpendence matrix element

Figures 2(a) and (2b): Addition of three branches Z_a and Z_b and Z_c between (a) distinct bus pairs and (b) no distinct bus pairs in the original system of known Z_{bus}

Which the columns are equal to the vectors $Z_{bus}^{(m-n)}$ and $Z_{bus}^{(p-q)}$ that is,

$$Z_{bus} A^T{}_c = \left[Z_{bus}^{(m-n)} \middle| Z_{bus}^{(p-q)} \right] \qquad (26)$$

Considerately, when more than two lines are to be added to the original system, the matrix Z is formed in the manner described above. For instance, to add three line impedances Z_a, Z_b, and Z_c between the buses (m) – (n), (p) – (q), and (R) - (s), respectively,

$$Z = \begin{bmatrix} Z_a & 0 & 0 \\ 0 & Z_b & 0 \\ 0 & 0 & Z_c \end{bmatrix} \begin{matrix} a \\ b \\ c \end{matrix} \begin{bmatrix} Z_{thmn} & Z_{mp} - Z_{mq} - Z_{np} + Z_{nq} & Z_{mr} - Z_{ms} - Z_{nr} + Z_{ns} \\ Z_{pm} - Z_{pn} - Z_{qm} + Z_{qn} & Z_{thpq} & Z_{pr} - Z_{ps} - Z_{qr} + Z_{qs} \\ Z_{rm} - Z_{rn} - Z_{sm} + Z_{sn} & Z_{rp} - Z_{sp} + Z_{sq} & Z_{thrs} \end{bmatrix} \quad (27)$$

Analysis of the Model, using 5 Bus System as a Case Study for the Calculations of Z_{bus} Elements from Y_{bus}.

When full numerical form of Z_{bus} is not explicitly required for an application, then it can readily calculate elements of Z_{bus} as needed if the upper and lower triangular factors of Y_{bus} are available. Consider post multiplying Z_{bus} by a vector with only one zero elements $1_m = 1$ in row m and all other elements equal to zero. When Z_{bus} is an NxN; we have, $Z_{bus}^{(m)}$ that;

$$\begin{matrix} 1 \\ 2 \\ \\ m \\ \\ N \end{matrix} \underbrace{\begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1m} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} & \cdots & Z_{2m} & \cdots & Z_{2N} \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ Z_{ml} & Z_{m2} & \cdots & Z_{mm} & \cdots & Z_{mV} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ Z_{NI} & Z_{N2} & \cdots & Z_{Nm} & \cdots & Z_{NN} \end{bmatrix}}_{Z_{bus}} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1_m \\ \vdots \\ 0 \end{bmatrix} = \begin{matrix} 1 \\ 2 \\ \\ m \\ \\ N \end{matrix} \underbrace{\begin{bmatrix} Z_{1m} \\ Z_{2m} \\ \vdots \\ Z_{mm} \\ \vdots \\ Z_{Nm} \end{bmatrix}}_{Z_{bus}^{(m)}} \quad (28)$$

Thus, post multiplying Z_{bus} by the vector shown extracts the mth column which we have called the vector as:

$$Z_{bus}^{(m)} \underset{=}{\Delta} \begin{bmatrix} column \\ of \\ Z_{bus} \end{bmatrix} = \begin{matrix} 1 \\ 2 \\ \\ m \\ \\ N \end{matrix} \begin{bmatrix} Z_{1m} \\ Z_{2m} \\ \vdots \\ Z_{mm} \\ \vdots \\ Z_{Nm} \end{bmatrix} \quad (29)$$

Since the product of Y_{bus} and Z_{bus} are equals to the 'units matrix', to obtain as:

$$Y_{bus} Z_{bus} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1_m \\ \vdots \\ 0 \end{bmatrix} = Y_{bus} Z_{bus}^{(m)} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1_m \\ \vdots \\ 0 \end{bmatrix} \quad (30)$$

If the lower-triangular L, and the upper triangular matrix U, of Y_{bus} are available, then it equation 30 given in the form as:

$$LuZ_{bus}^{(m)} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1_m \\ \vdots \\ 0 \end{bmatrix} \qquad (31)$$

It is now apparent that the elements in the column vector $Z_{bus}^{(m)}$ can be found from Equation 30 by forward elimination and back substitution if only some of the elements of Z_{bus} are required accordingly. For example suppose it is required to generate Z_{bus} elements, for five (5) bus system using convenience notation for all elements of L and U to obtain and expressed as:

$$A = Lu = \begin{bmatrix} L_{11} & 0 & 0 & 0 & 0 \\ I_{21} & I_{22} & 0 & 0 & 0 \\ I_{31} & I_{32} & I_{33} & 0 & 0 \\ L_{41} & L_{42} & L_{43} & L_{44} & 0 \\ L_{51} & L_{52} & L_{53} & L_{54} & L_{55} \end{bmatrix}_{\uparrow L} \begin{bmatrix} U_{11} & U_{12} & U_{13} & U_{14} & U_{15} \\ 0 & U_{22} & U_{23} & U_{24} & U_{25} \\ 0 & 0 & U_{33} & U_{34} & U_{35} \\ 0 & 0 & 0 & U_{44} & U_{45} \\ 0 & 0 & 0 & 0 & U_{55} \end{bmatrix}_{\uparrow U} \qquad (32)$$

By matrix operation row-column calculation we have;

$$\begin{bmatrix} L_{11} \times U_{11} + 0 \times 0 + 0 \times 0 + 0 \times 0 + o \times o, \; L_{11} \times U_{12} + 0 \times U_{22} \\ + 0 \times 0 + 0 \times 0 + 0 \times 0, \; L_{11} \times U_{13} + o \times o_{23} + \\ 0 \times U_{23} + 0 \times 0 + 0 \times 0, \; L_{11} \times U_{1} + 0 \times 0_{24} + 0 \times 0_{34} + \\ 0 \times U_{44} + 0 \times 0, \; L_{11} \times U_{15} + 0 \times U_{23} + 0 \times U_{25} \\ 0 \times U_{45} \quad 0 \times U_{55} \end{bmatrix}$$

$$\Downarrow$$

$$[L_{11}u_{11}; \quad L_{11}u_{12}; \quad L_{11}u_{13}; \quad L_{11}u_{14}; \quad L_{11}u_{15}] \qquad (33)$$

Similarly, the process can continue the same operations as;

$$\begin{bmatrix} L_{21}U_{11} + L_{22} \times 0 + L_{22} \times 0 + 0 \times 0 + 0 \times 0, + 0 \times 0; \\ L_{21} + U_{12} \quad L_{22} \times, U_{22} + 0 \times U_{33} + 0 \times 0 + 0 \times 0; \\ L_{21} \times U_{13} + L_{22} \times U_{23} + 0 \times 0; \quad 0 \times 0; \\ L_{21} \times U_{14} + L_{22} \times U_{24} + 0 \times U_{34} + 0 \times 0; \\ L_{21}U_{15} + L_{22}U_{25} + 0 \times U_{35} + 0 \times U_{45} + 0 \times U_{55} \end{bmatrix} \qquad (34)$$

$$\Downarrow$$

$[l_{21}U_{11}; l_{21}U_{12} + U_{22}; l_{21}U_{13} + l_{22}U_{23}; l_{21}U_{14} + l_{22}U_{24}; l_{21}U_{15} + l_{22}U_{25}]$

That is, continue the same operation as beforein order to obtain as:

$$A = Lu = \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} \begin{bmatrix} L_{11}u_{11}; \; L_{11}u_{12} \; L_{11}u_{13} \; L_{11}u_{14}; \; L_{11}u_{15} \\ L_{21}u_{11}; \; L_{21}u_{12}; \; L_{22}u_{22}; \; L_{21}u_{13} + L_{22}u_{23}; L_{21}u_{14} + L_{22}u_{24}; L_{21}u_{15}; L_{22}u_{25} \\ L_{31}u_{11}; L_{31}u_{12} + L_{32}u_{22}; L_{31}u_{13} + L_{32}u_{23}; L_{31}u_{14} + L_{33}u_{34}; L_{31}u_{15} + L_{32}u_{25} + L_{33}u_{45} \\ L_{41}u_{11}; L_{41}u_{12} + L_{41}u_{13} + L_{42}u_{23}; L_{41}u_{14} + L_{42}u_{24} + L_{43}u_{34} + L_{44}u_{44}; L_{41}u_{15} + L_{42}u_{25} + L_{44}u_{45} \\ L_{51}u_{11}; L_{51}u_{12}; L_{52}u_{22;}; L_{51}u_{13}; L_{52}u_{23} + L_{53}u_{33}; L_{51}u_{14} + L_{52}u_{24} + L_{53}u_{34}; L_{54}u_{44}; L_{51}u_{15} + L_{52}u_{25} + L_{53}u_{35} + L_{54}u_{45} + L_{55}u_{55} \end{bmatrix} (35)$$

For convenient lets request $u_{11} = u_{22} = u_{33} = u_{44} = u_{55} = 1$ into equation 32 to obtain as:

$$A = Lu = \begin{bmatrix} L_{11}u_{11}; & L_{11}u_{12} & L_{11}u_{13} & L_{11}u_{14}; & L_{11}u_{15} \\ L_{21}u_{22}; & L_{21}u_{12}+L_{22}; & L_{21}u_{13}+L_{22}u_{23}; & L_{21}u_{14}+L_{22}u_{24}; & L_{21}u_{15}; L_{22}u_{25} \\ L_{31}; & L_{31}u_{12}+L_{32}; & L_{31}u_{13}+L_{32}u_{23}+L_{33}; & +L_{31}u_{14}+L_{32}u_{24}+L_{33}u_{34}; & L_{31}u_{15}+L_{32}u_{25}+L_{33}u_{35} \\ L_{41}; & L_{41}u_{12}+L_{42}; & L_{41}u_{13}+L_{42}u_{23}; & +L_{41}u_{14}+L_{42}u_{24}+L_{43}u_{34}+L_{44}; & L_{41}u_{15}+L_{42}u_{25}+L_{43}u_{35}+L_{44}u_{45} \\ L_{51}; & L_{51}u_{12}+L_{52}u_{13}; & L_{52}u_{23}+L_{53}; & L_{51}u_{14}+L_{52}u_{24}+L_{53}u_{34}+L_{54}; & L_{51}u_{15}+L_{52}u_{25}+L_{53}u_{35}+L_{54}u_{45}+L_{55} \end{bmatrix} \quad (36)$$

Consider a five bus-system in fig. 5 below as:

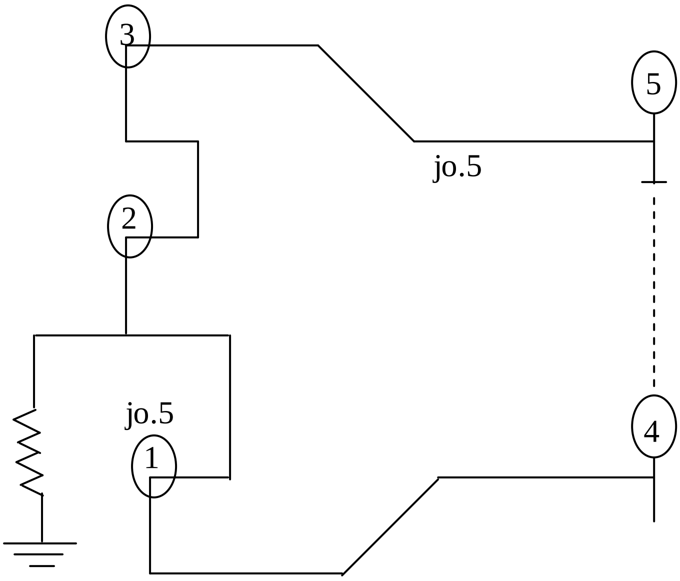

Fig. 5: A sample case for five bus-systems which has per unit impedance as marked. The symmetrical bus admittance matrix for the system is given as:

$$Y_{Bus} = \begin{bmatrix} -1/z11 & 1/z12 & 1/z13 & 1/z14 & -1/z15 \\ 1/z21 & -1/z22 & 1/z23 & 1/z24 & 1/z25 \\ 1/z31 & 1/z32 & -1/z33 & 1/z34 & 1/z35 \\ 1/z41 & 1/z42 & 1/z44 & -1/z44 & 1/z45 \\ 1/z51 & 1/z52 & 1/z54 & 1/z54 & -1/z55 \end{bmatrix} \quad (37)$$

From the bus admittance matrix for the system is figure 5 given as:

$$Y_{Bus} = \begin{bmatrix} -j30 & j10 & 0 & -j20 & 0 \\ j10 & -j26.2 & j16 & 0 & 0 \\ 0 & j16.0 & -j36.0 & j20 & j20 \\ j20 & 0 & 0 & j20 & 0 \\ 0 & 0 & j20.0 & 0 & -j20.0 \end{bmatrix} \quad (38)$$

Comparing equation (35) and (38) respectively, can be converted to have lower and upper triangular decomposition factorization matrix. For example it is required to generate Z_{45} of Z_{bus} for five buses system using convenient notation for the element L and U to obtain as:

$$LU = \underbrace{\begin{bmatrix} l_{11} & 0 & 0 & 0 & 0 \\ I_{21} & I_{22} & 0 & 0 & 0 \\ I_{31} & I_{32} & I_{33} & 0 & 0 \\ l_{41} & l_{42} & L_{43} & l_{44} & 0 \\ l_{51} & l_{52} & L_{53} & l_{54} & L_{55} \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} 1 & U_{12} & U_{13} & U_{14} & U_{15} \\ 0 & 0 & U_{23} & U_{24} & U_{25} \\ 0 & 0 & 1 & U_{34} & U_{35} \\ 0 & 0 & 0 & 1 & U_{45} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{U} \tag{39}$$

That is, $$\begin{bmatrix} Z_{15} \\ Z_{25} \\ Z_{35} \\ Z_{45} \\ Z_{55} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \tag{40}$$

Form equation (35) it is given as l_{11} U_{11} = -j30.0 and $u_{11} = u_{22} = u_{33} = u_{44} = u_{55} = 1$, l_{11} = - j30 this means the first entry in the lower (L) matrix is L_{11} = -j30 but other entries becomes zero (0) according to the entry l = [-j30, 0, 0, 0]

Similarly, $l_{11}U_{12}$ = j10
but l_{11} = j30
U_{12} = -0.3333

Similarly,
$l_{11}U_{13}$ = 0
U_{13} = 0/-j30 = 0
U_{13} = 0

Since $l_{11}U_{14}$ = j20.0
and -j30 U_{14} = j20 (41)

Hence, U_{14} = j20/-j30 = - 0.666667
U_{14} = - 0.66667
Recalled;
$l_{11}U_{15}$ = 0
$-j_{30}U_{15}$ = 0
U_{15}=0/-j30 = 0
U_{15} = 0 (42)

Then in a compact form it can be represented in the matrix as:
U = [1,-0.3333, 0,-0.66667, 0] (43)
It is found that the triangular factor of Y_{bus} can be summarized as:

$$L = \begin{bmatrix} -j30.0 & 0 & 0 & 0 & 0 \\ +j10 & -j22.866667 & 0 & 0 & 0 \\ 0 & j16.00 & -j24.804666 & 0 & 0 \\ j20 & j6.66667 & j4.664723 & -j3.845793 & 0 \\ 0 & 0 & j20.0 & j3.761164 & j0.195604 \end{bmatrix} \tag{44}$$

Similarly,

$$U = \begin{bmatrix} 1 & -0.33333 & 0 & -0.666667 & 0 \\ 0 & 1 & -0.99708 & -0.291545 & 0 \\ 0 & 0 & 1 & -0.18805 & -0.806300 \\ 0 & 0 & 0 & 1 & -0.977995 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (45)$$

$$\Downarrow_L$$

Using the triangular factors to calculate $Z_{th'45} = (Z_{44} - Z_{45}) - (Z_{34} - Z_{35})$ the thevenin impedance looking into the system between (4) and (5) from the figure 5 given.

Since Y_{bus} is symmetrical network, the row elements of U equals the column elements of L divided by their corresponding diagonal elements. With l's representing the numerical values of L, forward solution of the system equations [12, 13].

$$\underbrace{\begin{bmatrix} L_{11} & 0 & 0 & 0 & 0 \\ L_{21} & L_{22} & 0 & 0 & 0 \\ L_{31} & L_{32} & L_{33} & 0 & 0 \\ L_{41} & L_{42} & L_{43} & L_{44} & 0 \\ L_{51} & L_{52} & L_{53} & L_{54} & L_{55} \end{bmatrix}}_{\uparrow L} \underbrace{\begin{bmatrix} X_1 \\ X_2 \\ X_4 \\ X_5 \\ X_5 \end{bmatrix}}_{\uparrow X} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ -1 \end{bmatrix} \quad (46)$$

This yields the immediate values given as;

$X_1 = X_2 = X_3 = 0$

$X_4 = l_{44}^{-1} = (-j3.845793)^{-1} = j0.260024$

$$X_4 = \frac{-1 - 1_{54} X_4}{l55} = \frac{1 - j3.761164 \quad \times j0.260024}{-j0.195604} \quad (47)$$

$X_5 = -j0.112500$

By back substituting in the system of equations to obtain as:

$$\underbrace{\begin{bmatrix} 1 & U_{12} & U_{13} & U_{14} & U_{15} \\ 0 & 1 & U_{23} & U_{24} & U_{25} \\ 0 & 0 & 1 & U_{34} & U_{35} \\ 0 & 0 & 0 & 1 & U_{45} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{\uparrow U} \left[Z_{bus}^{(4-5)} \right] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ j0.260024 \\ -j0.112500 \end{bmatrix} \quad (48)$$

or

$$\underbrace{\begin{bmatrix} 0 & U_{12} & U_{13} & U_{14} & U_{15} \\ 0 & 1 & U_{23} & U_{24} & U_{25} \\ 0 & 0 & 1 & U_{34} & U_{35} \\ 0 & 0 & 0 & 1 & U_{45} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{\uparrow U} \underbrace{\begin{bmatrix} Z_{14} - Z_{15} \\ Z_{24} - Z_{25} \\ Z_{34} - Z_{35} \\ Z_{44} - Z_{45} \\ Z_{54} - Z_{55} \end{bmatrix}}_{\uparrow Z} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ j0.260024 \\ -j0.112500 \end{bmatrix} \quad (49)$$

Where U's represent the numerical values of U, in order to find from the two rows that is;
Z_{54} - Z_{55} = j0.1125 per unit
Z_{44} – Z_{45} = j0.26024 – U_{45} (Z_{54} - Z_{55}) =
j0.260024 – (-0.977995) (-j0.1125)
= j0.1500 per unit.

The desired thevenin impedance is therefore calculated as follows:
Zth_{45} = (Z_{44} – Z_{45}) – (Z_{54} - Z_{55}) = j0.1500 – (-j0.1125) per unit

From equation (49), it can be process in a matrix operations as:

$$\left.\begin{array}{l} 0x(Z_{14} - Z_{15}) + 0x(Z_{24} - Z_{25}) + 0x(Z_{34} - Z_{35}) \\ +0x(Z_{44} - Z_{45}) + 1x(Z_{45} - Z_{55}) \\ = -j0.11250 \\ \\ Z_{45} - Z_{55} = -j0.11250 \end{array}\right\} \quad (50)$$

Similarly,

$$\left.\begin{array}{l} 0x(Z_{14} - Z_{15}) + 0x(Z_{24} - Z_{25}) + 0x(Z_{34} - Z_{35}) \\ +0x(Z_{44} - Z_{45}) + U_{45}(Z_{45} - Z_{55}) = j0.260024 \end{array}\right\} \quad (51)$$

But,

$$\left.\begin{array}{l} U_{45} \text{ from U matrice} = (0.977995) \\ Z_{44} - Z_{45} = j0.260024 - U_{45}(Z_{54} - Z_{55}) \\ = j0.260024 - (-0.97795) \\ = (-j0.1125) \\ Z_{44} - Z_{45} = j0.15 \end{array}\right\} \quad (52)$$

Also,

$$\left.\begin{array}{l} 0x(Z_{14} - Z_{15}) + 0\,x(Z_{24} - Z_{25}) + 1\,x(Z_{34} - Z_{35}) \\ +U_{34}(Z_{44} - Z_{45}) + U_{35}(Z_{54} - Z_{55}) = 0 \\ Z_{34} - Z_{35} = -j0.0625 \end{array}\right\} \quad (53)$$

Continue the back substitution operation, in order to obtain as;
Z_{45} – Z_{25} = 0

Similarly Z_{14} – Z_{15} = + j0.10
To account for the addition of the line (2)-(5) and line (4)-(5) we need to calculate the corresponding vectors $Z_{bus}^{(2-5)}$ and $Z_{bus}^{(4-5)}$ using upward substitution. In the same manner it can be applied [14, 15].

$$\begin{bmatrix} 1 & U_{12} & U_{13} & U_{14} & U_{15} \\ 0 & 1 & U_{23} & U_{24} & U_{25} \\ 0 & 0 & 1 & U_{34} & U_{35} \\ 0 & 0 & 0 & 1 & U_{45} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} Z_{bus}^{(2-5)} \begin{bmatrix} X_1 = 0 \\ X_2 = 1 \\ X_3 = 0 \\ X_4 = 0 \\ X_5 = 0 \end{bmatrix} \quad (54)$$

or

$$\begin{bmatrix} 1 & U_{12} & U_{13} & U_{14} & U_{15} \\ 0 & 1 & U_{23} & U_{24} & U_{25} \\ 0 & 0 & 1 & U_{34} & U_{35} \\ 0 & 0 & 0 & 1 & U_{45} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Z_{12} - Z_{25} \\ Z_{22} - Z_{25} \\ Z_{32} - Z_{35} \\ Z_{42} - Z_{45} \\ Z_{52} - Z_{55} \end{bmatrix} \begin{bmatrix} X_1 & = & 0 \\ X_2 & = & 1 \\ X_3 & = & \\ X_4 & = & 0 \\ X_5 & = & 0 \end{bmatrix} \qquad (55)$$

With L^{rs} representing the numerical values of L, forwards with substitution of the systems.

$$\begin{bmatrix} L_{11} & 0 & 0 & 0 & 0 \\ I_{21} & I_{22} & 0 & 0 & 0 \\ I_{31} & I_{32} & I_{33} & 0 & 0 \\ L_{41} & L_{42} & L_{43} & L_{44} & 0 \\ L_{51} & L_{52} & L_{53} & L_{54} & L_{55} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} \qquad (56)$$

From the matrix operation, it is obtain as:
$L_{11}x_1 + 0xX_2 + xX_3 + 0xX_4 + X_5 = 0$

That is;
$L_{11}X_1 = 0$
$X_1 = 0/L_{11} = 0$

Also,
$L._{21}X_1 + L._{22}X_2 + 0.X_3 + 0.X_4 + 0.X_5 = 1$ (57)

$$X_2 = \frac{1}{-j22.866667} = \frac{1}{L_{22}}$$

$X_2 = +j0.04273$

Similarly,
$L_{31}X_1 + L_{32}X_2 + L_{33}X_3 + 0X_4 + 0X_5 = 0$ (58)

Thus, taking values for L_{32} and L_{33} from L triangular decomposition factor
Where;
$l_{32} = j16.00$, $l_{33} = -j24.804666$
Substituting into the equation to obtain as:

$$l_{32}\left(\frac{1}{L_{22}}\right) + L33\,X\,3 = 0$$

$X_3 = 0.69968/ - j24.80466$
$X_3 = j0.02820.8602$ (59)

In same similar manner X_4 and X_5 can be obtained as:
$X_4 = + j0.110019944$
$X_5 = -j0.112500$

Therefore formulating the upper triangular matrix while substituting
X_1, X_2, X_3, X_4 and X_5 respectively to obtain as:

$$\begin{bmatrix} 1 & U_{12} & U_{13} & U_{14} & U_{15} \\ 0 & 1 & U_{23} & U_{24} & U_{25} \\ 0 & 0 & 1 & U_{34} & U_{35} \\ 0 & 0 & 0 & 1 & U_{45} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Z_{12} - Z_{25} \\ Z_{22} - Z_{25} \\ Z_{32} - Z_{35} \\ Z_{42} - Z_{45} \\ Z_{52} - Z_{55} \end{bmatrix} \begin{bmatrix} j0.04373 \\ j0.04373 \\ j0.0282760 \\ j0.1100199 \\ -j0.11270 \end{bmatrix} \quad (60)$$

Similarly, for the matrix operation in equation (1), we have:

$$\left.\begin{array}{l} 0x(Z_{12} - Z_{15}) + 0x(Z_{22} - Z_{25}) + x(Z_{32} - Z_{35}) + \\ +0x(Z_{42} - Z_{45}) + 1X(Z_{52} - Z_{55}) = j0.11270 \\ (Z_{52} - Z_{55}) = -j0.11270 \end{array}\right\} \quad (61)$$

Similarly, the same process is continued in order to obtain the following; $(Z_{42} - Z_{45})$, $(Z_{32} - Z_{25})$, $(Z_{22} - Z_{25})$, $(Z_{12} - Z_{15})$, as: 0, - j0.06250; 0; 0 and $(Z_{52} - Z_{55})$ = - j0.11270

Now to account for the addition of line (2)-(5) and (4)-(5), we need to calculate the corresponding vector $Z_{bus}^{(2-5)}$ and $Z_{bus}^{(4-5)}$ using forward and backward substitution, then we have;

$$Z_{bus}^{(4-5)} \begin{bmatrix} Z_{12} - Z_{25} \\ Z_{22} - Z_{25} \\ Z_{32} - Z_{35} \\ Z_{42} - Z_{45} \\ Z_{52} - Z_{55} \end{bmatrix} \begin{bmatrix} j0.1 \\ 0 \\ -j0.0625 \\ j0.15 \\ -j0.1125 \end{bmatrix} \quad (62)$$

and

$$Z_{bus}^{(4-5)} \begin{bmatrix} Z_{12} - Z_{25} \\ Z_{22} - Z_{25} \\ Z_{32} - Z_{35} \\ Z_{42} - Z_{45} \\ Z_{52} - Z_{55} \end{bmatrix} \begin{bmatrix} j0.1 \\ 0 \\ -j0.0625 \\ j0.15 \\ -j0.1125 \end{bmatrix} \quad (63)$$

From Eq. (23) with m = 2, n = 5, p = 4, and q = 5 the elements of the Z matrix are determined to be:

$$\left.\begin{array}{l} Z = \begin{matrix} a \\ b \end{matrix}\left[\dfrac{(Z_{22} - Z_{25}) - (Z_{52} - Z_{55}) + Z_a \,\big|\, (Z_{24} - Z_{25}) - (Z_{54} - Z_{55})}{(Z_{42} - Z_{45}) - (Z_{52} - Z_{55}) \,\big|\, (Z_{44} - Z_{45}) - (Z_{54} - Z_{55}) Z_b} \right] \\ = \begin{matrix} a \\ b \end{matrix}\left[\dfrac{0 - (-j0.1125) + j0.04 \,\big|\, 0 - (-j0.1125)}{0 - (-j0.1125) \,\big|\, j0.15 - (-j0.1125) + j0.08} \right] \\ = \begin{matrix} a \\ b \end{matrix}\left[\dfrac{j0.1525 \,\big|\, j0.1125}{j0.1125 \,\big|\, j0.1125} \right] \end{array}\right\} \quad (64)$$

Using Eq. (23), we can calculate 1_a and I_b as the solution of

$$\begin{bmatrix} j0.1525 & j0.1125 \\ j0.1125 & j0.3425 \end{bmatrix}\begin{bmatrix} I_a \\ I_b \end{bmatrix} = \begin{bmatrix} V_2 - V_5 \\ V_4 - V_5 \end{bmatrix} = \begin{bmatrix} 1.0 & -0.98 \\ 0.989 & -0.98 \end{bmatrix} = \begin{bmatrix} 0.02 \\ 0.005 \end{bmatrix} \quad (65)$$

From which we find I_a = j0.158876 and I_b = j0.037587. According to Eqs. (17) and (26), the changes in the voltages at all buses are:

AV = - $I_a Z_{bus}^{(2-5)}$ -$I_b Z_{bus}^{(4-5)}$

$$= j0.158876\begin{bmatrix} 0 \\ 0 \\ -j0.0625 \\ 0 \\ -j0.1125 \end{bmatrix} - j0.037587\begin{bmatrix} j0.1 \\ 0 \\ -j0.0625 \\ j0.15 \\ -j0.1125 \end{bmatrix} = \begin{bmatrix} 0.003759 < 0^0 \\ 0 \\ 0.00781 < 0^0 \\ 0.005638 < 0^0 \\ 0.013645 < 0^0 \end{bmatrix} \quad (66)$$

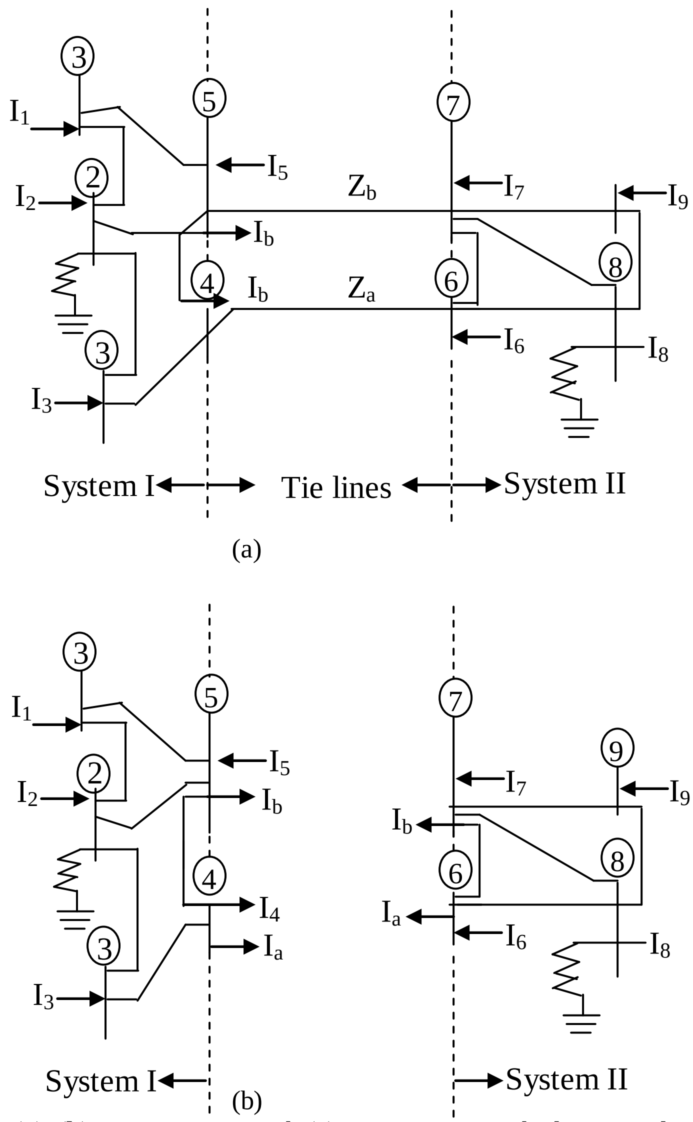

Fig 6(a), (b): Two systems with (a) interconnecting the line impedance Z_a and Z_b (b) equivalent compensating current injections.

V. CONCLUSION

Following to the significant electric power demand increase, transmission line operators and planners are required to increase transmission line power transfer capability in order to check system overload. This paper investigated network overload using development of an improved contingency analysis model for proper transmission expansion system on the view to address the power transfer capability (PTC) of the transmission lines addition or removal. This is achieved considering the system impedance matrix of Z_{bus} and Y_{us} that characterizes the system properties/behaviours. System can be adjusted or compensation for an overload line. The technique of generation or phase shifter can be calculated to compensate or correct an overloaded line for improved power quality. This model are developed to identify overloaded lines and to select one or more lines to be removed in order to reduce overload in power system.

This mean that a line can be added to or removed from the network in order to cause a shift in the power flow and then eliminate the overload. That is addition or removal of a line must not cause any other overloads on the systems as it's eliminate the original overload.

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