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A new model describing the column processes corresponding to higher order of flotation kinetics

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ABSTRACT: *The primary goal of the present paper is to give an extended model of some processes in camera flotation. This leads to a nonlinear first order system of hyperbolic equations. Such systems correspondence to the chemical reaction of higher order. From mathematical point of view this means that the system obtained contains polynomial nonlinearities. First we formulate a mixed problem for the hyperbolic system with boundary conditions corresponding to the processes on the boundary in the flotation camera. We present the mixed problem for the hyperbolic system in a suitable operator form and prove an existence of generalized solution by fixed point method. It is shown of how to reach the solution of the system in question by a sequence of successive approximations.*

KEYWORDS: *Camera Flotation, Higher Order Flotation Kinetics, Hyperbolic System, Successive Approximations.*

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I. INTRODUCTION

Many articles are devoted to the study of flotation processes. Without claiming completeness we note some of them [1]-[10]. In the present paper we introduce a generalized model describing the column flotation processes. The creation of this model was inspired by the article [11], where the authors consider the notion order of flotation kinetics by analogy with the order of the chemical reactions. Description of the processes in this case leads to a nonlinear first order hyperbolic system of partial differential equations with polynomial nonlinearities. Such type nonlinearities generate mathematical difficulties which we overcome by using the method developed in a series of papers [12]- [18].

Here we study the system of first order partial differential equations

$$\frac{\partial C_B(x,t)}{\partial t} = k_1 C_P{}^n(x,t) - k_2 C_B{}^n(x,t) - V_B \frac{\partial C_B(x,t)}{\partial x}$$

$$\frac{\partial C_P(x,t)}{\partial t} = -k_1 C_P{}^n(x,t) + k_2 C_B{}^n(x,t) + V_P \frac{\partial C_P(x,t)}{\partial x} \qquad (1)$$

$$(x,t) \in D = \{(x,t) \in R^2 : (x,t) \in [0,H] \times [0,T]\}.$$

Here the unknown functions are: $C_P(x,t)$ is the mineral concentration in the liquid, $C_B(x,t)$ is the mineral concentration on the bubbles, k_1 and k_2 are prescribed kinetic constants describing particle transitions from one phase to another, H >0 isthe height of the camera and $[0,T]$ is prescribed time interval; $V_P > 0$ is a particle sedimentation rate, $V_B > 0$ -the bubble lifting speed. We note that $V_B >> V_P$ (cf.[10]), but unlike some previous papers we do not neglect $V_P > 0$. The process in the camera are such that V_P is directed from top to bottom, while V_B the speed of the bubbles is directed upwards. Here we investigate the case $n > 1$, where n is the order of flotation kinetics. In previous papers we have considered the case *n=1.*

For system (1) one can formulate the following mixed (initial-boundary value) problem: to find the unknown concentration functions $C_P(x,t)$ and $C_B(x,t)$ in Π satisfying initial conditions

$$C_B(x,0)=0\,,\ C_P(x,0)=C_{P0}\,, \tag{2}$$

where $C_{P0}=const.>0$ is a prescribed initial concentration and boundary conditions

$$C_B(0,t)=0,\ \ C_P(0,t)=C_{P0}=const.>0 \qquad t\in[0,T]. \tag{3}$$

We follow the mathematical methods [12] for investigation of transmission lines and some applications (cf. also [12]- [18]). We present the mixed problem for the above hyperbolic system in an operator form. By using a suitable function space, we prove existence theorems for (1) - (3) by fixed point method (cf. [12]). Finally, we show of how to obtain a sequence of successive approximations tending to the solution of the system in question.

The system (1) can be rewrite as

$$\frac{\partial C_B(x,t)}{\partial t}+V_B\frac{\partial C_B(x,t)}{\partial x}=k_1C_P{}^n(x,t)-k_2C_B{}^n(x,t)$$

$$\frac{\partial C_P(x,t)}{\partial t}-V_P\frac{\partial C_P(x,t)}{\partial x}=-k_1C_P{}^n(x,t)+k_2C_B{}^n(x,t)$$

and introducing denotations

$$W=\begin{bmatrix}C_B(x,t)\\C_P(x,t)\end{bmatrix},\quad \frac{\partial W}{\partial t}=\begin{bmatrix}\dfrac{\partial C_B(x,t)}{\partial t}\\\dfrac{\partial C_P(x,t)}{\partial t}\end{bmatrix},\quad \frac{\partial W}{\partial x}=\begin{bmatrix}\dfrac{\partial C_B(x,t)}{\partial x}\\\dfrac{\partial C_P(x,t)}{\partial x}\end{bmatrix},A=\begin{bmatrix}V_B & 0\\0 & -V_P\end{bmatrix},\ K=\begin{bmatrix}k_1 & -k_2\\-k_1 & k_2\end{bmatrix}$$

we present the above system in a matrix form

$$\begin{bmatrix}\dfrac{\partial C_B(x,t)}{\partial t}\\\dfrac{\partial C_P(x,t)}{\partial t}\end{bmatrix}+\begin{bmatrix}V_B & 0\\0 & -V_P\end{bmatrix}\begin{bmatrix}\dfrac{\partial C_B(x,t)}{\partial x}\\\dfrac{\partial C_P(x,t)}{\partial x}\end{bmatrix}=\begin{bmatrix}k_1 & -k_2\\-k_1 & k_2\end{bmatrix}\begin{bmatrix}C_B{}^n(x,t)\\C_P{}^n(x,t)\end{bmatrix}$$

or

$$\frac{\partial W}{\partial t}+A\frac{\partial W}{\partial x}=KW\,. \tag{4}$$

Here the matrix $A=\begin{bmatrix}V_B & 0\\0 & -V_P\end{bmatrix}$ is in a diagonal form. Therefore the characteristic roots are $\lambda_1=V_B,\lambda_2=-V_P$ and we are able to formulate the main problem of the paper.

To solve the hyperbolic system (4) satisfying initial conditions

$$C_B(x,0)=0,\quad C_P(x,0)=C_{P0}=const.>0\ \ x\in[0,H] \tag{5}$$

and boundary conditions

$$C_B(0,t)=0,\quad C_P(0,t)=C_{P0},\quad t\in[0,T];\ C_B(\Lambda,t)=C_{B\Lambda},\quad C_P(\Lambda,t)=C_{P\Lambda},\quad t\in[0,T]$$

$C_B(\Lambda,t)+C_P(\Lambda,t)=\kappa(t)<1$, where $\overline{\kappa}=\sup\{\kappa(t):t\in[0,T]\}<1$.

II. AN OPERATOR FORMULATION OF THE MIXED PROBLEM

The mixed problem is: to find a solution $(C_P(x,t)\,,\,C_B(x,t))$ of the following system.
Following [12]- [18] we consider the Cauchy problem for the characteristics:

$$d\xi/d\tau=V_B,\ \xi(t)=x \text{ foreach } (x,t)\in D\ \Rightarrow\ \varphi_B(\tau;x,t)=V_B\tau+x-V_Bt\,, \tag{6}$$

$$d\eta/d\tau=-V_P,\ \eta(t)=x \text{ foreach } (x,t)\in D\ \Rightarrow \phi_P(\tau;x,t)=-V_P\tau+x+V_Pt\,. \tag{7}$$

Functions $\lambda_B(x,t)=V_B>0$ and $\lambda_P(x,t)=-V_P<0$ arecontinuousonesandimplyauniquenessto the leftfrom t of the solution $\xi=\varphi_B(t;x,t)$ of $d\xi/dt=V_B$, $\xi(t)=x$ andrespectively $\eta=\varphi_P(t;x,t)$ of $d\eta/dt=-V_P$, $\eta(t)=x$.

Denote by $\chi_B(x,t)$ the smallest value of τ such that the solution $\varphi_B(\tau;x,t)=V_B\tau+x-V_Bt$ of (6) still belongs to D and by $\chi_P(x,t)$ − the respective value of τ for the solution $\varphi_P(\tau;x,t)=-V_P\tau+x+V_Pt$ of (7).

If $\chi_B(x,t)>0$ then $\varphi_B(\chi_B(x,t);x,t)=0$ or $\varphi_B(\chi_B(x,t);x,t)=H$ and respectively if $\chi_P(x,t)>0$ then $\varphi_P(\chi_P(x,t);x,t)=0$ or $\varphi_P(\chi_P(x,t);x,t)=H$. In our case

$$\chi_B(x,t)=\begin{cases} t-\dfrac{x}{V_B} & \text{for } V_Bt-x>0\\ 0 & \text{for } V_Bt-x\le 0\end{cases}\qquad \chi_P(x,t)=\begin{cases} t-\dfrac{H-x}{V_P} & \text{for } V_Pt+x-H>0\\ 0 & \text{for } V_Pt+x-H\le 0\end{cases}.$$

Obviously $0\le\chi_B(x,t)\le t,\ 0\le\chi_P(x,t)\le t$. One can easy to see that

$$\varphi_B(\tau;x,t)=V_B\tau+x-V_Bt\ \Rightarrow\ \varphi_B(0;x,t)=x-V_Bt\,;$$

$$\varphi_P(\tau;x,t)=-V_P\tau+x+V_Pt\ \Rightarrow\ \varphi_P(0;x,t)=x+V_Pt\,.$$

Introduce the sets

$$D_{in,B}=\{(x,t)\in D:\ \chi_B(x,t)=0\}\equiv\{(x,t)\in D: x-V_Bt\ge 0\},$$

$$D_{in,P}=\{(x,t)\in D:\ \chi_P(x,t)=0\}\equiv\{(x,t)\in D: V_Pt+x-H\ge 0\},$$

$$D_{0B}=\{(x,t)\in D:\ \chi_B(x,t)>0,\ \ \phi_B(\chi_B(x,t);x,t)=V_B\left(V_Bt-x\right)/V_B+x-V_Bt=0\},$$

$$D_{HB}=\{(x,t)\in D:\ \chi_B(x,t)>0,\ \ \phi_B(\chi_B(x,t);x,t)=V_B\left(V_Bt-x\right)/V_B+x-V_Bt=H\}=\varnothing,$$

$$D_{0P}=\left\{(x,t)\in D:\chi_P(x,t)>0,\ \ \phi_P(\chi_P(x,t);x,t)=-V_P\left(t-\frac{H-x}{V_P}\right)+x+V_Pt=0\right\}=\varnothing,$$

$$D_{HP}=\left\{(x,t)\in D:\ \chi_P(x,t)>0,\ \ \varphi_P(\chi_P(x,t);x,t)=-V_P\left(t-\frac{H-x}{V_P}\right)+x+V_Pt=H\right\}.$$

Prior to present the mixed problem in operator form we introduce

$$\Phi_B(C_B,C_P)(x,t)=\begin{cases} C_{B0}(x-V_Bt),\ (x,t)\in D_{in,B} \\ \Phi_{0B}(C_B,C_P)(\chi_B(x,t)),(x,t)\in D_{0B}\end{cases}=\begin{cases}0,\ (x,t)\in D_{in,B}\\ C_{B0}\ \ ,(x,t)\in D_{0B}\end{cases}$$

and

$$\Phi_P(C_B,C_P)(x,t)=\begin{cases} C_{P0}(x+V_Pt)\ ,\ (x,t)\in D_{in,P}\\ \kappa(t)-C_B(H,\chi_P(x,t)),\ (x,t)\in D_{0P}\end{cases}.$$

So we assign to the above mixed problem the following system of operator equations

$$C_B(x,t)=0,\quad (x,t)\in D_{in,B};$$

$$C_B(x,t)=C_{B0}+\int_{t-\frac{x}{V_B}}^{t}k_1C_B{}^n(\chi_B(x,s),s)-k_2C_P{}^n(\chi_B(x,s),s)ds,\quad (x,t)\in D_{0B}$$

$$C_P(x,t)=C_{P0},\quad (x,t)\in D_{in,P} \tag{8}$$

$$C_P(\Lambda,t)=\kappa(t)-C_B{}^n(\Lambda,\chi_P(x,t))+\int_{t-\frac{H-x}{V_P}}^{t}\left(-k_1C_B{}^n(\chi_P(x,s),s)+k_2C_P{}^n(\chi_P(x,s),s)\right)ds,\ (x,t)\in D_{0P}.$$

Introduce the function sets:

$$M_B=\left\{C_B\in C([0,H]\times[0,T]:\left|C_B(x,t)\right|\le\hat{C}_Be^{\mu t},x\in[0,H]\right\},$$

$$M_P=\left\{C_P\in C([0,H]\times[0,T]:\left|C_P(x,t)\right|\le\hat{C}_Pe^{\mu t},x\in[0,H]\right\},$$

where $\hat{C}_B,\hat{C}_P$ and μ are positive constants.

It is easy to verify that the set $M_B\times M_P$ turns out into a complete metric space with respect to the metric:

$\rho((C_B,C_P),(\overline{C}_B,\overline{C}_P))=\max\left\{\rho(C_B,\overline{C}_B),\ \rho(C_P,\overline{C}_P)\right\}$, where

$$\rho(C_B,\overline{C}_B)=\sup\left\{e^{-\mu t}\left|C_B(x,t)-\overline{C}_B(x,t)\right|:(x,t)\in[0,H]\times[0,T]\right\},$$

$$\rho(C_P,\overline{C}_P)=\sup\left\{e^{-\mu t}\left|C_P(x,t)-\overline{C}_P(x,t)\right|:(x,t)\in[0,H]\times[0,T]\right\}.$$

Now we define an operator $T=(T_B,T_P):M_B\times M_P\to M_B\times M_P$ by the formulas

$$T_B(C_B,C_P)(x,t):=0,\ (x,t)\in D_{in,B},$$

$$T_B(C_B,C_P)(x,t):=\ C_{B0}+\int_{t-\frac{x}{V_B}}^{t}k_1C_B{}^n(\chi_B(x,t),s)-k_2C_P{}^n(\chi_B(x,t),s)ds,\ (x,t)\in D_{0B};$$

$$T_P(C_B,C_P)(x,t):=C_{0P},\quad (x,t)\in D_{in,P},$$

$$T_P(C_B,C_P)(x,t):=\kappa(t)-C_B{}^n(H,\chi_P(x,t))+\int_{t-\frac{H-x}{V_P}}^{t}\left(-k_1C_B{}^n(\chi_P(x,t),s)+k_2C_P{}^n(\chi_P(x,t),s)\right)ds \quad ,$$

$(x,t)\in D_{0P}$.

III. EXISTENCE THEOREM

We call a generalized solution (C_B,C_P) of (4),(5) if (C_B,C_P) is a solution of integral equations (8). The main purpose of the section is to prove an existence of solution of (8).

Theorem 1. Let the following conditions be fulfilled for sufficiently large $\mu>0$ and $0<\varepsilon<H$:

1) $\dfrac{(1+k_1)\hat{C}_B+k_2\hat{C}_P}{\mu}e^{\mu T}\le \hat{C}_B$;

2) $\left(\hat{\kappa}+\hat{C}_B e^{-\mu\frac{\varepsilon}{V_P}}+\dfrac{k_1\hat{C}_B+k_2\hat{C}_P}{\mu}\right)e^{\mu T}\le \hat{C}_P$;

3) $n\max\left\{e^{\mu T}\dfrac{(k_1\hat{C}_B+k_2\hat{C}_P)}{2\mu};\hat{C}_B e^{-\mu\frac{\varepsilon}{V_P}}+e^{\mu T}\dfrac{k_1\hat{C}_B+k_2\hat{C}_P}{2\mu}\right\}<1$;

4) $C_{B0}\le\hat{C}_B$; $C_{0P}\le\hat{C}_P$;

5) $\bar{\kappa}=\sup\{\kappa(t):t\in[0,T]\}<1$.

Then there exists a unique solution of (8) on the set $[0,H-\varepsilon]\times[0,T]$.

Proof: We show that the operator $T=(T_B,T_P):M_B\times M_P\to M_B\times M_P$ above introduced maps the set $M_B\times M_P$ into itself.

We notice that $T_B(C_B,C_P)(x,t)$ and $T_P(C_B,C_P)(x,t)$ are continuous functions.

First we have to show that $|T_B(C_B,C_P)(x,t)|\le\hat{C}_B e^{\mu t}$, $|T_P(C_B,C_P)(x,t)|\le\hat{C}_P e^{\mu t}$.

Indeed, $|\Phi_B(C_B,C_P)(x,t)|=0$ and therefore in view of $|C_B(\chi_B,s)|\le 1$, $|C_P(\chi_B,s)|\le 1$ and $n>1$, it follows $|C_B(\chi_B,s)|^n\le|C_B(\chi_B,s)|$; $|C_P(\chi_B,s)|^n\le|C_P(\chi_B,s)|$. Then

$$|T_B(C_B,C_P)(x,t)|\le C_{B0}+\int_{t-\frac{x}{V_B}}^{t}\left(k_1|C_B(\chi_B,s)|^n+k_2|C_P(\chi_B,s)|^n\right)ds\le$$

$$\le C_{B0}+\int_{t-\frac{x}{V_B}}^{t}\left(k_1|C_B(\chi_B,s)|+k_2|C_P(\chi_B,s)|\right)ds\le$$

$$\le\left((1+k_1)\hat{C}_B+k_2\hat{C}_P\right)\int_{t-\frac{x}{V_B}}^{t}e^{n\mu s}ds\le\left((1+k_1)\hat{C}_B+k_2\hat{C}_P\right)\frac{e^{\mu t}-e^{\mu(t-V_Bx)}}{n\mu}\le$$

$$\le\frac{(1+k_1)\hat{C}_B+k_2\hat{C}_P}{\mu}e^{\mu n t}e^{\mu t}\le\frac{(1+k_1)\hat{C}_B+k_2\hat{C}_P}{\mu}e^{\mu T}e^{\mu t}\le\hat{C}_B e^{\mu t}.$$

For the second component we have

$$|T_P(C_B,C_P)(x,t)|\le|\kappa(t)|+|C_B(H,\chi_P(x,t))|^n+\int_{t-\frac{H-x}{V_P}}^{t}k_1|C_B(\chi_P,s)|^n+k_2|C_P(\chi_P,s)|^n ds\le$$

$$\leq |\kappa(t)| + |C_B(H,\chi_P(x,t))| + \int_{t-\frac{H-x}{V_P}}^{t} k_1|C_B(\chi_P,s)| + k_2|C_P(\chi_P,s)|ds \leq$$

$$\leq \hat{\kappa}e^{\mu t} + \hat{C}_B e^{\mu\left(t-\frac{H-x}{V_P}\right)} + \frac{k_1\hat{C}_B + k_2\hat{C}_P}{\mu}e^{\mu t} \leq \left(\hat{\kappa} + \hat{C}_B e^{-\mu\frac{H-x}{V_P}} + \frac{k_1\hat{C}_B + k_2\hat{C}_P}{\mu}\right)e^{\mu t} \leq$$

$$\leq \left(\hat{\kappa} + \hat{C}_B e^{-\mu\frac{\varepsilon}{V_P}} + \frac{k_1\hat{C}_B + k_2\hat{C}_P}{\mu}\right)e^{\mu T}e^{\mu t} \leq \hat{C}_P e^{\mu t}$$

for sufficiently large $\mu > 0$.

The following inequalities are valid: for $(x,t) \in D_{0B}$ and $n > 2 \Rightarrow n-1 > 1$

$$|T_B(C_B,C_P)(x,t) - T_B(\overline{C}_B,\overline{C}_P)(x,t)| \leq$$

$$\leq \int_{t-\frac{x}{V_B}}^{t} k_1\left|C_B{}^n(\chi_B(x,t),s) - \overline{C}_B{}^n(\chi_B(x,t),s)\right| + k_2\left|C_P{}^n(\chi_B(x,t),s) - \overline{C}_P{}^n(\chi_B(x,t),s)\right|ds \leq$$

$$\leq k_1 n \int_{t-\frac{x}{V_B}}^{t} \left|C_B{}^{n-1}(\chi_B(x,t),s)\right|\left|C_B(\chi_B(x,t),s) - \overline{C}_B(\chi_B(x,t),s)\right|ds +$$

$$+ k_2 n \int_{t-\frac{x}{V_B}}^{t} \left|C_P{}^{n-1}(\chi_B(x,t),s)\right|\left|C_P(\chi_B(x,t),s) - \overline{C}_P(\chi_B(x,t),s)\right|ds \leq$$

$$\leq k_1 n \int_{t-\frac{x}{V_B}}^{t} \hat{C}_B e^{\mu t}\left|C_B(\chi_B(x,t),s) - \overline{C}_B(\chi_B(x,t),s)\right|ds +$$

$$+ k_2 n \int_{t-\frac{x}{V_B}}^{t} \hat{C}_P e^{\mu t}\left|C_P(\chi_B(x,t),s) - \overline{C}_P(\chi_B(x,t),s)\right|ds \leq$$

$$\leq \rho\left(C_B,\overline{C}_B\right) n k_1 \hat{C}_B \int_{t-\frac{x}{V_B}}^{t} e^{2\mu s}ds + \rho\left(C_P,\overline{C}_P\right) n k_2 \hat{C}_P \int_{t-\frac{x}{V_B}}^{t} e^{2\mu s}ds \leq$$

$$\leq \max\left\{\rho\left(C_B,\overline{C}_B\right), \rho\left(C_P,\overline{C}_P\right)\right\} n\left(k_1\hat{C}_B + k_2\hat{C}_P\right)\frac{e^{2\mu t} - e^{2\mu(t-V_B x)}}{2\mu} \leq$$

$$\leq e^{\mu t}\rho\left(\left(C_B,C_P\right),\left(\overline{C}_B,\overline{C}_P\right)\right)e^{\mu T}\frac{n(k_1\hat{C}_B + k_2)}{2\mu}.$$

Consequently

$$\rho\left(T_B(C_B,C_P), T_B(\overline{C}_B,\overline{C}_P)\right) \leq e^{\mu T}\frac{n(k_1\hat{C}_B + k_2\hat{C}_P)}{2\mu}\rho((C_B,C_P),(\overline{C}_B,\overline{C}_P)).$$

For the second component we have

$$|T_P(C_B,C_P)(x,t) - T_P(\overline{C}_B,\overline{C}_P)(x,t)| \leq \left|C_B{}^n(H,\chi_P(x,t)) - \overline{C}_B{}^n(H,\chi_P(x,t))\right| +$$

$$\leq \left|C_B{}^n(H,\chi_P(x,t)) - \overline{C}_B{}^n(H,\chi_P(x,t))\right| +$$

$$\leq \int_{t-\frac{H-x}{V_P}}^{t} k_1\left|C_B{}^n(\chi_P,s)-\overline{C}_B{}^n(\chi_P,s)\right|+k_2\left|C_P{}^n(\chi_P,s)-\overline{C}_P{}^n(\chi_P,s)\right|ds \leq$$

$$\leq n\left|C_B\right|^{n-1}\left|C_B(H,\chi_P(x,t))-\overline{C}_B(H,\chi_P(x,t))\right|+$$

$$+n\int_{t-\frac{H-x}{V_P}}^{t} k_1\left|C_B\right|^{n-1}\left|C_B(\chi_P,s)-\overline{C}_B(\chi_P,s)\right|+k_2\left|C_P\right|^{n-1}\left|C_P(\chi_P,s)-\overline{C}_P(\chi_P,s)\right|ds \leq$$

$$\leq \rho\left(C_B,\overline{C}_B\right)n\hat{C}_B e^{(n-1)\mu\chi_P}+\rho\left(C_B,\overline{C}_B\right)k_1\int_{t-\frac{H-x}{V_P}}^{t} e^{2\mu s}ds+\rho\left(C_P,\overline{C}_P\right)k_2\int_{t-\frac{H-x}{V_P}}^{t} e^{2\mu s}ds \leq$$

$$\leq \rho\left(C_B,\overline{C}_B\right)n\hat{C}_B e^{\mu\left(t-\frac{\Lambda-x}{V_P}\right)}+\rho\left(C_B,\overline{C}_B\right)n k_1\hat{C}_B\int_{t-\frac{H-x}{V_P}}^{t} e^{\mu s}ds+\rho\left(C_P,\overline{C}_P\right)n k_2\hat{C}_P\int_{t-\frac{H-x}{V_P}}^{t} e^{\mu s}ds \leq$$

$$\leq e^{\mu t}\rho\left(C_B,\overline{C}_B\right)n\hat{C}_B e^{-\mu\frac{H-x}{V_P}}+\rho\left(C_B,\overline{C}_B\right)k_1\frac{e^{2\mu t}-e^{2\mu\left(t-\frac{H-x}{V_P}\right)}}{2\mu}+\rho\left(C_P,\overline{C}_P\right)k_2\frac{e^{2\mu t}-e^{2\mu\left(t-\frac{H-x}{V_P}\right)}}{2\mu} \leq$$

$$\leq e^{\mu t}\left[\rho\left(C_B,\overline{C}_B\right)n\hat{C}_B e^{-\mu\frac{H-x}{V_P}}+\rho\left(C_B,\overline{C}_B\right)nk_1\hat{C}_B e^{\mu t}\frac{1-e^{-\mu\frac{H-x}{V_P}}}{2\mu}+\rho\left(C_P,\overline{C}_P\right)nk_2\hat{C}_P e^{\mu t}\frac{1-e^{-\mu\frac{H-x}{V_P}}}{2\mu}\right] \leq$$

$$\leq e^{\mu t}\left(n\hat{C}_B e^{-\mu\frac{H-x}{V_P}}+e^{\mu T}\frac{nk_1\hat{C}_B+nk_2 n\hat{C}_P}{2\mu}\right)\rho((C_B,C_P),(\overline{C}_B,\overline{C}_P)) \leq$$

$$\leq \left(n\hat{C}_B e^{-\mu\frac{\varepsilon}{V_P}}+e^{\mu T}\frac{nk_1\hat{C}_B+nk_2 n\hat{C}_P}{2\mu}\right)\rho((C_B,C_P),(\overline{C}_B,\overline{C}_P))$$

which implies

$$\rho\left(T_P(C_B,C_P),T_P(\overline{C}_B,\overline{C}_P)\right)\leq\left(e^{\mu T}\frac{nk_1\hat{C}_B+nk_2 n\hat{C}_P}{2\mu}\right)\rho((C_B,C_P),(\overline{C}_B,\overline{C}_P)).$$

Since

$$\rho\left(T_B(C_B,C_P),T_B(\overline{C}_B,\overline{C}_P)\right)\leq \rho\left((C_B,C_P),\left(\overline{C}_B,\overline{C}_P\right)\right)e^{\mu T}\frac{n(k_1\hat{C}_B+k_2\hat{C}_P)}{2\mu}<1$$

and

$$\rho\left(T_P(C_B,C_P),T_P(\overline{C}_B,\overline{C}_P)\right)\leq \rho\left((C_B,C_P),\left(\overline{C}_B,\overline{C}_P\right)\right)\left(n\hat{C}_B e^{-\mu\frac{\varepsilon}{V_P}}+e^{\mu T}\frac{nk_1\hat{C}_B+nk_2 n\hat{C}_P}{2\mu}\right),$$

it follows

$$\max\left\{\rho\left(T_B(C_B,C_P),T_B(\overline{C}_B,\overline{C}_P)\right),\rho\left(T_P(C_B,C_P),T_P(\overline{C}_B,\overline{C}_P)\right)\right\}\leq L\rho\left((C_B,C_P),\left(\overline{C}_B,\overline{C}_P\right)\right),$$

where

$$L=n\max\left\{e^{\mu T}\frac{(k_1\hat{C}_B+k_2\hat{C}_P)}{2\mu};\hat{C}_B e^{-\mu\frac{\varepsilon}{V_P}}+e^{\mu T}\frac{k_1\hat{C}_B+k_2\hat{C}_P}{2\mu}\right\}<1.$$

In this way we have shown that T is a contractive operator. The fixed point of T is a solution of the mixed problem above formulated.

The main Theorem is thus proved.

IV. CONCLUSION REMARKS

Here we show the process of obtaining of successive approximations. We can begin with the first approximation choosing the constants $C_B^{(0)}(x,t) = C_{B0}$, $C_P^{(0)}(x,t) = C_{P0}$. Then

$$C_B^{(m+1)}(x,t) = C_{B0} + \int_{t-\frac{x}{V_B}}^{t} k_1\left(C_B^{(m)}(\chi_B(x,t),s)\right)^n - k_2\left(C_P^{(m)}(\chi_B(x,t),s)\right)^n ds, \quad (x,t) \in \Pi_{0B}$$

$$C_P^{(m+1)}(\Lambda,t) = \kappa(t) - C_B^{(m)}(\Lambda, \chi_P(x,t)) + \int_{t-\frac{H-x}{V_P}}^{t} \left(-k_1\left(C_B^{(m)}(\chi_P(x,t),s)\right)^n + k_2\left(C_P^{(n)}(\chi_P(x,s),t)\right)^n\right) ds,$$

$(x,t) \in \Pi_{0P}$.

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