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Multiagent Control with Reinforced-CloudConsensus

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Abstract
This paper addresses a consensus problem for a network of autonomous multi-agents with linear dynamics and bounded disturbances under cloud-reinforced control strategy. Consensus is achieved by having the agents asynchronously upload (download) data to (from) a shared warehouse, rather than directly exchanging data with other agents. Well posedness of the closed-loop system is demonstrated by showing that there exists a lower bound for the time interval between two consecutive agent accesses to the warehouse.
Index Terms: *Networked control system, Cloud-reinforced control, Multiagent systems, Consensus.*

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I. INTRODUCTION

Nowadays, heterogeneous and geographically distributed devices can be connected with cheap and reliable wireless technologies. In turn, this motivated the study of networked control systems (NCS). Specifically, consensus algorithms have been investigated [1], [2], [3] and tailored for platooning and formation control [4], [5]. On the other hand, several recent papers consider the possibility of distributed wireless sensors and actuators in NCS, devoting the research effort in coordinating the data packets and guaranteeing desired performances [8], [9]. Motivated by the need of saving hardware and software resources and reducing the transmitted data, event-triggered and self-triggered control strategies have been introduced [10]–[12], and later extended to multi-agent coordination [13]–[15]. These strategies do not require a fixed sampling period for the feedback loop, but the control input is updated only when a specific condition related to the stability or to some control performance is violated

Coordination of networked multi-agent systems is the subject of a large body of research work, because such systems constitute a suitable model for a large number of phenomena in robotics, biology, physics, and social sciences [28]–[30]. In most realistic scenarios, the agents in a multi-agent system have limited communication capabilities. This happens, for example, when they communicate over a wireless medium, which is a shared resource with limited throughput capacity. In some cases, inter-agent communication is completely or almost completely interdicted. This challenge arises, for example, in the coordination of a fleet of autonomous underwater vehicles (AUVs) [31]. Because of their severely limited communication, sensing, and localization capabilities, underwater vehicles are virtually isolated systems. Underwater communication and positioning may be implemented by means of battery-powered acoustic modems, but such devices are expensive, limited in range, and power- hungry. Inertial sensors for underwater positioning are prohibitively expensive in most practical scenarios. Moreover, GPS is not available underwater, and a vehicle needs to surface whenever it needs to get a position fix [32].

When such limitations arise, coordination strategies that rely on continuous information exchanges among the agents cannot be implemented. To address this challenge, the idea of triggered control [33], [34] has been tailored to multi-agent systems. Triggered control was introduced to limit the amount of communication within the parts of a feedback control system (plant, sensors, actuators). In the context of multi-agent systems, triggered control is used to limit the communication among different agents. Various flavors of triggered control have been applied to multi-agent systems: namely, with event-triggered control, inter-agent communication is triggered when a given state condition is satisfied [35]; with self-triggered control, the agents schedule when to exchange data in a recursive fashion, so that there is no need to monitor a condition between consecutive communication instances [36]. However, even these triggered control schemes require that the agents exchange information, and, therefore, are not well-suited for those scenarios where direct inter-agent communication is interdicted. The use of a sharedinformation repository in multi-agent control is subject to recent, but growing, research attention. In [37], the authors employ asynchronous communication with a base station to address a multi-agent coverage control problem. In [38], the authors present a cloud-supported approach to multi-agent optimization.

In this paper, we present a multi-agent control scheme where inter-agent communication is completely replaced by the use of a shared information warehouse hosted on a cloud. Differently than in traditional event-triggered coordination schemes, here each agent schedules its own cloud accesses independently, and does not need to be alert for information broadcast by other agents. When an agent accesses the warehouse, it uploads some data packets, and downloads other packets that were previously deposited by other agents. Therefore, each agent receives only outdated information about the state of the other agents. The control law and the rule for scheduling the cloud accesses are designed to guarantee that the closed-loop system is well-posed and achieves the control objective, in spite of only using outdated information.

II. ELEMENTS OF GRAPH THEORY

In this section, some preliminary knowledge of graph theory [50] is introduced to facilitate the subsequent analysis.

Let $G=(V,E,A)$ be a directed graph of order n, where $V=\{s_1,\dots,s_n\}$ is the set of nodes, $E\subseteq V\times V$ is the set of edges, and $A=[a_{ij}]\in\Re^{n\times n}$ is a weighted adjacency matrix. The node indexes belong to a finite index set $I=\{1,2,\dots,n\}$. An edge of G is denoted by $e_{ij}=(s_i,s_j)$, where the first element s_i of the e_{ij} is said to be the tail of the edge and the other s_j to be the head. The adjacency elements associated with the edges are positive, that is $e_{ij}\in E\Leftrightarrow a_{ij}>0$. If a directed graph has the property that $a_{ij}=a_{ji}$ for any $i,j\in I$, the directed graph is called undirected. The Laplacian with the directed graph is defined as $L=\Delta-A\in\Re^{n\times n}$, where $\Delta=[\Delta_{ij}]$ is a diagonal matrix with $\Delta_{ii}=\sum_{j=1}^{n}a_{ij}$. An important fact of L is that all the row sums of L are zero and thus 1 is an eigenvector of L associated with the zero eigenvalue. The set of neighbors of node s_i is denoted by $N_i=\{s_j\in V:(s_i,s_j)\in E\}$. A directed path is a sequence of ordered edges of the form $(s_{i1},s_{i2}),(s_{i2},s_{i3}),\dots$, where $s_{ij}\in V$ in a directed graph. A directed graph is said to be strongly connected, if there is a directed path from every node to every other node. Moreover, a directed graph is said to have spanning trees, if there exists a node such that there is a directed path from every other node to this node.

A. *Basic results*

Lemma 1 ([58]): If the graph G *has a spanning tree, then its Laplacian* L *has the following properties:*

1. *Zero is a simple eigenvalue of* L*, and* 1_n *is the corresponding eigenvector, that is* $L1_n=0$
2. *The rest* $n-1$ *eigenvalues all have positive real parts. In particular, if the graph* G *is undirected, then all these eigenvalues are positive and real.*

Lemma 2 ([52]): Consider a directed graph $\mathcal{G}$*. Let* $\mathcal{D}\in\Re^{n\times|\mathcal{E}|}$ *be the 01-matrix with rows and columns indexed by the nodes and edges of* $\mathcal{G}$*, and* $\mathcal{E}\in\Re^{|\mathcal{E}|\times n}$ *be the 01-matrix with rows and columns indexed by the edges and nodes of* G*, such that*

$$\mathcal{D}_{uf}=\begin{cases}1 & \text{if the node } u \text{ is the tail of the edge f}\\ 0 & \text{otherwise}\end{cases}$$

$$\mathcal{E}_{fu}=\begin{cases}1 & \text{if the node } u \text{ is the head of the edge f}\\ 0 & \text{otherwise}\end{cases}$$

where $|\mathcal{E}|$ is the number of the edges. Let $Q=\operatorname{diag}\{q_1,q_2,\dots,q_{|E|}\}$, where $q_p(p=1,\dots,|\mathcal{E}|)$ is the weight of the pth edge of G (i.e. the value of the adjacency matrix on the pth edge). Then the Laplacian of $\mathcal{G}$ can be transformed into $\mathbb{L}=\mathcal{D}Q(\mathcal{D}^T-\mathcal{E})$.

B. *Laplacian spectrum of graphs*

This section is a concise review of the relationship between the eigenvalues of a Laplacian matrix and the topology of the associated graph.We refer the reader to [49] for a comprehensive treatment of the topic. We list a collection of properties associated with undirected graph Laplacians and adjacency matrices, which will be used in subsequent sections of the paper.

A graph $\mathcal{G}$ is defined as

$$\mathcal{G}=(\mathcal{V},\mathcal{A}) \tag{1}$$

where $\mathcal{V}$ is the set of nodes (or vertices) $\mathcal{V}=\{1,\dots,N\}$ and $\mathcal{A}\subseteq\mathcal{V}\times\mathcal{V}$ the set of edges (i,j) with $i\in\mathcal{V},j\in\mathcal{V}$. The degree d_j of a graph vertex j is the number of edges which start from j. Let $d_{\max}(\mathcal{G})$ denote the maximum vertex degree of the graph $\mathcal{G}$.

C. *Properties of adjacency matrix*

We denote $\mathbf{A}(\mathcal{G})$ by the (0,1) adjacency matrix of the graph $\mathcal{G}$. Let $\mathbf{A}_{ij}\in\mathbb{R}$ be its i,j element, then $\mathbf{A}_{i,i}=0,\forall i=1,\dots,N$, $\mathbf{A}_{i,j}=0$ if $(i,j)\notin\mathcal{A}$ and $\mathbf{A}_{i,j}=1$ if $(i,j)\in\mathcal{A},\forall i,j=1,\dots,N,i\neq j$. We will focus on undirected graphs, for which the adjacency matrix is symmetric.

Let $\mathcal{S}(\mathbf{A}(\mathcal{G})) = \{\lambda_1(\mathbf{A}(\mathcal{G})), \dots, \lambda_N(\mathbf{A}(\mathcal{G}))\}$ be the spectrum of the adjacency matrix associated with an undirected graph $\mathcal{G}$ arranged in non-decreasing semi-order.

- **Property 1:** $\boldsymbol{\lambda_N(\mathbf{A}(\mathcal{G})) \leq d_{\max}(\mathcal{G})}$.

This property together with Proposition 1 implies

- **Property 2:** $\boldsymbol{\gamma_i \geq 0, \forall \gamma_i \in \mathcal{S}(d_{\max} I_N - \mathbf{A})}$.

We define the Laplacian matrix of a graph $\mathcal{G}$ in the following way:

$$L(\mathcal{G}) = \mathbf{D}(\mathcal{G}) - \mathbf{A}(\mathcal{G}) \tag{2}$$

where $\mathbf{D}(\mathcal{G})$ is the diagonal matrix of vertex degrees d_i (also called the valence matrix). Eigenvalues of Laplacian matrices have been widely studied by graph theorists. Their properties are strongly related to the structural properties of their associated graphs. Every Laplacian matrix is a singular matrix. By Gershgorin's theorem [51], the real part of each nonzero eigenvalue of $L(\mathcal{G})$ is strictly positive.

For undirected graphs, $L(\mathcal{G})$ is a symmetric, positive semidefinite matrix, which has only real eigenvalues. Let $\mathcal{S}(L(\mathcal{G})) = \{\lambda_1(L(\mathcal{G})), \dots, \lambda_N(L(\mathcal{G}))\}$ be the spectrum of the Laplacian matrix L associated with an undirected graph $\mathcal{G}$ arranged in non-decreasing semi-order. Then,

- **Property 3:**

1. $\lambda_1(L(\mathcal{G})) = 0$ with corresponding eigenvector of all ones, and $\lambda_2(L(\mathcal{G}))$ iff $\mathcal{G}$ is connected. In fact, the multiplicity of 0 as an eigenvalue of $L(\mathcal{G})$ is equal to the number of connected components of $\mathcal{G}$.
2. The modulus of $\lambda_i(L(\mathcal{G})), i = 1, \dots, N$ is less then N.

The second smallest Laplacian eigenvalue $\lambda_2(L(\mathcal{G}))$ of graphs is probably the most important information contained in the spectrum of a graph. This eigenvalue, called the algebraic connectivity of the graph, is related to several important graph invariants, and it has been extensively investigated.

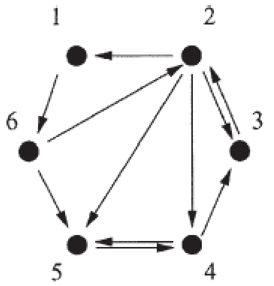
Let $L(\mathcal{G})$ be the Laplacian of a graph $\mathcal{G}$ with N vertices and with maximal vertex degree $d_{\max}(\mathcal{G})$. Then properties of $\lambda_2(L(\mathcal{G}))$ include

- **Property 4:**

1. $\lambda_2(L(\mathcal{G})) \leq (N/(N-1)) \min\{d(v), v \in \mathcal{V}\}$;
2. $\lambda_2(L(\mathcal{G})) \leq v(\mathcal{G}) \leq \eta(\mathcal{G})$;
3. $\lambda_2(L(\mathcal{G})) \geq 2\eta(\mathcal{G})(1 - \cos(\pi/N))$;
4. $\lambda_2(L(\mathcal{G})) \geq 2\left(\cos\frac{\pi}{N} - \cos 2\frac{\pi}{N}\right)\eta(\mathcal{G}) - 2\cos\frac{\pi}{N}\left(1 - \cos\frac{\pi}{N}\right) d_{\max}(\mathcal{G})$

where $v(\mathcal{G})$ is the vertex connectivity of the graph $\mathcal{G}$ (the size of a smallest set of vertices whose removal renders $\mathcal{G}$ disconnected) and $\eta(\mathcal{G})$ is the edge connectivity of the graph $\mathcal{G}$ (the size of a smallest set of edges whose removal renders $\mathcal{G}$ disconnected) [53].

Further relationships between the graph topology and Laplacian eigenvalue locations are discussed in [55] for undirected graphs. Spectral characterization of Laplacian matrices for directed graphs can be found in [51], see also Fig. 1



$$L = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & -1 \\ -\frac{1}{4} & 1 & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 1 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & -\frac{1}{2} & 0 & 0 & -\frac{1}{2} & 1 \end{pmatrix}$$

Fig. 1. Sample graph and Laplacian

A lemma about Laplacian L associated with a balanced digraph G is given hereafter:

Lemma 3: If G is balanced, then there exists a unitary matrix

$$V = \begin{bmatrix} \frac{1}{\sqrt{n}} & \cdot & \cdots & \cdot \\ \frac{1}{\sqrt{n}} & \cdot & \cdots & \cdot \\ \vdots & \vdots & & \vdots \\ \frac{1}{\sqrt{n}} & \cdot & \cdots & \cdot \end{bmatrix} \in C^{m \times n} \tag{3}$$

such that

$$V^* L V = \begin{bmatrix} 0 & \\ & H \end{bmatrix} = \Lambda \in C^{n \times n},\ H \in C^{(n-1)\times(n-1)} \tag{4}$$

Moreover, if G has a globally reachable node, $H + H^*$ is positive definite.
Proof: Let $V = [\zeta_1, \zeta_2, \ldots, \zeta_n]$ be a unitary matrix where $\zeta_i \in C^n (i = 1, \ldots, n)$ are the column vectors of V and

$$\zeta_1 = (1/\sqrt{n})1 = (1/\sqrt{n}, 1/\sqrt{n}, \ldots, 1/\sqrt{n})^T$$

Notice that if G is balanced, it implies that $\zeta_1^* L = 0$. Then we have

$$\begin{aligned} V^*LV &= V^*L[\zeta_1, \zeta_2, \ldots, \zeta_n] \\ &= \begin{bmatrix} \zeta_1^* \\ \zeta_2^* \\ \vdots \\ \zeta_n^* \end{bmatrix} [0_n, L\zeta_2, \ldots, L\zeta_n] \\ &= \begin{bmatrix} 0 & 0_{n-2}^T \\ \cdot & H \end{bmatrix} \end{aligned}$$

Furthermore, if G has a globally reachable node, then $L + L^T$ is positive semi-definite, see Theorem 7 in [54]. Hence, $V^*(L + L^T)V$ is also positive semidefinite. From the results of [56] we know that 'zero' is a simple eigenvalue of L and, therefore, $H + H^*$ is positive definite.
As closing remarks, the Laplacian matrix satisfies the property $L = CC^T$. It is well-known fact that this property holds regardless of the choice of the orientation of $\mathcal{G}$. Let x_i denote a scalar real value assigned to v_i. Then $x = [x_1, \ldots, x_n]^T$ denotes the state of the graph $\mathcal{G}$. We define the Laplacian Potential of the graph as follows

$$\Psi_{\mathcal{G}}(x) = \frac{1}{2} x^T L x \qquad (5)$$

From this definition, the following property of the Laplacian potential of the graph follows:
Lemma 4: [60] The Laplacian potential of a graph is positive definite and satisfies the following identity:

$$x^T L x = \sum_{j \in \mathbb{N}_i} \left(x_j(t) - x_i(t)\right)^2 \qquad (6)$$

Moreover, given a connected graph, $\Psi_{\mathcal{G}}(x) = 0$ *if and only if* $x_i = x_j, \forall i, j$.
It follows from Lemma 4, the Laplacian potential of the graph $\Psi_{\mathcal{G}}(x)$ is a measure of the total disagreement among all nodes. If at least two neighboring nodes of $\Psi_{\mathcal{G}}$ disagree, then $\Phi_{\mathcal{G}} > 0$. Hence, minimizing $\Psi_{\mathcal{G}}$ is equivalent to reaching a consensus which signifies a fundamental key in the design of consensus protocols.
Remark 1: It well know from [57] that for a connected graph that is undirected, the following well-known property holds [50]:

$$\min_{x \neq 0 \, 1^t x = 0} \frac{x^t L x}{\|x\|^2} = \lambda_2(L) \qquad (7)$$

The proof follows from a special case of Courant-Fischer Theorem in [61]. A connection between $\lambda_2(\hat{L})$ with $\hat{L} = \frac{1}{2}(L + L^t)$, called the Fiedler eigenvalue of $(\hat{L})$[62] and the performance (that is, worst case speed of convergence) of protocol (.) on digraphs is established in [59].

D. Notation

Throughout this paper, $\mathfrak{R}^n$ is used to denote the n-dimensional Euclidean space equipped with $\|.\|$, the standard L_2 norm on vectors or their induced norms on matrices and $\mathfrak{R}^{m \times n}$ is the set of all $m \times n$ real matrices. Let I_r be the unit matrix of order r. The superscript ' T ' denotes matrix transposition and ' $\cdot$ ' denotes the transpose of corresponding elements introduced by symmetry. $X > 0$ means that X is real symmetric and positive definite; Moreover, $X > Y$ means $X - Y > 0$. Given a matrix W, let $\rho(W)$ denote its spectral radius. For any positive integer N, let $\mathbb{N} = \{1, \ldots, N\}$. diag $(W_1, \ldots, W_N)$ is a block diagonal matrix with main diagonal block matrices $W_j, j \in \mathbb{N}$ and the off-diagonal blocks are zero matrices. The Kronecker product [50] of $A \in \mathfrak{R}^{p \times q} := [a_{ij}]$ and $B \in \mathfrak{R}^{m \times n}$ is denoted by $A \otimes B$ and is a $pm \times qn$ matrix defined by

$$A \otimes B := [a_{ij} B]$$

The Kronecker product further facilitates the manipulation of matrices by the following expansion properties

1. $(A \otimes B)(C \otimes D) = AC \otimes BD$,
2. $(A \otimes B)^T = A^T \otimes B^T$,
3. Let $A \in \mathfrak{R}^{r \times s}$ and $B \in \mathfrak{R}^{N \times N}$. Then $(I_N \otimes A)(B \otimes I_s) = (B \otimes I_r)(I_N \otimes A) = B \otimes A$

E. Communication Graph

The topology of a communication network can be expressed by a graph, either directed or undirected, according to whether the information flow is unidirectional or bidirectional [50]. A weighted directed graph (digraph) $G = (V, E, A)$ be a directed graph of order N, where $V = \{v_1, \ldots, v_N\}$ is the set of nodes, $E \subseteq V \times V$ is the set of edges, and $A = [a_{ij}] \in \mathfrak{R}^{N \times N}$ is a weighted adjacency matrix. The node indexes belong to a finite index set $I = \{1, 2, \ldots, N\}$. An edge of G is denoted by $e_{ij} = (v_i, v_j)$, where the first element v_i of the v_{ij} is said to be the tail of the edge and the other v_j to be the head. The adjacency elements associated with the edges are positive, that is $e_{ij} \in E \Leftrightarrow a_{ij} > 0$. If a directed graph has the property that $a_{ij} = a_{ii}$ for any $i, j \in I$, the directed graph is called undirected. The set of neighbors of node v_i is denoted by $\mathbb{N}_i = \{v_j \in \mathbb{V}: (v_i, v_j) \in E\}$, which is the index set of the agents from which the ith agent can obtain necessary information. The Laplacian with the directed graph is defined as $L = [\ell_{ij}]_{N\times N}$, where

$$\ell_{ij} = \begin{cases} -1 & \text{if } j \in \mathbb{N}_i \\ |\mathbb{N}_i| & \text{if } j = i \\ 0 & \text{otherwise} \end{cases}$$

where $\mathbb{N}_i$ denotes the number of neighbors of the i th agent (the in-degree of agent i). It turns out that $L = \Lambda - A \in \Re^{n\times n}$, where $\Lambda = [\Lambda_{ij}]$ is a diagonal matrix with $\Lambda_{ii} = \sum_{j=1}^{n} a_{ij}$. An important fact of L is that all the row sums of L are zero and thus $1_n = [1,1,\ldots,1]$ is an eigenvector of L associated with the zero eigenvalue.
Lemma 5: Given integers n, N and $A \in \Re^{n\times n}, R \in \Re^{N\times N}$. Let $A_0 = I_N \otimes A$ and $R_0 = R \otimes I_n$. Then $R_0 A_0 = A_0 R_0$. Proof: Using the expansion properties, we obtain

$$\begin{aligned} R_0 A_0 &= (R \otimes I_n)(I_N \otimes A) = (R I_N) \otimes (I_n A) \\ &= (I_N R) \otimes (A I_n) = (I_N \otimes A)(R \otimes I_n) = A_0 R_0 \end{aligned}$$

III. DISTRIBUTED EVENT-TRIGGERED TRACKING CONTROL

Recently some great advances have been achieved in cooperative control of multiagent systems. The research focus is mainly on communication environments which consequently require distributed control design. To this day, some control techniques have been proposed according to different communication conditions, such as time-varying networks [86], [95], subject to measurement noise [89], [92], time delays [88], [50], or disturbances [94], [98].

A future control design may equip agents with embedded micro-processors to collect information from neighboring agents so as to update the controller according to some pre-designed rules. Motivated by this observation, some protocols were proposed to deal with distributed algorithms of communication and controller actuation scheduling [84], [97], [99]. Since micro-processors are generally resource- and energy-limited, an event-triggered control was designed based on measurement errors for execution in [97]. A timing issue was investigated through the use of a distributed event-triggered feedback scheme in networked control systems in [99]. Very recently, some distributed event-triggered control strategies were proposed for multi-agent systems [83], [84], [93]. All these control design methods possess a common characteristics that the controller is updated only when the measurement error magnitude exceeds a certain threshold.

In [84] and [83], centralized and decentralized event-triggered multiagent control protocols were developed for a first-order agreement problem, which were proven to be input-to-state stable (ISS) [91]. The centralized cooperative controller was actuated according to a global event-trigger rule while the decentralized one was updated at a sequence of separate eventtimes encoded by a local trigger function for each agent. Furthermore, a centralized event-triggered cooperative control was constructed for higher-dimensional multi-agent consensus with a weighted topology in [93], an event-triggered cooperative control was proposed for first-order discrete-time multi-agent systems in [85], and a neighbor-based tracking control together with a distributed estimation was proposed for leader-follower multi-agent systems in [87].

In what follows, we follow [81] and consider a distributed event-triggered tracking control problem for leader-follower multi-agent systems in a fixed directed network topology with partial measurements and communication delays. In collective coordination of a group of autonomous agents, the leader-follower problem has been considered for tracking a single or multiple leaders in [82], [87], [89], [90], [96]. In reality, some state information of the leader cannot be measured, therefore a decentralized observer design plays a key role in cooperative control of leader-follower multiagent systems. Within this context, an observer-based dynamic tracking control was proposed to estimate the unmeasurable state (i. e., velocity) of an active leader in [87] by collecting real-time measurements from neighbors. In this paper, inspired by the event-triggered scheduling strategy in multi-agent systems, we consider a dynamic tracking problem with event-triggered strategy involved in the control update. During the event-triggered tracking control process, we assume that every follower agent broadcasts its state information only if needed, which requires the follower agent to update its state only if some measure of its state error is above a specified threshold.

It is noted in the literature about event-triggered control of multi-agent systems that, event-triggered cooperative controllers often keep constant between two consecutive broadcasts. However, in this paper we concern with the scenario of an independent active leader, who does not need the event-triggered control updates. Thus, a more sophisticated event-triggered strategy needs to be developed to continuously update every agent's partial control input, subject to its local computational resources availability. We adopt a decentralized event-triggered strategy to update the local controllers, and finally take into account the communication delays in the tracking control design.

IV. PROBLEM DESCRIPTION

The multi-agent system under study is a group of n follower-agents (called followers for simplicity and labeled $1, \ldots, n$) and one active leader-agent (called leader and labeled 0). The followers are moving based on the information exchange in their individual neighborhood while the leader is self-active hence moving independently.

Thus, the information flow in the leader-follower multi-agent system can be conveniently described by a directed graph $\bar{G}$. We recall the information about graph theory from [50] or Section II.

The dynamics of the i th follower is assumed to be a first-order linear system:

$$\dot{x}_i(t) = u_i(t),\ i = 1, \ldots., n \tag{8}$$

where $x_i(t) \in \Re^l$ and $u_i(t) \in \Re^l$ are, respectively, the state and the control input. The active leader is described by a second-order linear system with a partially unknown acceleration:

$$\begin{aligned} \dot{x}_0(t) &= v_0(t), \\ \dot{v}_0(t) &= u_0(t) + \delta(t) \\ y_0(t) &= x_0(t) \end{aligned} \tag{9}$$

where $x_0(t) \in \Re^l, v_0(t) \in \Re^l$ and $u_0(t) \in \Re^l$ are, respectively, the position, velocity and acceleration, the disturbance $\delta(t) \in \Re^l$ is bounded with an upper bound δ, and $y_0(t)$ is the only measured output.

Since only the position of the leader can be measured, each follower has to collect information from its neighbors and estimate the leader's velocity during the motion process. In [87], a distributed observer-based dynamic tracking control was proposed for each follower :

$$\begin{aligned} \dot{v}_i(t) &= u_0(t) - \gamma k \left[\sum_{j \in \mathbb{N}_i} a_{ij}(x_i - x_j) + a_{i0}(x_i - x_0) \right] \\ u_i(t) &= v_i(t) - k \left[\sum_{j \in \mathbb{N}_i} a_{ij}(x_i - x_j) + a_{i0}(x_i - x_0) \right] \end{aligned} \tag{10}$$

where $v_i(t)$ is the 'estimate' of the leader's velocity $v_0(t)$ and a_{i0} is the leader's adjacency coefficient. The dynamic tracking control (10) assumes that the relative position measurements $(x_i - x_0)$ are transmitted in continuous time.

In practice, however, communication (especially wireless communication) takes place over digital networks therefore information is transmitted at discrete time instants. When the follower finds that a local "error" signal exceeds a given threshold, it broadcasts its state information to all neighboring agents. Under this scenario, the event-triggered dynamic tracking control is more preferable than that proposed in (10).

In the leader-follower problem under investigation, the active leader is independent and needs not broadcast its information in any event-triggered fashion. However, follower $i's$ control, $u_i(t)$, has to be designed based on the latest states received from its neighboring followers and also the sate $x_0(t)$ if it is linked to the leader. Therefore, a new control protocol needs to be designed to solve the leader-following problem with an event-triggered scheduling strategy. The event-triggered tracking problem is said to be solved if one can find a distributed event-triggered control strategy such that

$$\|(x_i - x_j)\| \leq \xi,\ i = 1, \ldots., n \tag{11}$$

for some constant $\xi = \xi(\bar{\delta})$ as $t \to \infty$.

A. Control design scheme

In consensus control, it turns out that typical information available for a follower is its relative positions with the neighbors. It is usually assumed that the relative-position measurement

$$y_{ij}(t) = x_i(t) - x_j(t) \tag{12}$$

is performed in continuous time, which implicitly implies that the multi-agent communication network bandwidth is unlimited or every agent has abundant energy.

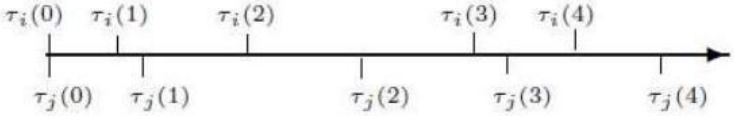


Fig. 2. The event times for follower i and follower j

However, when followers transmit their state information in discrete time, distributed tracking control needs to be redefined to take into account event-triggered strategies. In order to model the event-triggers for followers, assume that there are n monotone increasing sequences of event times

$$\tau_i(s)(s = 0,1, \cdots\cdots, i = 1, \cdots, n)$$

Let $\hat{x}_i(t) = x_i(\tau_i(s)), t \in [\tau_i(s), \tau_i(s+1))$, be the measured state of follower i. The measured relative-position measurements $y_{ij}(t)$ depend on the measured states $\hat{x}_i(t)$ and $\hat{x}_j(t), j \in \mathbb{N}_i$, that is,

$$\hat{y}_{ij}(t) = \hat{x}_i(t) - \hat{x}_j(t),\ i, j = 1, \dots, n \tag{13}$$

It should be noted that the event times $\tau_i(s)$ are mutually independent among followers and may take different values, as illustrated by Fig. 2.

Furthermore, if the communication between agent i and agent j (or the leader) has a time-varying delay $r(t)$, then the measured relative-position measurement is described by

$$\hat{y}_{ij}(t - r(t)) = \hat{x}_i(t - r(t)) - \hat{x}_j(t(t) - r) \tag{14}$$

where $r(t)$ is a continuously differentiable function satisfying $0 < r(t) < \bar{r} < \infty$.

Due to unavailable measurement of the leader's velocity $v_0(t)$, each follower can have an estimate $v_i(t)$ by fusing the information obtained from its neighbors. When communication delay is not considered, the velocity estimate $v_i(t)$ is given with the measurements $\hat{y}_{ij}(t)$ and $y_{i0}(t)$, as follows:

$$\dot{v}_i(t) = u_0(t) - \gamma k\left[\textstyle\sum_{j\in\mathbb{N}_i} a_{ij}\hat{y}_{ij}(t) + a_{i0}y_{i0}(t)\right] \tag{15}$$

where a_{ij} denotes the adjacency coefficient between follower i and follower j, constant $0 < \gamma < 1$, and the gain k is to be designed. Moreover, an event-triggered tracking control is designed as follows:

$$u_i(t) = v_i(t) - k\left[\textstyle\sum_{j\in\mathbb{N}_i} a_{ij}\hat{y}_{ij}(t) + a_{i0}y_{i0}(t)\right] \tag{16}$$

where the gain k is the same as above. It is noted that both the velocity estimate $v_i(t)$ and the control input $u_i(t)$ use the broadcasted measurements $\hat{y}_{ij}(t)$ from neighboring followers and the continuous-time measurement $y_{i0}(t)$ from the leader.

When communication delay is involved in the multi-agent coordination, a distributed event-triggered tracking control with time delays can be similarly formulated, as follows:

$$\begin{aligned} u_i(t) &= v_i(t) - k\left[\sum_{j\in\mathbb{N}_i} a_{ij}\hat{y}_{ij}(t-r) + a_{i0}y_{i0}(t-r)\right] \\ \dot{v}_i(t) &= u_0(t) - \gamma k\left[\sum_{j\in\mathbb{N}_i} a_{ij}\hat{y}_{ij}(t-r) + a_{i0}y_{i0}(t-r)\right] \end{aligned} \tag{17}$$

Next, we analyze the convergence of the tracking errors for all followers under distributed event-triggered control in both cases with and without communication delays

B. Without communication delays

For simplicity in exposition, we define the error term

$$e_i(t) = \hat{x}_i(t) - x_i(t) = \hat{x}_i(\tau_i(s)) - x_i(t),\ t \in [\tau_i(s), \tau_i(s+1))$$

The event-time $\tau_i(s)$ is implicitly defined by an event-trigger, $f_i(e_i(t), e_j(t) \mid j \in \mathbb{N}_i) = 0$, which will be given below. Hence, $\hat{x}_i(t) = e_i(t) + x_i(t)$.

With this variable change, the control (16) together with the velocity estimation (15) is applied to system (8), which yields the following closed-loop system:

$$\begin{aligned} \dot{x} &= v - k(L+B)x + kB\mathbf{1}x_0 - kLe, \\ \dot{v} &= u_0\mathbf{1} - \gamma k(L+B)x + \gamma kB\mathbf{1}x_0 - \gamma kLe \end{aligned} \tag{18}$$

where

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \in \Re^n, v = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} \in \Re^n, e = \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_n \end{bmatrix} \in \Re^n$$

respectively, denote the position, velocity estimation, measurement error of the leader-follower multi-agent system, $L = D?A \in \Re^{n\times n}, A = [a_{ij}] \in \Re^{n\times n}$, and $D \in \Re^{n\times n}$ are, respectively, the Laplacian matrix, adjacency matrix and degree matrix of the directed subgraph $G.B = \text{diag}\{a_{10}, \dots, a_{n0}\}$ is a diagonal matrix representing the leader-follower adjacency relationship, and $1 = \text{col}\,[1, \dots, 1] \in \Re^n$.

From the algebraic graph theory [50], it is known that L always has a zero eigenvalue associated with the right eigenvector 1. Moreover, if the subgraph G is balanced, L has a zero eigenvalue associated with the left eigenvector 1 . This leads to

$$-(L+B)x + B1x_0 = -(L+B)(x - x_0 1) := -H(x - x_0\mathbf{1})$$

It follows from Section II that

- vertex 0 is a globally reachable vertex of the directed graph $\bar{\mathcal{G}}$ and if its subgraph $\mathcal{G}$ is balanced, then
  $$\lambda_* = \text{m}\ \{\lambda\text{: eigenvalues of } (H + H^t)\} > 0 \tag{19}$$
- ? H is a stable matrix whose eigenvalues have negative real-parts;
- $\mathcal{G}$ is balanced and $(H + H^t)$ is symmetric positive-definite matrix

Proceeding to examine the stability of system (18), we introduce the change of variables:

$$\bar{x} = x - x_0 1,\ \bar{v} = v - v_0 1 \tag{20}$$

so that system (18) is expressed by

$$\begin{aligned} \dot{\bar{x}} &= \bar{v} - kH\bar{x} - kLe, \\ \dot{\bar{v}} &= -\gamma kH\bar{x} - \gamma kLe - 1 \otimes \delta \end{aligned} \tag{21}$$

or in compact form:

$$\begin{aligned} &\dot{\xi} = \Xi\xi + \Gamma e + d \\ &\xi = [\bar{x}^t, \bar{v}]^t, \Xi = \begin{bmatrix} -kH & I \\ -\gamma kH & 0 \end{bmatrix}, \Gamma = \begin{bmatrix} -kL \\ -\gamma kL \end{bmatrix}, d = \begin{bmatrix} 0 \\ 1 \otimes \delta \end{bmatrix} \end{aligned} \tag{22}$$

Define a candidate ISS Lyapunov function

$$V(\xi) = \xi^t P\xi,\ P = \begin{bmatrix} I & -\gamma I \\ \cdot & I \end{bmatrix}, 0 < \gamma < 1$$

The main result is established by the following theorem

Theorem 1: Assume that vertex 0 is a globally reachable vertex of the directed graph $\bar{\mathcal{G}}$, if its subgraph $\mathcal{G}$ is balanced and the gain k satisfies

$$k > \frac{1}{2\gamma(1-\gamma^2)\lambda_*} \tag{23}$$

Then, control (16) and estimation (15) solve the event-triggered tracking problem. Moreover, if the disturbance bound $\bar{\delta} = 0$, then $\lim_{t\to\infty} ||\xi(t)|| = 0$.

Proof: Computing the derivative $\dot{V}(\xi)$ along the solutions of (22) yields

$$\begin{aligned} \dot{V}(\xi) &= \xi^t[P\Xi + \Xi^t P]\xi + 2\xi^t P\Gamma e + 2\xi^t Pd \\ &= -\xi^t Q\xi + 2\xi^t P\Gamma e + 2\xi^t Pd \end{aligned} \tag{24}$$

where

$$Q = \begin{bmatrix} k(1-\gamma^2)(H + H^t) & -I \\ \cdot & 2\gamma I \end{bmatrix}$$

It is easy to see with the help of Schur complements that $Q > 0$ if k satisfies (23). Further computations show that the minimum eigenvalue of Q is given by

$$\sigma_* = \frac{1}{2}\left[(1-\gamma^2)k\lambda_* + 2\gamma - \sqrt{[(1-\gamma^2)k\lambda_* - 2\gamma] + 4}\right. \tag{25}$$

When k satisfies (23), then $\sigma_* > 0$. Since the eigenvalues of P are $1+\gamma, 1-\gamma$, it follows that

$$(1-\gamma) \parallel \xi \parallel^2 \leq V(\xi) \leq (1+\gamma) \parallel \xi \parallel^2 \tag{26}$$

Taking advantage of (25) and (26), we get along the solutions of (22) that:

$$\begin{aligned} \dot{V}(\xi) &\leq -\sigma_* \parallel \xi \parallel^2 + 2\xi^t P\Gamma e + 2\xi^t Pd \\ &\leq -\sigma_* \parallel \bar{x} \parallel^2 - \sigma_* \parallel \bar{v} \parallel^2 - 2(1-\gamma^2)k \sum_i \sum_{j\in\mathbb{N}_i} \bar{x}(e_i - e_j) + 2(1+\gamma) \parallel \xi \parallel \bar{\delta} \\ &\leq -\sigma_* \parallel \bar{v} \parallel^2 - \sigma_* \sum_i \left[\|\bar{x}_i\|^2 - \frac{2(1-\gamma^2)k \parallel \bar{x} \parallel}{\sigma_*} \sum_{j\in\mathbb{N}_i} (\|e_i\| - \|e_j\|) \right] + 2(1+\gamma) \parallel \xi \parallel \bar{\delta} \end{aligned} \tag{27}$$

Enforcing the condition

$$\sum_{j\in\mathbb{N}_i} (\|e_i\| + \|e_j\|) \leq \varepsilon \frac{\sigma_* \|\bar{x}_i\|}{2(1-\gamma^2)k},\ 0 < \varepsilon < 1 \tag{28}$$

we have

$$\begin{aligned} \dot{V}(\xi) &\leq -(1-\varepsilon)\sigma_* \parallel \xi \parallel^2 + 2(1+\gamma) \parallel \xi \parallel \bar{\delta} \\ &\leq -\frac{1}{2}(1-\varepsilon)\sigma_* \parallel \bar{x} \parallel^2 + 2\frac{(1+\gamma)^2\bar{\delta}^2}{(1-\varepsilon)\sigma_*} \end{aligned} \tag{29}$$

Thus, for follower i an event-trigger can be defined by

$$f_i(e_i(t), \{e_j(t) \mid j \in \mathbb{N}_i\}) = \sum_{j\in\mathbb{N}_i} (\|e_i\| + \|e_j\|) - \varepsilon \frac{\sigma_* \|\bar{x}\|}{(1-\gamma^2)k} \tag{30}$$

When the event-trigger $f_i(e_i(t), \{e_j(t) \mid j \in \mathbb{N}_i\}) = 0$, condition (28) is enforced. Given the event-trigger (30), then from (26) and (29) we have

$$\dot{V}(\xi) \leq -\frac{(1-\varepsilon)\sigma_*}{2(1+\gamma)} V(\xi) + \frac{2(1+\gamma)^2\bar{\delta}^2}{(1-\varepsilon)\sigma_*} \tag{31}$$

With $t_o = 0$, we obtain

$$V(\xi) \leq e^{-\frac{(1-\varepsilon)\sigma_*}{2(1+\gamma)}t} V(\xi(0)) + \frac{4(1+\gamma)^3\bar{\delta}^2}{(1-\varepsilon)^2\sigma_*^2} \tag{32}$$

which implies

$$\lim_{t\to\infty} \parallel \xi \parallel \leq \psi,\quad \psi^2 = \frac{4(1+\gamma)^3\bar{\delta}^2}{(1-\varepsilon)^2\sigma_*^2}$$

Additionally, if $\bar{\delta} = 0$, then $\lim_{t\to\infty} \parallel \xi \parallel = 0$, which completes the proof.
Remark 2: For simplicity in the exposition, the event-trigger condition (28) can be replaced by a centralized one
$\parallel e \parallel \leq \varepsilon \frac{\sigma_* \parallel \xi \parallel}{2(1-\gamma^2)\parallel L \parallel}$ (33)
Evidently, the trigger condition (33) is conservative, however it helps in simulation experimentation. Suppose that this condition (33) is satisfied and $\bar{\delta} = 0$, then there exists at least one agent for which the next inter-event interval is bounded from below by a time τ_D, determined by

$$\tau_D = \frac{1}{\parallel \Xi \parallel - \parallel \Gamma \parallel} \ln \left[\frac{1+\phi}{1+\frac{\parallel \Gamma \parallel}{\parallel \Xi \parallel}\phi} \right]$$
$$\phi(\tau_D, 0) = \frac{\varepsilon \sigma_*}{2(1-\gamma^2) \parallel L \parallel}$$
$$\frac{\parallel e \parallel}{\parallel \xi \parallel} \leq \phi(t, \phi_o), \phi_o = \phi(0, \phi_o) \quad (34)$$

and $\phi(t, \phi_o)$ is the solution of
$\dot{\phi} == \parallel \Xi \parallel (1+\phi)\left[1+\frac{\parallel\Gamma\parallel}{\parallel\Xi\parallel}\phi\right]$
(35)
C. *With communication delays*
In this case, we take into consideration model (17) along with $\hat{x}_i(t) = e_i(t) + x_i(t)$ and manipulate to obtain:

$$\dot{x}_(t) = v(t) - k(L+B)x(t-r) + kB1x_o(t-r) - kLe(t-r)$$
$$\dot{v}_i(t) = u_0 1 - \gamma k(L+B)x(t-r) + \gamma kB1x_o(t-r) - kLe(t-r) \quad (36)$$

Using the change of variables (20), algebraic manipulations yield a further simplified closed-loop system in the form of time-delayed differential equations:

$$\dot{\bar{x}} = \bar{v} - kH\bar{x}(t-r) - kLe(t-r),$$
$$\dot{\bar{v}} = -\gamma kH\bar{x}(t-r) - \gamma k\,\mathrm{L}e(t-r) - 1 \otimes \delta \quad (37)$$

or in compact form:

$$\dot{\xi} = \Xi_1 \xi(t) + \Xi_2 \xi(t-r) + \Gamma e(t-r) + d$$
$$\xi = [\bar{x}^t, \bar{v}]^t, \Xi_1 = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}, \Xi_2 = \begin{bmatrix} -kH & 0 \\ -\gamma kH & 0 \end{bmatrix},$$
$$\Gamma = \begin{bmatrix} -kL \\ -\gamma kH \end{bmatrix}, d = \begin{bmatrix} 0 \\ 1 \otimes \delta \end{bmatrix} \quad (38)$$

Before proceeding further, the standard theorem of stability must be recalled. The main results can then be readily derived and left for the time being.

V. LYAPUNOV-RAZUMIKHIN THEOREM

Here the idea is based on the following argument: because the future states of the system depend on the current and past states values, the Lyapunov function should become functional- more details in Lyapunov Krasovskii method- which may complicate the condition formulation and the analysis. To avoid using functional; Razumikhin made his theorem, which is, based on formulating Lyapunov functions not functionals. First one should build a Lyapunov function $V(x(t))$ which is zero when $x(t) = 0$ and positive otherwise, then the theorem does not require $\dot{V} < 0$ always but only when the $V(x(t))$ for the current state becomes equals to $\bar{V}$ which is given by

$$\bar{V} = \max_{\theta\in[-\tau,0]} V(x(t+\theta)) \quad (39)$$

the theorem statement is given by [100]:
Suppose f is a functional that takes time t and initial values x_t and gives a vector of n states $\dot{x}$ and u, v, w are class $\mathcal{K}$ functions $u(s)$ and $v(s)$ are positive for $s > 0$ and $u(0) = v(0) = 0, v$ is strictly increasing. If there exists a continuously differentiable function $V: \Re \times \Re^n \to R$ such that

$$u(\parallel x \parallel) \leq V(t,x) \leq v(\parallel x \parallel) \quad (40)$$

and the time derivative of V along the solution $x(t)$ satisfies $\dot{V}(t,x) \leq -w(\parallel x \parallel)$ whenever $V(t+\theta, x(t+\theta)) \leq V(t, x(t))\theta \in [-\tau, 0]$, then the system is uniformly stable
If in addition $w(s) > 0$ for $s > 0$ and there exists a continuous non-decreasing function $p(s) > s$ for $s > 0$ such that $\dot{V}(t,x) \leq -w(\parallel x \parallel)$ whenever $V(t+\theta, x(t+\theta)) \leq p(V(t, x(t)))$
for $\theta \in [-\tau, 0]$, then the system is uniformly asymptotically stable. If in addition $\lim u(s)_{s\to\infty} = \infty$ then the system is globally asymptotically stable.
The argument behind the theorem is like this: $\bar{V}$ is serving as a measure for the V in the interval $t-\tau$ to t then if $V(x(t))$ is less than $\bar{V}$ then it's not necessary that $\dot{V} < 0$, but if $V(x(t))$ becomes equals to $\bar{V}$ then $\dot{V}$ should be

< 0 such that V will not grow.
The procedure can be explained more by the following discussion: consider system and a selected Lyapunov function $V(x)$ which is positive semi-definite. By taking the time derivative of this Lyapunov function we get $\dot{V}$ According to Razumikhin theorem this term does not always need to be negative, but if we added the following term $a\big(V(x) - V(x_t)\big)$ $a > 0$ to V then the term

$$\dot{V} + a\big(V(x) - V(x_t)\big) \quad (41)$$

should always be negative then by looking at this term we find that this condition is satisfied if $\dot{V} < 0$ and $V(x) \leq V(x_t)$ meaning that the system state are not growing in magnitude and it is approaching the origin (stable system) Or $a\big(V(x) < V(x_t)\big)$ and $\dot{V} > 0$ but $\dot{V} < \big|a\big(V(x) - V(x_t)\big)\big|$ then although $\dot{V}$ is positive and states are increasing but the Lyapunov function is limited by an upper bound and it will not grow without limit. The third case is that both of them are negative and it's clear that it is stable. This condition insures uniformly stability meaning that the states may not reach the origin but it is contained is a domain say ϵ which obey the primary definition of the stability.
To extends this theorem for asymptotic stability we can consider adding the term $p(V(x(t))) - V(x_t)$ where $p(.)$ is a function that has the following characteristics

$$p(s) > s$$

and then the condition becomes

$$\dot{V} + a\big(p(V(x(t))) - V(x_t)\big) < 0, a > 0 \quad (42)$$

By this when the system reaches some value which make $p(V(x(t))) = V(x_t)$ requires $\dot{V}$ to be negative but at this instant $V(x(t) < V(x_t)$ then in the coming τ interval the $V(x)$ will never reaches $V(x_t)$ and the maximum value in this interval is the new $V(x_t)$ which is less than the previous value and with the time the function keeps decreasing until the states reach the origin.

VI. CLOUD-REINFORCED CONTROL SCHEME

We present a multiagent control scheme where inter-agent communication is completely replaced by the use of a shared information repository (archive, warehouse) hosted on a cloud. Differently than in traditional event-triggered coordination schemes, here each agent schedules its own cloud accesses independently, and does not need to be alert for information broadcast by other agents. When an agent accesses the repository, it uploads some data packets, and downloads other packets that were previously deposited by other agents. Therefore, each agent receives only outdated information about the state of the other agents. The control law and the rule for scheduling the cloud accesses are designed to guarantee that the closed-loop system is well-posed and achieves the control objective, in spite of only using outdated information. Our analysis extends the use of the edge Laplacian [39], [40] to second-order directed networks, which allows us to consider control tasks with asymmetric information flow among the agents, such as leader-following tasks. With respect to the related works [41]-[44], this report introduces cloud reinforcement for multi-agent systems with second-order dynamics. Moreover, differently than [43], [44], here we consider additive disturbances (both persistent and vanishing) on the agent dynamics.

In comparison to centralized solutions for multiagent coordination, the proposed cloud-reinforced control scheme presents several important advantages: the computational burden can be distributed between the agents and the cloud according to the available resources; the architecture can be made resilient to failures of individual subsystems; fall-back local control laws can be used to put the agents in a fail safe state in case the communication with the cloud is temporarily lost; the framework can be also used for tasks that require the agents to perform local computations between two consecutive cloud accesses. We wish to emphasize that the proposed cloud-reinforced control scheme is scalable with the number of agents. Indeed, each agent can carry its own computational resources, while performing only local computations. The amount of such computation does not scale with respect to the number of agents added to the overall system. Indeed, at any cloud access, only the data referred to a single agent is communicated and processed. The only centralized resource that grows with respect to the number of agents is the memory of the cloud, which scales linearly. Moreover, the proposed setup differs from existing control schemes for asynchronous consensus algorithms with communication delays, for example [45], in that the delay in the information acquisition is not an undesired exogenous phenomenon, but it is induced by the control policy itself. In particular, the proposed scheduling policy aims at prolonging as much as possible the interval between two consecutive cloud connections of the same agent, in order to reduce the total number of communication instances.

TABLE I
DATA STORED IN THE CLOUD AT ARBITRARY TIME INSTANT t ≥ 0

Agent	1	2	…	N
lastaccess	ti,ℓ1	ti,ℓ2	…	tN,ℓN
distance	di,ℓ1	di,ℓ2	…	dN,ℓN
speed	si,ℓ1	si,ℓ2	…	sN,ℓN
control	ui,ℓ1	ui,ℓ2	…	uN,ℓN
nextaccess	ti,ℓ1 +1	ti,ℓ2 +1	…	tN,ℓN +1

VII. CLOUD WAREHOUSE

In our view, we consider that the agents cannot exchange any information directly, but can only upload and download information on a shared warehouse hosted on a cloud, which is accessed intermittently by each agent and asynchronously by different agents. The topology of the information exchanges happening through the cloud is described by a network graph $\mathcal{G} = (\mathcal{V}; \mathcal{E}; \omega)$ where $\mathcal{V} = \{1, \dots N\}$ with $N \in \mathbb{N}, \mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ under the constraint $(i, j) \notin \mathcal{E}$ for all $i \in \mathcal{V}$ and $\omega: \mathcal{E} \rightarrow \mathbb{R}_+$. Each element of $\mathcal{V}$ is called vertex and each vertex represents one of the agents, and each agent i downloads the information uploaded by its neighbors $j \in \mathcal{N}_i$ in the graph.

Throughout this work, the network graph $\mathcal{G}$ is considered time-invariant and contains a spanning tree.

When an agent accesses the cloud, it also has access to a sampled measurement of its own state. The time instants when agent i accesses the cloud are denoted as $t_{i,k}$ with $k \in \mathbb{N}$, and by convention $t_{i,0} = 0$ for all the agents. For convenience, we denote as $\ell_i(t)$ the index of the most recent access time of agent i before time t, that is,

$$\ell_i(t) = \mathrm{m}\ \{k \in \mathbb{N}: t_{i,k} \leq t\} \quad (43)$$

The measurement obtained by agent i upon the time instant $t_{i,k}$ is denoted as $z_{i,k}$. The control signals $u_i(t)$ are held constant between two consecutive cloud accesses:

$$u_i(t) = u_{i,k}(t) \forall t \in [t_{i,k}, t_{i,k+1}) \quad (44)$$

The data contained in the cloud at a representative time instant is represented in Table I.

Remark 3: Note in Table I that the $i - th$ column corresponds to the latest packet uploaded by agent i. The elements $d_{i,k}$ and $s_{i,k}$ are generic measurement variables and problem-dependent. The time dependence of the functions ℓ_i is omitted to keep the notation acute.

When an agent accesses the cloud, it uploads data that other agents may download later, when they, in turn, access the cloud. Typically, when agent i accesses the cloud at time $t_{i,k}$, it uploads a packet containing the following information: the current time $t_{i,k}$, the measurements $d_{i,k}$ and $s_{i,k}$, the value $u_{i,k}$ of the control input that is going to be applied in the time interval $[t_{i,k}, t_{i,k+1})$, and the time $t_{i,k+1}$ of the next access. This packet overwrites the packet that was uploaded on the previous access, thus avoiding that the amount of data contained in the cloud grow over time. When agent i accesses the cloud at time $t_{i,k}$, it downloads and stores the latest packet uploaded by each agent $j \in \mathcal{N}_i$.

This information, together with the measurements $d_{i,k}$ and $s_{i,k}$, is used by agent i to compute its control input $u_{i,k}$ for the upcoming time interval $[t_{i,k}, t_{i,k+1})$, and to schedule the next cloud access $t_{i,k+1}$.

In case that the cloud is endowed with some computational capabilities, it may also compute some global information about the state of the system for the agents to download. In such situation, the cloud provides a positive scalar $\sigma_{i,k}$ which represents an upper bound on x, the overall state for possible performance improvement.

VIII. CONCLUSIONS

This report has provided a theoretical framework for analysis of consensus algorithms for networked multi-agent systems with fixed or dynamic topology and directed information flow. The role of "cooperation" in distributed coordination of networked autonomous systems has been clarified. The main tools have been laid down and several theoretical results are sorted out.

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Awatef K. Ali, et. al. "Multiagent Control with Reinforced-Cloud Consensus."*American Journal of Engineering Research (AJER)*, vol. 11(01), 2022, pp. 154-167.