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Expert Decision-Making Systems

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Annotation. *An approach to making complex technical and economic decisions is proposed in cases where there is insufficient (or no) information about the experimental statistical data necessary for the construction of regression models of criterion functions. To solve the problem under consideration, a multicriteria optimization approach is taken using a nonlinear trade-off scheme. A model example is given.*

Key words*: vector optimization, regression models, expert judgment method, approximation polynomials, least squares method, interpolation method, nonlinear trade-off scheme.*

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I. INTRODUCTION

The adoption of complex technical and economic decisions is carried out using formalized decision-making systems. When creating such systems, one often has to face the fact that there is not enough experimental and statistical data to obtain the necessary mathematical models. The situation is aggravated when a decision is made according to several conflicting quality criteria.

In the face of an acute shortage of experimental data, we propose to obtain the necessary information ("quasi-experimental" data) from experts - specialists with sufficient experience in making decisions for the class in question. One of the specific aspects is the extreme limitation (and sometimes complete absence) of experimental statistical data, which could be used to determine mathematical models of criterion functions.

In these difficultly formalized conditions, one has to resort to non-traditional approaches, one of which is considered in this article. Naturally, in this case, we can only talk about rough calculations, about an approximate determination of the main trends in the choice of factors affecting the criterion functions.

II. FORMULATION OF THE PROBLEM

To solve the problem, you must have the following initial data:

1. Mathematical models:

$$y_k = f_k(x),\ k \in [1, s],$$

where $y = \{y_k\}_{k=1}^{s}$ is the vector of quality criteria by which multicriteria decisions are made; $f(x) = \{f_k(x)\}_{k=1}^{s}$ – vector of criterion functions; $x = \{x_i\}_{i=1}^{n}$ – n-dimensional vector of independent variables (optimization arguments of criterion functions).

2. Constraints on independent variables, $x \in X$,

where $X = \{x \mid x_{i\,\min} \le x_i \le x_{i\,\max},\ i \in [1, n]\}$.

The qualitative composition of the vector x is quite diverse and, accordingly, the dimension n of this vector is generally large. Full consideration of the parameters x would lead to an unjustified complication of the criterion functions $f(x)$ and to excessive difficulties in solving the optimization problem. Therefore, it is natural to choose only the most informative parameters x – coordinates of the space in which the optimization of the criteria y will be carried out, while the other parameters are considered fixed and given. The selection is usually done with the assistance of experts.

3. Limitations by criteria $y \in M$,

where $M = \{ y | 0 \le y_k \le A_k , k \in [1; s] \}$.

Arguments $x_{i\,\min}$, $x_{i\,\max}$, $x \in X$ and criteria A_k , $y \in M$ constraints are set based on physical considerations.

If all this is there, then there are all the prerequisites for optimizing the criterion functions, i.e. to determine the optimal compromise values of the parameters $x^* = \{ x^*_i \}_{i=1}^{n}$.

III. SOLUTION METHOD

The decision is made by optimizing the criteria functions included in the system. In view of the obvious inconsistency of the criteria, it is necessary to resort to specific methods of the theory of multicriteria (vector) optimization. If the scalar convolution method is used, then the mathematical model for solving the vector optimization problem is represented in the form

$$x^* = \arg \min_{x \in X} Y[y(x)],$$

where $Y(y)$ is a scalar function that has the meaning of a scalar convolution of the vector of partial criteria, the form of which depends on the chosen compromise scheme.

In [1-3], scalar convolution according to a nonlinear trade-off scheme is proposed. For the minimized criteria, it has the form:

$$Y[y(x)] = \sum_{k=1}^{s} A_k [A_k - y_k(x)]^{-1},$$

This convolution makes it possible to formally obtain Pareto-optimal solutions adequate to the given situations [4].

The type of criteria functions, $y_k = f_k(x), k \in [1, s]$, depends on what information the researcher has to build the model. The spectrum is wide – from complete knowledge of the mechanisms of phenomena (deterministic model) to complete uncertainty ("black box"). Between these information poles there is a probabilistic level of uncertainty.

It is usually difficult to obtain a deterministic mathematical model $f(x)$ of any characteristic of the decision-making process due to the complexity of the physical and other processes occurring in the object. Therefore, the criterion function $f_k(x), k \in [1, s]$ is approximated on the set of arguments $x \in X$ by some approximating function $F_k(x, a), k \in [1, s]$, known up to a vector of constants (coefficients) $a = \{ a_j \}_{j=1}^{m}$.

IV. REGRESSION MODELS

When choosing the type of function, $F_k(x, a), k \in [1, s]$ keep in mind the following. It was established [1] that the best results are obtained if the regression model is built on the basis of some known information about the mechanisms of the studied phenomena. Then the model is called meaningful. If there is no such information, then you have to work in the class of formal regression models and pay for the lack of information with a large amount of computation.

There are two conflicting requirements for the formal model. On the one hand, the approximating function must be simple enough so that the computational processes do not turn out to be excessively cumbersome. On the other hand, the approximating dependence should have sufficient predictive and accurate properties. In most practical cases, both of these requirements are met in the class of second-order regression polynomials:

$$F_k(x,a) = a_0 + \sum_{i=1}^{n} a_i x_i + \sum_{i,j=1,i<j}^{n} a_{ij} x_i x_j, k \in [1,s],$$

where a_0, a_1, a_{ij} are the coefficients. This function is quite well adapted to the topography of the objective function, $f_k(x)$, it is able to express such features as ravine, etc. In practice, various truncations of the regression polynomial are used, mainly linear approximations.

DETERMINATION OF REGRESSION COEFFICIENTS

The determination of the coefficients *a* can be performed both by interpolation methods and by the method of least squares (MLS). The interpolation formulas provide for the exact coincidence of the approximating and objective functions at the control points (interpolation nodes), the number of which *N*, as well as the number of unknown constants *a*, is equal to *m*. Coefficients *a* are determined by solving a certain system of equations:

$$F_k(x^{(u)}, a) = f_k(x^{(u)}), u \in [1, N = m],$$

where $x^{(u)}$ are the nodes of interpolation.

It is assumed that the values of the objective function at the approximation nodes $f_k(x^{(u)}), u \in [1, N]$ are known. Determination of these values with the help of experts is a key point in this work and is discussed below.

MLS provides *N* control points (approximation nodes), and the number *N* can be greater, less or equal (as a special case) to the number of constants *m*. The unknown coefficients of the approximating function are determined from the condition

$$E_k(a) = \sum_{u=1}^{N} [F_k(x^{(u)}, a) - f_k(x^{(u)})]^2 = \min_a .$$

Using the necessary condition for the minimum of the function, we obtain the system of so called in the LS theory *normal* equations

$$\frac{\partial E_k(a)}{\partial a_j} = 0, \ j \in [1, m],$$

the solution of which determines the coefficients of the approximating function. Note that regardless of the number of selected reference points *N*, the system of normal equations is *always* definite.

EXPERT EVALUATIONS

The specificity of the problem under consideration is that it is very difficult to obtain the values of the objective functions at the control points. As always, in those cases when the task is difficult to formalize, one has to resort to the methods of expert assessments.

A qualified specialist (expert) with sufficient experience in making decisions for this class can perform a **thought experiment**. He must imagine what, in his opinion, the levels of criterion functions will be at various reference points of the factors $x \in X$. Thus, the method is based on an individual opinion (postulate), expressed by a specialist-expert about the estimated value, based on his professional experience. The main disadvantage of postulation is subjectivity and the possibility of arbitrariness.

The procedure of the method of expert assessments processing can reduce this disadvantage. The method lies in the fact that to assess a certain quantitative characteristic, the postulates of not one, but several persons competent in this issue are used. It is assumed that the "true" value of the unknown quantitative characteristic is within the range of expert assessments and the "generalized" collective opinion is more reliable. When solving complex problems, it is necessary to take into account that the number of experts is usually limited, and the degree of their competence in this matter may be different. Failure to take into account these circumstances reduces the reliability and accuracy of the sought estimates.

A curious analogy is given by Olaf Helmer: “We receive information about events with the help of various devices, sometimes inaccurate, and we do not refuse this information, considering only the degree of its accuracy and reliability. Specialists-experts can also be regarded as a kind of "device" that gives information about the likelihood of certain upcoming events or hypotheses explaining the events taking place. You should not refuse such information. You should only try to determine the degree of accuracy and reliability of this information, just as it is done for other measuring devices." Continuing this analogy, we can say that the task of processing expert statements is similar to the task of integrating devices with different accuracy classes.

It is natural to assume that the accuracy and reliability of the expert assessment procedure will increase significantly if the statements of each expert are perceived with a coefficient (weight) depending on the degree of his competence in this issue. This weight can be established either on the basis of assessments of the expert's previous activities, or according to self-assessment data, or taking into account the qualifications, erudition, position or academic title of the expert. A more reliable procedure is in which the competence of an expert is assessed directly in the process of solving a specific problem.

An unknown quantitative characteristic is considered as a random variable, the reflection of the distribution law of which is the expert's postulate. To establish the final assessment, the statements of all experts are studied in aggregate and processed as a kind of initial statistical material. The processing should be done using the concepts of mathematical statistics.

In [1], a procedure for processing the data of expert assessments is proposed, in the course of which refined aggregated assessments are obtained, as well as (as a by-product) coefficients of confidence in the opinion of individual experts.

Applying this method to the processing of expert estimates of criterion functions at each of the N nodal points of the domain of definition, we obtain s vectors of estimates (quasi-experimental data):

$$\{ f_1 (x^{(u)})\}_{u=1}^{N};$$
$$\{ f_2 (x^{(u)})\}_{u=1}^{N};$$
$$\ldots\ldots\ldots\ldots \quad \ldots\ldots\ldots\ldots \quad ..$$
$$\{ f_s (x^{(u)})\}_{u=1}^{N},$$

which serve as the basis for determining the vectors of constants a by the method of interpolation, or LSM. This is how mathematical regression models are defined

$$y_1 = f_1(x) \approx F_1(x);$$
$$y_2 = f_2(x) \approx F_2(x);$$
$$\ldots\ldots\ldots\ldots \quad \ldots\ldots\ldots\ldots \quad \ldots\ldots\ldots\ldots \quad ..$$
$$y_s = f_s(x) \approx F_s(x),$$

which are involved in the optimization procedure. Since the information about the objective functions at the nodal points of the approximation area was obtained not experimentally, but by means of expert assessment, then the models $F_k(x)$, $k \in [1, s]$ are called *expert* regression models.

ILLUSTRATIVE EXAMPLE

The problem of making a decision regarding the choice of the construct of the designed bridge is considered. The quality of the bridge is assessed according to three criteria:

1. $y_1(x)$ – design reliability;
2. $y_2(x)$ – the capacity of the bridge;
3. $y_3(x)$ – the cost of design and construction of the bridge.

By the term "reliability" we mean the probability of finding the defining parameters of all elements of the bridge within the limits admissible under the operating conditions: $y_1(x) \in [0;1]$ (the unit corresponds to 100% reliability).

The desired bridge capacity is assigned a value of one. In this way, $y_2(x) \in [0;1]$. The maximum permissible value of the cost of designing and building a bridge is estimated by the unit: $y_3(x) \le 1$. The criteria $y_1(x)$ $y_2(x)$ and are to be maximized, and the criterion $y_3(x)$ is to be minimized.

The optimization argument x is determined by the variants of the chosen construct of the designed bridge and, therefore, is a discrete quantity. Three options are considered:
1. Arched structure. It corresponds to the value of the optimization argument x_1.
2. Cable-stayed design option. It corresponds to the value of the optimization argument x_2
3. Suspension bridge. This option is assigned the value of the optimization argument x_3.

Each of the options for the meaning of the argument has a sense of the probability of its choice in the decision-making process. In this way, $x_1, x_2, x_3 \in [0;1]$. The unit corresponds to the unconditional choice of the l-th option with zero values of the remaining arguments, i.e. $x_l = 1, x_p = 0, p \ne l$.

The criterion functions $y_k(x)$, $k \in [1;3]$ will be approximately represented by the linear parts of the approximation polynomial without a free term:

$$y_1(x) \approx F_1(x) = a_1 x_1 + a_2 x_2 + a_3 x_3;$$
$$y_2(x) \approx F_2(x) = b_1 x_1 + b_2 x_2 + b_3 x_3;$$
$$y_3(x) \approx F_3(x) = c_1 x_1 + c_2 x_2 + c_3 x_3.$$

It will be shown below that this form allows experts to quickly determine the regression coefficients.

Arguments x_1, x_2, x_3 are included in expressions for control points (approximation nodes) used to determine the regression coefficients of criterion functions. In our example, the number of control points is the same as the number of unknown coefficients. This makes it possible to use the interpolation method to calculate the coefficients (in case of a mismatch, it would be necessary to use the OLS, which would complicate the calculations).

Consider an approximate criterion function $F_1(x)$ (structural reliability) at three nodal points. First nodal point: $x_1 = 1, x_2 = 0, x_3 = 0$. This corresponds to the choice of the arch bridge design option. In this case

$$F_1(x_1) = a_1 \times 1 + a_2 \times 0 + a_3 \times 0,$$

i.e. $F_1(x_1) = a_1$.

The experts are invited to assess the relative reliability $F_1(x_1)$ of the arch structure of the bridge. We will assume that the generalized expert opinion is obtained by processing individual statements. Let the expert judgment

$$F_1(x_{1)} = 0{,}6.$$

This means that thus the coefficient of the criterion function of the reliability of the bridge structure $a_1 = 0{,}6$.

Let us turn to the second nodal point: $x_1 = 0, x_2 = 1, x_3 = 0$. This corresponds to the choice of the cable-stayed bridge structure. In this case

$$F_1(x_2) = a_2.$$

If the expert assessment of the reliability of the cable-stayed bridge structure

$$F_1(x_2) = 0{,}8,$$

then the coefficient $a_2 = 0{,}8$.

The third nodal point $x_1 = 0, x_2 = 0, x_3 = 1$ corresponds to the choice of the suspension structure of the bridge. If the experts assessed the reliability of the suspended structure as

$$F_1(x_3) = 0{,}2,$$

then the coefficient $a_3 = 0{,}2$.
Thus, the approximate criterion function of the bridge structure reliability has the form

$$F_1(x) = 0{,}6x_1 + 0{,}8x_2 + 0{,}2x_3.$$

Similarly, we obtain an expression for the approximate criterion function of the bridge capacity:

$$F_2(x) = 0{,}5x_1 + 0{,}8x_2 + 0{,}2x_3.$$

These two criterion functions are maximizable, i.e. the larger the better. The situation is different with the criterial function $F_3(x)$ of the cost of designing and building a bridge. It appears to be minimized, i.e. the smaller it is, the better.
Applying the above-described expert evaluation procedure for the case of the minimized criterion, we obtain an expression for the approximate criterial function of the cost of designing and building a bridge:

$$F_3(x) = 0{,}6x_1 + 0{,}4x_2 + 0{,}8x_3.$$

For further calculations, it is necessary to resolve the issue of a unified method of extremization of criterion functions [5]. We will assume that all considered criterion functions should be minimized. Then the criterion function corresponding to the reliability of the bridge structure will look like this:

$$F_1^{'}(x) = 1 - F_1(x) = 1 - 0{,}6x_1 - 0{,}8x_2 - 0{,}2x_3$$

(the unit corresponds to 100% structural reliability).
We represent the minimized criterion function corresponding to the capacity of the bridge in the form:

$$F_2^{'}(x) = 1 - F_2(x) = 1 - 0{,}5x_1 - 0{,}8x_2 - 0.2x_3$$

(the unit corresponds to the desired throughput).
The system of criterion functions, the minimization of which leads to a decision on the choice of the bridge design, has the form:

$$F_1'(x) = 1 - 0{,}6x_1 - 0{,}8x_2 - 0{,}2x_3;$$

$$F_2'(x) = 1 - 0{,}5x_1 - 0{,}8x_2 - 0{,}2x_3;$$

$$F_3(x) = 0{,}6x_1 + 0{,}4x_2 + 0{,}8x_3.$$

The scalar convolution of these functions according to a nonlinear trade-off scheme, taking into account obvious limitations $A_1 = A_2 = 1$, is represented as:

$$Y(x) = \frac{1}{1 - F_1'(x)} + \frac{1}{1 - F_2'(x)} + \frac{1}{1 - F_3(x)} =$$

$$= \frac{1}{0{,}6x_1 + 0{,}8x_2 + 0{,}2x_3} + \frac{1}{0{,}5x_1 + 0{,}8x_2 + 0{,}2x_3} + \frac{1}{1 - 0{,}6x_1 - 0{,}4x_2 - 0{,}8x_3}.$$

After minimizing the scalar convolution of the criteria with respect to the optimization arguments by the Nelder-Mead method, we obtain

$$x_1{}^* = 0{,}0571 \quad ; x_2{}^* = 0{,}9980 \quad ; x_3{}^* = 0{,}00006 \quad .$$

This result can be interpreted as follows. If the expert assessment is carried out according to three criteria (reliability, capacity and cost), then with almost 100% probability a decision should be made on the choice of the cable-stayed bridge structure. The likelihood of choosing an arched structure is much less likely and the probability of choosing a suspended bridge structure is negligible.

V. CONCLUSION

This study makes it possible to identify the main trends in the development of multi-criteria decision-making systems in the absence (or lack) of experimental data. One should be aware that expert models are less informative than regression models determined using real-life experiments. However, firstly, expert assessments still reflect, albeit with possible distortions, the real reality, and, secondly, expert models are only initial approximations and can be improved as statistical data accumulate.

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