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Effect of Capillary Pressure on Estimation of Relative Permeability Using JBN Method

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ABSTRCT: The following investigation focuses on the impact of capillary pressure on estimation of two phases relative permeability curves. Accurate relative permeability is a crucial parameter for evaluating reservoir performance. The unsteady state core flooding tests, which is considered in this study, is mostly used to measure oil-water relative permeabilities.

The Johnson, Bossler and Neumann (JBN) method is the conventional method for estimating relative permeabilities from field core. The limitations in the JBN method create an error in relative permeability curves and make it unrepresentative of a typical core flooding test results. There are always capillary pressure effects taking place during core flood tests. Ignoring of capillary pressure by JBN method will influence the calculation of relative permeability curves and final saturation levels.

One dimensional numerical model with uniform initial saturation has been implemented in this study using Eclipse 100 software to understand the relationship between relative permeability and capillary pressure. Pressure drop and recovery data obtained from 1-D numerical simulations are used to estimate the relative permeabilities by JBN method. Many scenarios have been studied by running the simulation at constant injection rate and varying the input capillary pressure.

The results obtained have shown the influence of capillary pressure on estimating relative permeability curves. It is shown that increase in capillary pressure increases the water relative permeability. Furthermore, the results demonstrate that the water flooding curves differ greatly in shape and position according to the corresponding values of capillary pressure.

Comparisons of relative permeability curves have shown that the capillary pressure dominates the displacement process. Capillary pressure gradient will increase the fractional flow of water and this increase in fractional flow of water results in lower frontal water saturation, higher frontal velocity and subsequently leading to a decrease in oil recovery.

Key words: Relative permeability, JBN method, Eclipse 100 model.

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I. INTRODUCTION

Reservoir engineering studies generally require some indispensable parameters such as reservoir fluid flow and rock properties. Maximizing recovery and development strategy success depend on understanding the type of fluid and rock characteristics.

Relative permeability is a dominant factor controlling the movement of two immiscible fluid phases in porous media. Availability of accurate and representative relative permeability data is of significant concern to reservoir engineers as dearth of these data indicates poor forecasting of production, ultimate recovery and difficulties in reservoir management. The most important parameters required for reservoir engineering studies include the absolute permeability, capillary pressure and relative permeability to the fluids [1].

Relative permeability and capillary pressure of porous media are crucial properties for evaluating accurate reservoir performance. In reservoir simulation studies, relative permeability and capillary pressure lab data are required as input parameters for reservoir simulator to predict reservoir performance. Relative permeability data are incorporated in oil recovery forecasts and feasibility study of enhanced oil recovery methods [2].

Capillary pressure is the pressure difference existing across the curved interface of two immiscible fluids at equilibrium. Capillary pressure is used for determining the hydrocarbon distribution through the porous media. Surface forces of capillary pressure can either support or resist the displacement process in the pores of porous medium. Inexpensive computer power which has allowed millions of multicomponent phase equilibrium and physical property calculations to be performed within seconds using a state equation as the thermodynamic basis. [3]

II. OBJECTIVES

The objectives of this study are:

a. To develop a core flood model for unsteady state core flood test to estimate relative permeability.

b. To investigate the problems associated with estimation of relative permeability by analyzing the core flooding "unsteady state" by using JOHNSON, BOSSLER and NAUMANN method (JBN) which does not consider the capillary pressure. This limitation in analyzing the core flood data come up with the end capillary effect.

c. To demonstrate the important aspect of the error caused due to ignoring the capillary pressure and the effect of the injection rate on the feasibility of using the JBN to calculate the relative permeability curve.

III. JBN METHOD

The Buckley and Leverett theory (1941) was modified by Welge in 1952 to facilitate estimation of relative permeability in laboratory core flooding displacement tests. The work of Welge was extended by Johnson-Bossler-Naumann (JBN) 1958 for estimation of the relative permeability from unsteady state core flood test data which is consider in this study [4].

There are three important assumptions for JBN method [4]:

- Total flow velocity is the same throughout the cross section of linear porous body.
- Flow velocity is high enough to achieve Buckley and Leverett displacement.
- Capillary effect is negligible at high injection rates.

To overcome the capillary end effect the experiment should be done at high enough displacement rate. At higher rate the flow will be unstable in the experiments and the concept of relative permeability will not hold. Cumulative recoveries of oil and water versus time are measured at the outlet face of the core during the JBN method to estimate the relative permeability curve. Some of the mathematical relations which have been developed by Welge are required for calculation of two phase relative permeability by JBN method as follow [4]:.

$W_i = \frac{1}{f'} = \frac{1}{df_{/dS}} \dots \text{Eq 1}$
$\frac{f}{1-f} = \frac{f}{fo} = \frac{K_{rw}\mu_o}{K_{ro}\mu_w}.$ Eq 2
$(f)_{o_2} = \frac{ds_{av}}{dW_i}.$ Eq 3
$S_{av} = S_2 + W_i(f_o)_2$ Eq 4
$f_o u = -\frac{kk_{ro}}{\mu_o} \frac{\partial p}{\partial x}.$ Eq 5
$\Delta x = v \Delta t f' \dots \text{Eq } 6$
The pressure drop across the core which has length L is shown as the integral
$p = -\int_0^L \frac{\delta p}{\delta x} dx.$ Eq 7
Substituting $\frac{\partial p}{\partial x}$ from Eq5 will give,
$\Delta p = \frac{u\mu_o}{k} \int_0^L \frac{f_o}{k_{ro}} dx$ Eq 7a
By rearranging and substituting equations 6 and 7a, the following equation is obtained:

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$$\int_{0}^{f'_{2}} \frac{f_{o}}{k_{ro}} df' = \frac{\Delta p k f'_{2}}{L u \mu_{o}} = f'_{2} \frac{\frac{u_{s}}{\Delta p_{s}}}{\frac{u_{/\Delta p}}{u_{/\Delta p}}} = \frac{f'_{2}}{l_{r}}....Eq7b$$

(f')

Where Ir is the relative injectivity which is the ratio of the intake capacity at any given flood stage to the intake capacity of the system at the start of the flood. From measurements of flow rate and pressure drop in a water flood susceptibility test, relative injectivity function for a given type of reservoir rock can be determined [4]. Ordinary differentiation is used for equation 7b with respect to f'_2 since f' is the only independent variable [4],

$$\frac{d\left(\frac{J-2}{l_r}\right)}{df'_2} = \frac{f_o}{k_{ro}}...Eq 8$$

When f'_{2} is equal to the reciprocal of the cumulative volume injection, the equation 19 will be written as [4],

$$\frac{d\left(\frac{1}{W_i lr}\right)}{d\left(\frac{1}{W_i}\right)} = \frac{f_o}{k_{ro}}...$$
Eq 8a

From the equation 19a individual relative permeability of oil can be calculated. The outlet face saturation S_2 is obtained by rearranging equation4 [4]:

 $S_2 = S_{av} - W_i(fo_2)....Eq 4a$

The relative permeability of the water at S_2 is calculated by solving equation 2[17]:

$$k_{rw} = \frac{(1-f_o)}{f_o} \frac{\mu_w}{\mu_o} k_{ro}$$
.....Eq 9

Jones and Roszelle in 1978 extended the JBN method for estimating relative permeabilities by presenting a graphical technique to perform the essential differentiation of the production data and the late time data analysis by their method. They figured out that the fractional flow of displacing phase concave downward when it is plotted against saturation. Jones and Roszell method could also be used for experiments conducted at constant pressure drop across the core, constant rate or changeable pressure drop and flow rate [5].

In 1984 Tao and Watson developed a Monte Carlo error analysis for JBN. The two sources of error in relative permeability are estimation error which related to the error included in the process of measured data to estimate relative permeability and modeling error which is attributed to the degree where the mathematical model fails to exhibit the physical experiment. They postulated that the use of various viscosity ratios did not affect very much the accuracy of relative permeability and the injection rate as well. The error will increase only when oil production or pressure drop are reduced. They also developed the algorithms for computer implementation for JBN method. They pointed out that the relative permeability can be estimated fairly accurately by using linear regression or optimal spline algorithm [6].

In 1986 Kerig and Watson included high flexible cubic splines for estimating relative permeability from unsteady state experiment. The error in estimating relative permeability was greatly reduced by using cubic splines and very accurate result can be obtained [7].

In 1988 Watson et al. introduced B-spline for use as functional representations of relative permeability curves. They indicated that serious error may be detected when relative permeability curves are performed with function having too few parameters. They used both hypothetical and real experiments data for core flood test and also pointed out that without acceptable number of parameters; large errors estimation can occur [8].

IV. METHODOLOGY

The study starts with the introduction of the relative permeability and the methods of measuring the relative permeability from core flooding tests. A literature review of the estimation of relative permeability from core flooding tests and influence of capillary pressure on the accuracy of the estimated relative permeability is presented.

One dimensional numerical simulation model of imbibitions unsteady state test will be performed using input relative permeability and capillary pressure data. After running the simulation, the data will be collected to estimate the relative permeability by JBN method. The data required from the simulation model to calculate the relative permeability curves are cumulative recovery of oil, V_o , pressure drop across the core, Δ_p , and total water injected W_i .

The following equations are used for the calculation:

 $\overline{S_w} = S_{wi} + V_o \dots \text{Eq 10}$ $(f)_{o_2} = \frac{d\overline{S_w}}{dw_i} \dots \text{Eq 11}$

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Water saturation at the outlet face of the core is calculated based on Welge method

$$S_{w2} = \overline{S_w} - f_{o2}W_i....Eq 2$$

Relative injectivity is calculated I_r from total flow rate and pressure drop to estimate individual relative permeability by:

$$I_r = \frac{\left(\frac{q}{\Delta P}\right)}{\left(\frac{q}{\Delta P}\right)i}...$$
Eq 13

Where $\left(\frac{q}{\Delta_p}\right) \mathbf{i}$ is at initial condition.

The oil relative permeability is given by:

$$k_{ro} = f_{o2} \frac{d\left(\frac{1}{W_i}\right)}{d\left(\frac{1}{W_i lr}\right)}.$$
 Eq 14

The water relative permeability is calculated by:

$$k_{rw} = \frac{(1 - f_{02})}{f_{02}} \frac{\mu_w}{\mu_0} k_{r0}...$$
Eq 15

The water saturation during the simulation process will be observed to see the influence of capillary end effect on the breakthrough. The result from the simulation will be compared with input relative permeability data to check the error between the input relative permeability and the one that will be measured by the JBN method. Number of grid cells, injection flow rate and input capillary pressure will be varied until satisfactory results is obtained.

V. NUMERICAL SIMULATION RESULTS AND DISCUSSION

5.1 ECLIPSE 100

Numerical simulation is performed in this study using a commercial black oil simulator "Schlumberger ECLIPSE 100". The recovery and pressure drop data from input relative permeability and capillary pressure were obtained by the Eclipse simulator. Four scenarios have been studied to investigate the effect of capillary pressure on estimation relative permeability curves.

5.2 SIMULATION OF CORE FLOODING TO DETERIMINE OIL-WATER RELATIVE PERMEABILITIES

Core flooding tests is used to evaluate and determine relative permeabilities which are required for simulation studies. Linear core was used to determine oil and water relative permeability with unsteady state method by reducing it to irreducible oil saturation.

Flow is uni-directional and the core is homogeneous and isotropic. Properties of the core are tabulated in the Table 1. The relative permeability data used in the model are shown in Figure 1 and the values are tabulated in Table 2.

Parameter	Unit	Value
К	md	124
μο	ср	8
μw	ср	0.51
Q	cc/min	5
Injection pressure	atm	40
Swi	%	0.303
Ø	%	0.28
L	cm	25
A	cm ²	25

Table 1: Core properties.

Table 2: Oil/water relative permeability

SW	Krw	Kro
0.303	0	0.722
0.342	0.022	0.485

0.381	0.036	0.325
0.426	0.057	0.193
0.463	0.079	0.128
0.492	0.104	0.088
0.522	0.132	0.056
0.556	0.167	0.035
0.586	0.209	0.021
0.621	0.262	0.012
0.653	0.314	0.0065
0.685	0.369	0.0035
0.707	0.409	0.0017
0.738	0.469	0.0017



Figure 1: Relative permeability versus water saturation

Capillary pressure for all the scenarios are illustrated in Figure 2 and values are tabulated in Table 3

SW	PC1	PC2	PC3
0.303	2.274282	4.548563	1.137141
0.342	1.595325	3.190649	0.797662
0.381	1.326606	2.653211	0.663303
0.426	1.181933	2.363865	0.590966
0.463	1.082878	2.165755	0.541439
0.492	1.006928	2.013856	0.503464
0.522	0.944602	1.889203	0.472301
0.556	0.891019	1.782038	0.44551
0.586	0.843318	1.686635	0.421659
0.621	0.799638	1.599275	0.399819
0.653	0.758656	1.517312	0.379328
0.685	0.719339	1.438678	0.35967
0.707	0.680785	1.361569	0.340392
0.738	0.642096	1.284192	0.321048

Table 1:	Capillary	pressures



Figure 2: Capillary pressures versus saturation

The core sample modelled has 100 cells in X direction, 1 cell in Y direction and 1 cell in Z direction. One production well and one injection well will be integrated in the model. One dimensional Cartesian grid will be used in this model as shown in the Figure 3. The model has length of 25 cm and area of 25 cm2.



Figure 3: One Dimensional Cartesian Grid

Recovery and pressure drop at given time are used to calculate the relative permeabilities by JBN method which gives the relative permeability curves as a function of saturation at the outlet face of the core sample after the beginning of the displacement. The relative permeability curves are generated by following steps:

Collect the data from the simulator for constant water injection rate water, and then injected water volumes and oil recovery are converted into the pore volume injected.

Given the initial water saturation, average water saturation as a function of pore volume is calculated from equation 10.

> The fractional flow of oil is measured at the outlet of the core by plotting the average water saturation versus total water injected at each time step using equation 11 "i.e., the slope of the curve".

Water saturation at the outlet face of the core at each time step is calculated based on Welge method using equation 12.

Initial Pressure drop across the core is calculated using Darcy law by the following equation :

$$(\Delta P)i = \frac{q\mu_o L}{\kappa_A} \quad \dots \quad \text{Eq 16}$$

Final pressure drop at each time steps is calculated from taking the difference of pressure at the injection well and production well.

The reciprocal of the relative injectivity is calculated by the following equation :

 $\frac{1}{l_r} = \frac{\left(\frac{q}{\Delta P}\right)i}{\left(\frac{q}{\Delta P}\right)}.$ Eq 17

Since the flow rate is constant during displacement, equation 17 becomes:

 $\frac{1}{l_r} = \frac{\Delta P}{(\Delta P)i}.$ Eq 18

Substituting equation 16 in equation 18 gives:

 $\frac{1}{l_r} = \frac{K\Delta P}{\mu_o(L/A)q}.$ Eq 19

From simulation results, slope of the $\frac{1}{W_i}$ as a function of $\frac{1}{W_i l_r}$ is calculated and individual relative permeability of oil is obtained using equation 14.

Water relative permeability at each time steps is calculated using equation 15.

To obtain the data for estimating representative relative permeability core flood simulation is carried out for four scenarios. After doing the sensitivity study for the number of core cells I decided to use 100 cells in 1D model.

All simulation scenarios that are considered in this chapter were design to study the effect of capillary pressure on measuring relative permeability curves. Results have been obtained by simulation for estimation relative permeability curves by JBN method. Different results were acquired according to different capillary pressure.

5.2.1 Case 1

The first scenario is to measure relative permeabilities by JBN method based on the measurement of pressure drop across and the cumulative production of oil and water

The core model is flooded with water at constant flow rate of 0.0833cc/sec and variation of pressure drop during the displacement is measured. Since the rate is constant equation 19 becomes:

$$\frac{1}{I_r} = \frac{K\Delta P}{\mu_o(L/A)q} = 0.186 \,\Delta P(\text{atm})$$

Results from the JBN method are compared with values that used to generate the simulation recovery and pressure drop data. The graphs of total water injection per pore volume versus total oil recovery per pore volume and average saturation versus the total water injection are show in the Figure 4& Figure 5 respectively.

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Figure 4: Water injected versus oil recovery



Figure 5: Average saturation versus total water injected

Initial water saturation is assumed to be immobile; hence oil will be produced at the same rate of water injected for an incompressible system. Water saturation gradient exist form inlet to the end of the system when the water breakthrough occurs.

Figure 4& Figure 5 shows that the slope of curve is equal 1 before breakthrough time, hence the fraction flow of oil is equal the slope of curve at any given injection the oil fractional flow calculated. After breakthrough, water saturation continuously increases as water move through the core. From the graph, slope is decreasing after the breakthrough. This means that fractional flow of oil is decreasing and fractional flow of water is increasing as the water started producing from the outlet face of the core.

Pressure drop across the core during the displacement test is increasing from initial pressure drop until the point where the breakthrough happened and water started producing after that is decreasing as shown in Figure 6.



Figure 6: Pressure drop versus time

The error introduced by the assumption of JBN method is easy to evaluate as the simulation was run by known relative permeabilities. The input relative permeabilities and the calculated relative permeabilities from 5 cc/min imbibitions simulation are compared as shown in the Figure 7.



Figure 7: Relative permeability curves from JBN and input relative permeability curves

At the start of the displacement process water saturation is low, and because of this, there will be an error in the estimated relative permeability which is due to capillary pressure effect. The errors decrease when the water saturation increase and capillary pressure effect is reduced. The oil relative permeability calculated by JBN was small at the beginning of displacement compared with input data in the model.

The saturation at the outlet face of the core is unchanged during the displacement until water breakthrough. Also the water saturation at the outlet is always less than the average saturation. The saturation profile from the core model is shown for different time steps in Figure 8.

Water arrives at the outlet of the core and accumulates until water phase pressure exceeds the oil phase pressure. Finally accumulation of water at the end of the core leads to sufficient water phase pressure for flow. Increase in water saturation decreases the oil phase permeability. Error is created when accumulation of water at the outlet of core delays water breakthrough. This end effect results in large value of relative injectivity and give error for oil relative permeability. Due to end effect of capillary pressure, water saturation is changing in non-uniform manner and also effects the JBN assumption.



Figure 8: Water saturation versus distance

5.2.2 Case 2

JBN method requires the capillary pressure to be vanished or minimized In this case the input capillary pressure for the simulation model is neglected (Pc=0). The cumulative water injection versus cumulative oil recovery is compared with the first cases as illustrated in Figure 9. The graph shows that oil recovery is enhanced as the capillary pressure is neglected.



Figure 9: Water injection versus oil recovery

The average water saturation is compared with first case as illustrated in Figure 10. The slope of average water saturation versus cumulative water injected pore volume provides an estimate of oil fractional flow.

From the graph the slope is increasing when capillary pressure is neglected comparing to the first case and oil fractional flow is increasing. Ideally, oil fractional flow will decrease from one to zero monotonically.



Figure 10: Water injection versus average saturation

Pressure drop across the core versus time for both cases are compared as illustrated in the Figure 11. The graph shows the pressure drop is greater when neglecting capillary pressure and the water breakthrough time is delayed.



Figure 11: Pressure drop versus time

The input relative permeability curves are compared for both cases as shown in Figure 12. Water relative permeability is smaller when capillary pressure is neglected which leads to improve oil recovery.



Figure 12: Relative permeability curves from JBN & input relative permeability curves for Case 2

Water saturation versus distance is compared for different time steps, the shock front happened early when neglecting capillary pressure and gave stable displacement and better recovery as shown in Figure 13.



Figure 13: Water saturation versus distance

5.2.3 Case 3

The third scenario is when capillary pressure is twice of the first case (PC2) as shown in the Table 3. This is done to investigate the effect of increasing the capillary pressure on the data collected from the model. Higher capillary pressure gives higher water cut and decrease the fractional flow of oil. This phenomenon can be seen clearly as illustrated in Figure 14for average water saturation versus cumulative water injection and from

plotting cumulative oil recovery against the cumulative water injection as shown in Figure 15 comparing with the other scenarios.



Figure 2: Water injection versus average saturation



Figure 15: Water injection versus oil recovery

When water is injected to the core, capillary pressure acts on the water phase and hence, early breakthrough. However when capillary pressure is zero in the core flood simulator, a sharp decrease in water saturation occur and water will move in piston-like displacement and improve oil recovery. Pressure drop across the core for the three cases is illustrated in Figure 16.



Figure 16: Pressure drop versus time

Relative permeability curves calculated by JBN method for the three scenarios compared with the input relative permeability curves are illustrated in Figure 17. Fractional flow of water is increased due to the presence of capillary pressure gradient. Water relative permeability increases with decrease in oil relative permeability. Water saturations profiles are illustrated Figure 18.



Figure 17: Relative permeabilities from JBN & input relative permeabilities for case3



Figure 18: Water saturation versus distance

5.2.4 Case 4

Case 4 is when capillary pressure is half of the case PC3 as shown in the Table 3. It can be observed that water flooding curves differ greatly in shape and position according to the corresponding values of capillary pressure as illustrated in the Figure 19, Figure 20 and Figure 21. All the flooding curves have the tendency to shift upward with the decreasing values of the capillary pressure and consequently increasing the slope and oil fractional flow increased. Result of this behavior in core flooding tests demonstrates qualitatively interchangeable capillary pressure on estimation relative permeability from core flooding tests.







Figure 30: Water injection versus average saturation



Figure 41: Pressure drop versus time

Relative permeability curves for all cases calculated by JBN method versus input relative permeability curves and water saturation profile are illustrated in Figure 22 & Figure 23. The shape of relative permeability curves are influenced by capillary pressure, subsequently impacting the average saturation and oil recovery. A decrease in capillary pressure, results in a decrease in water relative permeability and a corresponding increase in oil relative permeability.



Figure 22 Relative permeabilities from JBN & input relative permeabilities for case4



VI. CONCLUSIONS & RECOMMENDATIONS

6.1. Conclusions

In this study, numerical simulation has been used to investigate the effect of capillary pressure on estimation relative permeability by a conventional method (JBN) from unsteady state displacement tests which

are accepted to be the closest to the flow mechanism in the reservoirs. Influences of capillary pressure in computation of relative permeability and saturation on core flooding displacement tests have been addressed. The effect of capillary pressure on fluid flow in one dimensional cause errors in the analysis of displacement data by conventional methods for estimation of relative permeability curves.

All four study cases were investigated at various capillary pressures and their calculated relative permeability curves by JBN method were plotted along with the input relative permeability curves for comparison purposes. Main conclusions based on the simulation results are:

1. Capillary pressure plays a dominate role in displacement processes and it is responsible for trapping a large portion of oil within the pore structure of the reservoir rocks.

2. Relative permeability calculated by JBN method is not accurate due to capillary pressure effect. Relative permeability of oil is decreases due to this effect.

3. Fractional oil flow is decreasing as capillary pressure increases and fractional flow of water increases. This increase in water fractional flow results in a lower frontal water saturation and a higher frontal velocity.

4. The inclusion of capillary pressure also decreases water flood oil recovery and affects displacement performance.

6.2. Recommendations

For future study, it is recommended to carry out core flooding experiment in the laboratory and calculate the relative permeability from the data collect in the lab. In the lab, distribution grooves at both ends of the core are used to distribute the fluid evenly over the core face. Also the saturation distribution as a function of time can be measured accurately by scanning the core during displacement process.

In order to reflect experimental cores used in lab, it is recommended to represent cores in simulation using radial grids. An evenly distributed groove for distribution of fluid should also be represented when carrying out simulation. This can be done by introducing a layer of high permeability grid blocks, with zero capillary pressure at the inlet and outlet of the core model. The simulator should be properly model at inlet and outlet end plugs with end effect and non-linear nature of the flow near the ends.

Nomenclature

menerature		
Ø	\Rightarrow	porosity
P_c	\Rightarrow	Capillary pressure
Pnw	\Rightarrow	Capillary pressure of non wetting phase
P_w	\Rightarrow	Capillary pressure of wetting phase
σ	\Rightarrow	Surface tension
r	\Rightarrow	Radius of the pore
θ	\Rightarrow	Contact angle
V	\Rightarrow	Fluid velocity
K	\Rightarrow	Permeability
μ	\Rightarrow	Viscosity of the fluid
q	\Rightarrow	Flow rate through the porous medium
А	\Rightarrow	Cross-sectional area across which flow occurs
k _{eff}	\Rightarrow	Effective permeability
k _{abs}	\Rightarrow	Absolute permeability
k_{ro}	\Rightarrow	Oil relative permeability
k_o	\Rightarrow	Effective permeability of oil
k_{rw}	\Rightarrow	Water relative permeability
k_w	\Rightarrow	Effective permeability of water
k_{rg}	\Rightarrow	Gas relative permeability
k_g	\Rightarrow	Effective permeability of gas
Sw_i	\Rightarrow	Initial water saturation
S_{or}	\Rightarrow	Irreducible oil saturation
ΔP	\Rightarrow	Pressure drop
q_t	\Rightarrow	Total flow rate
$f\mathbf{w}$	\Rightarrow	Fractional flow of water
$\frac{fW}{S_w}$	\Rightarrow	Average water saturation
μο	\Rightarrow	Viscosity of oil

- Viscosity of water μw ⇒ ⇒ Reservoir dip angle α Gravitational constant g ⇒ \tilde{S}_{wc} ⇒ Connate water saturation S_w Water saturation ⇒ Swf Water saturation at the Buckley-Leverett front ⇒ Fractional of displacing phase in flowing stream ⇒ Fractional of displaced phase in flowing stream fo ⇒ Cumulative injection in pore volume W_i ⇒ Average velocity v ⇒ L ⇒ Length Relative injectivity I_r ⇒
- $V_o \Rightarrow$ Cumulative oil recovery

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