American Journal of Engineering Research (AJER)
e-ISSN: 2320-0847 p-ISSN : 2320-0936
Volume-10, Issue-12, pp-194-201
www.ajer.org
Research Paper Open Access

One-Port Power Conservative Equivalent Circuit For Dc Networks With Dependent Current Sources

M. Artemenko[1], L. Batrak[2], Y. Kutafin[1]

[1]*Department of Acoustic and Multimedia Electronic Systems of the National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute",Kyiv, Ukraine*

[2]*Department of Electronic Devices and Systems of the National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute" Kyiv, Ukraine*

ABSTRACT: *In this work, the new one-portpower conservativesubstitution circuit for DC networks consisting of resistors, independent and dependent current sources is presented, which is equivalent to the primary circuit both in terms of external volt-ampere characteristic and in terms of the total consumers power at an arbitrary load current. It differs from the Barbi's circuit equivalent by the dependent voltage source controlled by the load voltage, the output terminals of which are connected in series with the Norton's current source. Formulas are derived for calculating the gain of this dependent voltage source both analytically on the basis of the matrix-vector model of the primary circuit, and on the basis of short-circuit and open-circuit experimental data. Based on the parameters of the proposed equivalent circuit, formulas are obtained for calculating the matched load resistance that maximizes the efficiency of the loaded primary network. The methods of the theory of electrical circuits and matrix algebra were applied. The results obtained can be used for energy optimization of DC networks, which amplifying and converting elements are modeled by dependent current sources.*

KEYWORDS: *power conservative equivalent circuit, Norton's theorem, matching of load resistance*

Date of Submission: 16-12-2021 Date of acceptance: 31-12-2021

I. INTRODUCTION

The equivalent circuit concept or the Thévenin's and Norton's theorems about the equivalent source (generator) is one of the key principles of the linear electric circuit theory [1,2]. In its traditional sense, this concept makes it possible to replace an linear active DC electric network (DCEN) of arbitrary complexity with a two-element Thévenin's [3] or Norton's [4] circuit equivalent, which accurately reproduces the volt-ampere characteristic of the primary network at a pair of accessible terminals. Based on the circuit equivalents a method of equivalent generator was developed and improved in [5-7] to calculate the current or voltage of one branch of the primary network, which is represented by the load of the corresponding circuit equivalent [8]. However, as was noted in [9, 10], the energy properties of the primary network these circuit equivalents reflect inadequately.

Only recently the interpretation of the equivalent circuit concept was developed by I. Barbi in [11]. He put forward the idea of creating a one-port substitution circuit with the minimum possible number of internal elements, which would be equivalent to the primary circuit not only in volt-ampere characteristic, but also in total internal power losses (so-called "power conservative equivalent circuit" [11]). He also proposed in [11, 13] the first three-element power conservative circuit equivalents shown in Fig. 1 (in Fig. 1a for DCEN with resistors and independent voltage sources and in Fig. 1b for networks with resistors and independent current sources). In particular, the circuit in Fig. 1b differs from the well-known Norton's circuit equivalent with source current I_N and conductance G_N by the presence of resistor R_{BN}, which resistance is calculated by the formula $R_{BN} = P_{SC} / I_N^2$, where P_{SC} is the total power of all primary circuit consumers in the short circuit mode of output terminals [13]. In [12], L. Coraddini extended the new energy content of the equivalent circuit concept to the class of DCEN, which contains independent sources of both voltage and current types. He also proposed several new one-port circuit equivalents of this class and investigated the conditions of load matching to achieve maximum efficiency. Developing his own ideas, I. Barbi formulated (although without proof) in [13] a theorem about the structure of the generalized one-port circuit equivalent for DCEN, which consist of resistors and independent sources of two types. This circuit equivalent consists of five elements [13] and differs from the

Coraddini's circuits.

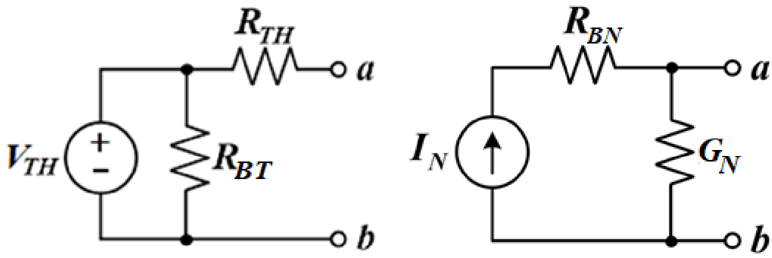

Figure. 1. Power conservative three-element substitution circuits of DCEN: a) with resistors and independent voltage sources [11]; b) with resistors and independent current sources [13]

All power conservative circuit equivalents presented in [11-13] are designed for DCEN, formed by solely of independent DC sources and resistors. This does not allow the modeling of linear active DC circuits with dependent sources that reflect the amplifying and converting elements of real networks in linear mode of operation [8]. In [14] the first power conservative substitution circuit for DCEN with dependent voltage sources was proposed which differs from the of Barbi's circuit equivalent by the presence of dependent current source controlled by the load current and new analytical dependence for its parameter was obtained. However, the energy characteristics of DCEN with voltage and current sources differ significantly [12]: if the of internal power losses in DCEN with voltage sources increases with increasing load current, then in DCEN with current sources it decreases.

The goal of this work is the development of a new one-port power conservative equivalent circuit for the DCEN class, containing both independent and dependent current sources and resistors, and studying the conditions of matching its load to achieve maximum efficiencyof primary network.

II. JUSTIFICATION OF STRUCTURE AND PARAMETERS OF THE POWER CONSERVATIVE EQUIVALENT CIRCUIT FOR DC NETWORKS WITH DEPENDENT CURRENT SOURCES

We will establish the power equivalence of the primary circuit, consisting of resistors, dependent and independent current sources (Fig. 2), and its one-port substitution circuit containing a minimum number of elements, provided the same analytical dependence of total power consumption from load current *I*.

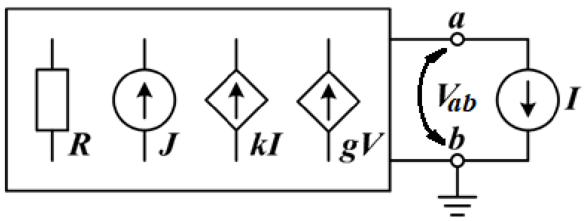

Fig. 2. Primary circuit consisting of resistors, dependent and independent current sources with pair of accessible terminals ab and load current *I*

To do this, select two external terminals *ab* in the primary circuit (Fig. 2) and connect to them an ideal current source with a variable value of *I*, which will simulate the load current. Then denote as *Vab*the voltage between the external terminals, which coincides with the load voltage, and ground node *b*. The node voltage equations for the primary circuit in matrix form are given by:

$$\begin{cases} G_0 V_{ab} + \mathbf{g}_0^T \mathbf{v} = J_0 - I; \\ \mathbf{g} V_{ab} + \mathbf{G}\mathbf{v} = \mathbf{j}, \end{cases} \quad (1)$$

where G_0, J_0 are the intrinsic conductance and reference current of the primary circuit external node *a*; $\mathbf{g}_0^T$ is row vector of mutual conductances between external node *a* and internal nodes; ^{T} - transposition sign; **V** iscolumnvectorof voltages betweeninternal nodes and the reference node *b*; **g** iscolumnvectorof mutual conductances between internal node *a* and external nodes; **G** is the matrix of conductances between the primary circuit internal nodes;**j** iscolumnvectorof internal node reference currents.

Let's express the vector of internal voltages from the second (1)

$$\mathbf{v} = \mathbf{G}^{-1}\mathbf{j} - \mathbf{G}^{-1}\mathbf{g}V_{ab} = \mathbf{R}\mathbf{j} - \mathbf{R}\mathbf{g}V_{ab}; \mathbf{G}^{-1} = \mathbf{R} \quad (2)$$

and substitute its value in the first (1):

$$(G_0 - \mathbf{g}_0^T \mathbf{R}\mathbf{g})V_{ab} = (J_0 - \mathbf{g}_0^T \mathbf{R}\mathbf{j}) - I. \quad (3)$$

Denoting $G_N = G_0 - \mathbf{g}_0^T \mathbf{R}\mathbf{g}$; $I_N = J_0 - \mathbf{g}_0^T \mathbf{R}\mathbf{j}$, we obtain analytical expressions for the internal conductance and

source current of the Norton's circuit equivalent, which coincide with those obtained in [13]. Also these values may be found by short circuit and open circuit experiments on output terminals *ab*. In the short circuit (SC) mode the value of the load current is equal to the SC current $I = I_{SC}$, the external voltage is zero $Vab = 0$ and from (3) we obtain that $I_N = I_{SC}$. In the open circuit (OC) mode the external voltage is equal to the OC voltage $V_{ab} = V_{OC}$, the load current is zero $I = 0$, and from (3) we obtain that $G_N = I_{SC}/V_{OC}$. As you know [8], relations between the parameters of the Thévenin's and Norton's circuit equivalents are $R_{TH} = 1/G_N$; $V_{TH} = V_{OC} = I_N/G_N$.
According to the power balance, the total power of all primary circuit consumers is equal to the total power of all sources that can be found using (2):

$$P_1 = \mathbf{j}^T\mathbf{v} + (J_0 - I)V_{ab} = \mathbf{j}^T\mathbf{R}\mathbf{j} + (J_0 - \mathbf{j}^T\mathbf{R}\mathbf{g})V_{ab} - IV_{ab} = P_{SC} + P_+ - IV_{ab}, \quad (4)$$

where $P_{SC} = \mathbf{j}^T\mathbf{R}\mathbf{j}$ is short-circuit total power of all internal consumers at $V_{ab}=0$, the expression for which coincides with that obtained in [13]; $P_+ = (J_0 - \mathbf{j}^T\mathbf{R}\mathbf{g})V_{ab}$ is additional power component of internal losses, proportional to the load voltage; $I_+ = (J_0 - \mathbf{j}^T\mathbf{R}\mathbf{g}) = P_+/V_{ab}$ is current coefficient of proportionality between the additional component of internal power loss and load voltage. This current coefficient differs from the Norton's source current by value of the difference current

$$I_\Delta = I_+ - I_N = (J_0 - \mathbf{j}^T\mathbf{R}\mathbf{g}) - (J_0 - \mathbf{g}_0^T\mathbf{R}\mathbf{j}) = (\mathbf{g}_0^T\mathbf{R} - \mathbf{g}^T\mathbf{R}^T)\mathbf{j} \quad (5)$$

In the absence of dependent current sources, nodal conductance matrix of the primary circuit is symmetric with respect to the main diagonal, so $\mathbf{G} = \mathbf{G}^T$; $\mathbf{R} = \mathbf{R}^T$; $\mathbf{g}_0 = \mathbf{g}$ wherefore $I_\Delta = 0$; $I_+ = I_N$ and the Barbi's circuit equivalent (Fig. 1b) adequately represents the total power loss of primary circuit.

In the presence of dependent current sources, their parameters are entered into the matrix $\mathbf{G}$ and vectors $\mathbf{g}_0^T$, $\mathbf{g}$ in an asymmetric manner, so in the general case $\mathbf{G} \neq \mathbf{G}^T$; $\boldsymbol{R} \neq \boldsymbol{R}^T$; $\mathbf{g}_0 \neq \mathbf{g}$; and $I_\Delta \neq 0$.
As follows from (4), the maximum power of DC networks with dependent current sources occurs at the maximum value of the output voltage in OC mode at zero load current:

$$P_{OC} = P_{SC} + I_+ V_{OC}.$$

This value differs from OC power of electric networks with solely independent current sources [13]

$$P_{OC} = P_{SC} + G_N V_{OC}^2 = P_{SC} + I_{SC} V_{OC}.$$

As follows from the above equality, in these networks, only 3 of the 4 values P_{SC}, P_{OC}, I_{SC}, V_{OC} determined in the open circuit and short circuit experiments are independent. Therefore, the Barbi's circuit equivalent (Fig. 1b), which should provide these four specified values, may be consisting of three elements.
Therefore, linear DCEN with dependent current sources has an important characteristic feature: due to the asymmetry of the nodal conductivities matrix current coefficient of proportionality between the additional component of internal power loss and load voltage is not equal to Norton's source current. As a result, in such networks all 4 specified values determined in the SC and OC experiments are independent. This requires the introduction of an additional, fourth element in the known structure of three-element Barbi's substitution scheme (Fig. 1b), which will ensure its power equivalence.

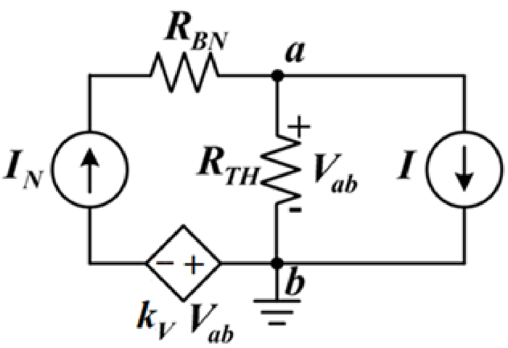

Figure 3. The proposed one-port power conservative circuit equivalent for primary DCEN with dependent current sources

The proposed one-port substitution circuit (Fig. 3) differs from the Barbi's one by a dependent voltage source $V = k_V V_{ab}$ controlled by the load voltage V_{ab}. We find the value of this source gain k_V from the condition of matching the total power of all consumers for primary circuit described by formula (4) and proposed substitution circuit. The circuit (Fig. 3) contains a single ideal current source I_N which voltage has three components: the voltage of the Barbi's resistor with value $I_N R_{BT}$, the load voltage V_{ab} and the dependent source voltage with value $k_V V_{ab}$. The analytical dependence of proposed circuit's internal power losses is determined by the expression:

$$P_2 = I_N(I_N R_{BN} + V_{ab} + V_{ab}k_V) - IV_{ab} = P_{SC} + I_N V_{ab}(1 + k_V) - IV_{ab} \qquad (6)$$

From comparison (4) and (6) we set the expression for the value of the voltage dependent source transfer ratio:

$$k_V = I_+ / I_N - 1 = I_\Delta / I_N \ . \qquad (7)$$

The value of this transfer ratio can also be determined by the results of short circuit and open circuit experiments. From (6) we obtain that in the open-circuit mode, when $I = 0$, $V_{ab} = V_{OC}$, the total consumers power may be found by $P_{OC} = P_{SC} + I_{SC}V_{OC} + k_V I_{SC} V_{OC}$, therefore, the dependent sourcegain can be determined by

$$k_V = \frac{P_{OC} - P_{SC}}{I_{SC}V_{OC}} - 1 . \qquad (8)$$

Thus, the introduction into the Barbi's substitution circuit [13] of a dependent voltage source controlled by the load voltage with the specified with the specified connection and gain provides its power equivalence to the primary DC circuit with dependent current sources. This makes it possible to investigate the energy properties of primary circuit on its simplified one-port four-element substitution circuit, for example, to optimize the load parameters to achieve maximum efficiency.

III. LOAD MATCHING OF THE PROPOSED CIRCUIT EQUIVALENT TO ACHIEVE THE MAXIMUM EFFICIENCY

Let's find the optimal value of the load current $I = \mu I_N = \mu I_{SC}$that provides the maximum efficiency of proposed substitution circuit (Fig. 3). The load voltage of this circuit is $V_{ab} = (I_N - I)R_{TH} = (1-\mu)I_N R_{TH} = (1-\mu)V_{OC}$, and load poweris given by $P_L = IV_{ab} = \mu(1-\mu)I_{SC}V_{OC}$. In the load supply mode the single current source power can be found from (6) and (8) in the form

$$P_S = P_{SC} + I_N V_{ab}(1 + k_V) = P_{SC} + I_N V_{ab}\frac{P_{OC} - P_{SC}}{I_{SC} - V_{OC}} = P_{SC} + (1-\mu)(P_{OC} - P_{SC}) = \mu P_{SC} + (1-\mu)P_{OC} . \qquad (9)$$

The expression for the efficiency of proposed substitution circuit is transformed as follows:

$$\eta = \frac{P_L}{P_S} = \frac{\mu(1-\mu)I_{SC}V_{OC}}{\mu P_{SC} + (1-\mu)P_{OC}} = \frac{I_{SC}V_{OC}}{P_{SC}/(1-\mu) + P_{OC}/\mu} . \qquad (10)$$

The minimum value of the denominator (10) occurs at the optimal meaning of the coefficient μ_0 that is determined from the equality $P_{SC}\mu_0^2 = P_{OC}(1-\mu_0)^2$ in the form

$$\mu_0 = \frac{\sqrt{P_{OC}}}{\sqrt{P_{SC}} + \sqrt{P_{OC}}} ; \ 1 - \mu_0 = \frac{\sqrt{P_{SC}}}{\sqrt{P_{SC}} + \sqrt{P_{OC}}} ;. \qquad (11)$$

Substitution of these meanings in (10) gives the value of maximum possible efficiency that can be reached by load matching in primary network and its power conservative equivalent circuit

$$\eta_0 = \frac{I_{SC}V_{OC}}{\sqrt{P_{SC}} + \sqrt{P_{OC}}} . \qquad (12)$$

The optimum meaning of loading resistance that provided maximum value of efficiency (12) is given by

$$R_{L0} = \frac{V_{ab}\big|_{\mu=\mu_0}}{I\big|_{\mu=\mu_0}} = \frac{(1-\mu_0)R_{TH}}{\mu_0} = R_{TH}\sqrt{\frac{P_{SC}}{P_{OC}}} . \qquad (13)$$

The expressions (12), (13) are completely coincide with the results of research [12] of more complex one-port power equivalent circuits for the substitution of DCEN with independent sources of both voltage and currentas well as for the substitution scheme of the DCEN with independent and dependent voltage sources [14]. This gives grounds to assert that above mentioned maximum value of efficiency (12) and optimal value of load resistance (13) based on the results of short circuit and open circuit experiments ofprimary network are valid for any linear DCEN.

The optimal value of load resistance can also be expressed through the parameters of the substitution circuit (Fig. 3) by using (13) and expressions for open-circuit power P_{OC} andR_{BN}resistance:

$$R_{L0} = \frac{R_{TH}}{\sqrt{1 + (k_V + 1)I_{SC}V_{OC}/P_{SC}}} = \frac{R_{TH}}{\sqrt{1 + (k_V + 1)R_{TH}/B_{BN}}} . \qquad (14)$$

As you might expect, the optimal value of load resistance depends only on the resistive parameters and the voltage source transfer ratio of the proposed substitution circuit.

IY. VERIFICATION OF THE OBTAINED ANALYTICAL EXPRESSIONS BY VIRTUAL EXPERIMENT

We illustrate the processes of finding the parameters of the proposed one-port substitution circuit and its load matching to obtain the maximum efficiency on the example of a linear DCEN with dependent current sources of both types in Fig. 4.

The system of scalar equations for this network by the method of nodal voltages has the form

$$\begin{aligned}
&(G_4+G_5)V_{ab} - G_4 v_2 - G_5 v_3 = -I;\\
&G_6 v_1 = -J_1;\\
&-G_4 V_{ab} + (G_1+G_3+G_4)v_2 - G_3 v_3 - g v_3 + k_1 I = J_1;\\
&-G_5 V_{ab} + G_3 v_2 + (G_2+G_3+G_5)v_3 - k_1 I = J_2.
\end{aligned} \qquad (15)$$

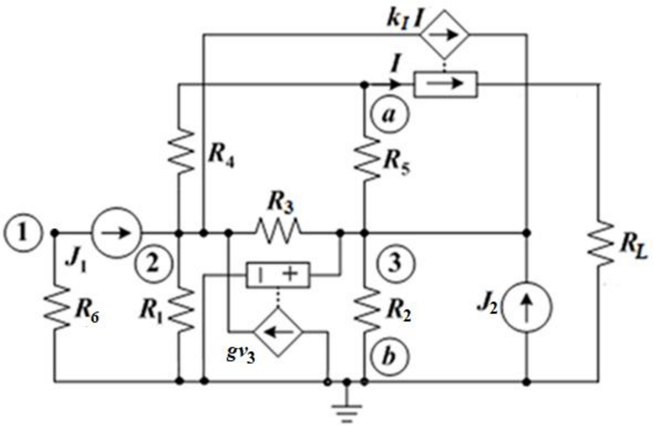

Figure 4. Scheme of the studied primary network with independent current sources

After substituting the numerical values of the element parameters $R_i = 10/i$, Ohm, $i = 1,2,\ldots,6$; $J_j = j$, A, $j = 1, 2$; g = 0.3 *mho*; k_I= 1, we form a matrix-vector system of equations, in which matrix and vectorblocks are selected according to the notation of (1)

$$10^{-1}\times\begin{Vmatrix} 9 & 0 & -4 & -5\\ 0 & 6 & 0 & 0\\ -13 & 0 & 12 & -1\\ 4 & 0 & -7 & 5\end{Vmatrix}\times\begin{Vmatrix} V_{ab}\\ v_1\\ v_2\\ v_3\end{Vmatrix}=\begin{Vmatrix} -1\\ -1\\ 1\\ 2\end{Vmatrix}.$$

From comparison with (1) we set the values of scalar, vector and matrix parameters of the primary circuit model (dimensions of all quantities are in *SI* units):

$$G_0 = 0.9;\ J_0 = 0;$$

$$\mathbf{g}_0 = 10^{-1}\times\begin{Vmatrix} 0\\ -4\\ -5\end{Vmatrix};\ \mathbf{g} = 10^{-1}\times\begin{Vmatrix} 0\\ -13\\ 4\end{Vmatrix};\ \mathbf{j} = \begin{Vmatrix} -1\\ 1\\ 2\end{Vmatrix};\ \mathbf{G} = 10^{-1}\times\begin{Vmatrix} 6 & 0 & 0\\ 0 & 12 & -1\\ 0 & -7 & 5\end{Vmatrix}.$$

Taking into account the quasi-diagonal structure of the matrix **G**, the inverse matrix is calculated in the form

$$\mathbf{R} = \mathbf{G}^{-1} = 10\times\begin{Vmatrix} 1/6 & 0 & 0\\ 0 & 5/53 & -1/53\\ 0 & -7/53 & 12/53\end{Vmatrix} = \frac{5}{159}\begin{Vmatrix} 53 & 0 & 0\\ 0 & 30 & 6\\ 0 & 42 & 72\end{Vmatrix}.$$

We calculate the parameters of Norton’s and Thévenin’s circuit equivalents according to (3):

$$I_N = I_{SC} = I_0 - \mathbf{g}_0^T\mathbf{Rj} = = -\frac{5\times 10^{-1}}{159}\begin{Vmatrix} 0 & -4 & -5\end{Vmatrix}\begin{Vmatrix} 53 & 0 & 0\\ 0 & 30 & 6\\ 0 & 42 & 72\end{Vmatrix}\begin{Vmatrix} -1\\ 1\\ 2\end{Vmatrix} = \frac{183}{53} = 3{,}45285\ ;$$

$$G_N = G_0 - \mathbf{g}_0^T\mathbf{R}\,\mathbf{g} = \left(9 + \frac{1}{318}\begin{Vmatrix} 0 & 330 & 384\end{Vmatrix}\begin{Vmatrix} 0\\ -13\\ 4\end{Vmatrix}\right)\times 10^{-1} = \frac{9}{256} = 0{,}03396226;\ R_{TH} = 1/G_N = \frac{256}{9} = 29{,}(4);$$

$$V_{TH} = V_{OC} = I_N / G_N = \frac{183}{53} \div \frac{18}{530} = \frac{305}{3} = 101,(6).$$

Calculating the total power of all consumers in the short circuit mode

$$P_{SC} = \mathbf{j}^{\mathbf{T}}\mathbf{Rj} = \frac{5}{159}\begin{Vmatrix}-1\\1\\2\end{Vmatrix}\begin{Vmatrix}53 & 0 & 0\\0 & 30 & 6\\0 & 42 & 72\end{Vmatrix}\begin{Vmatrix}-1\\1\\2\end{Vmatrix} = \frac{2335}{159} = 14,6855,$$

we determine the resistance of the resistor proposed by I.Barbi [13]to improve the Norton's circuit equivalent

$$R_{BN} = \frac{P_{SC}}{I_N^2} = \frac{2335}{159} \Big/ \left(\frac{183}{53}\right)^2 = \frac{2335 \times 53}{3 \times 183^2} = 1,2317975 .$$

By the value of current coefficient

$$I_+ = J_0 - \mathbf{j}^T\mathbf{Rg} = -\frac{5}{159}\begin{Vmatrix}-1\\1\\2\end{Vmatrix}^T\begin{Vmatrix}53 & 0 & 0\\0 & 30 & 6\\0 & 42 & 72\end{Vmatrix}\begin{Vmatrix}0\\-13\\4\end{Vmatrix}\times 10^{-1} = \frac{147}{53} = 2,77358$$

and I_N we calculate the gainof the proposed dependent voltage source

$$k_V = \frac{I_+}{I_N} - 1 = \frac{147}{53} \Big/ \frac{183}{53} - 1 = -\frac{12}{61} = -0,196721 .$$

To check the power equivalence of the primary circuit and different substitution circuits with calculated parameters we plot graphs (Fig. 5) of experimentally removed watt-ampere characteristics (i. e. dependences of the total consumers powers as the functions of load current) of electrical networks in Fig. 1b, 3, 4. As expected, the watt-ampere characteristics of the proposed circuit equivalent (Fig. 3) and the primary network (Fig. 4) completely coincide in the current range 0 - I_{SC} that indicates power equivalence of these networks. Graphs of watt-ampere characteristics of Barbi's circuit equivalent (Fig. 1b) and the primary circuit has only one common point, which corresponds to the short-circuit current I_{SC} at which the control voltage of the proposed dependent voltage source is zero.

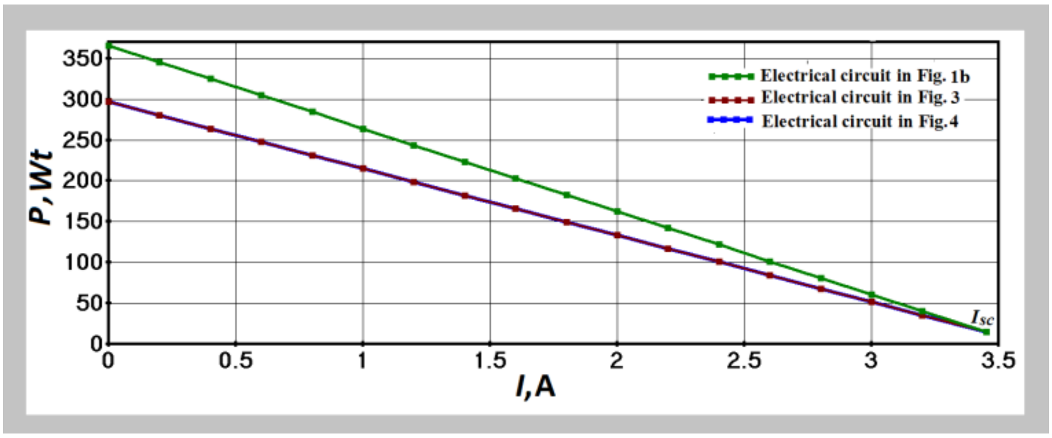

Figure 5. Graphs of watt-ampere characteristics for electrical circuits in Figure 1b, 3, 4.

The optimal value of load resistance can be found by (14)

$$R_{L0} = \frac{R_{TH}}{\sqrt{1 + (k_V + 1)R_{TH} / R_{BN}}} = \frac{256}{9\sqrt{1 + \frac{49}{61} \times \frac{265 \times 3 \times 183^2}{9 \times 2335 \times 53}}} = \frac{256}{9\sqrt{1 + \frac{49 \times 183}{467}}} = 6,551 .$$

Setting the value of open circuit power

$$P_{OC} = P_{SC} + I_+ V_{OC} = \frac{2335}{159} + \frac{147}{53} \times \frac{305}{3} = \frac{47170}{159} = \frac{890}{3} = 296,(6).$$

we calculate the predicted value of the optimal efficiency by (12):

$$\eta_0 = \frac{I_{SC}V_{OC}}{(\sqrt{P_{SC}} + \sqrt{P_{OC}})^2} = \frac{\frac{183}{53}\times\frac{305}{3}}{\left(\sqrt{\frac{2335}{159}} + \sqrt{\frac{890}{3}}\right)^2} = \frac{61\times 305}{53\left(\sqrt{\frac{2335}{159}} + \sqrt{\frac{890}{3}}\right)^2} = 0{,}791761.$$

To experimentally confirm these values, we obtain the analytical dependence of the efficiency vs load resistance for the proposed circuit equivalent and plot on its graph the discrete values of virtual experiment results with the primary circuit (Fig. 4). The analytical dependence of the efficiency vs the load resistance R_L is obtained from (10) by substituting $\mu = R_L / (R_{TH} + R_L)$:

$$\eta = \frac{I_{SC}V_{OC}}{P_{SC}/(1-\mu) + P_{OC}/\mu} = \frac{I_{SC}V_{OC}}{(R_{TH} + R_L)\times(P_{SC}/R_L + P_{OC}/R_{TH})}.$$

When substituting the numerical data this dependence takes the form

$$\eta = \frac{351{,}037735849}{(29{,}44444 + R_L)\times(14{,}6855 / R_L + 10{,}0752468)}.$$

The graph of this dependence (Fig. 6) with the plotted points of the virtual experiment confirms the coincidence of the maximum coordinates with the calculated values.

Thus, by creating a new one-port substitution circuit that is equivalent to the primary circuit by internal power, the concept of the substitution circuit energy equivalence is extended to the DCEN class, consisting of dependent and independent current sources and resistors.

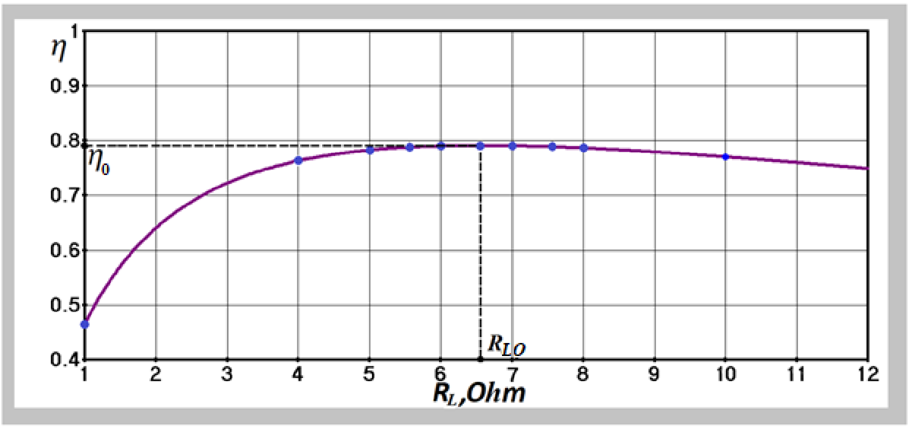

Fig. 6. Graph of the analytical dependence of efficiency vs load resistance with plotted points of the virtual experiment

The specificity of this class of networks is the asymmetry of the matrix of nodal conductivities, as a result of which the coefficient of proportionality between the component of the power of internal losses and the load voltage is not equal to the Norton current source value. This requires introducing of additional element into the known Barbi circuit equivalent. It was proposed to choose it in the form of dependent voltage source, controlled by load voltage, the output terminals of which are connected in series with the Norton current source. New analytical dependences of this dependent voltage source gain based on the vector and matrix coefficients of the nodal voltage macromodel of the primary circuit as well as based on short circuit and open circuit experimental data were established. The virtual experiment confirmed the complete coincidence of the watt-ampere characteristics of the proposed circuit equivalent and the primary circuit in the entire range of changes in load currents, which indicates their equivalence in power. The establishment of an energy-adequate replacement circuit for DCEN class with dependent sources made it possible to find conditions for matching its load to achieve the maximum efficiency of the primary circuit. Experimental data confirmed the complete coincidence of the coordinates of the maximum efficiency of the primary circuit with the calculated forecast values calculated on the basis of the parameters of the proposed substitution circuit.

IV. CONCLUSION

1. The first in the class of DCEN with dependent current sources one-port substitution circuit is proposed, that is equivalent to the primary circuit both in terms of volt-ampere characteristic on external terminals and in terms of internal power losses at arbitrary load. It differs from the Barbi's circuit equivalent by the presence of a voltage-controlled load-dependent voltage source, the output terminals of which are connected in series with the Norton current source.

2. New analytical dependences of dependent voltage source gain based on the vector and matrix coefficients of the nodal voltage macromodel of the primary circuit as well as based on short circuit and open circuit experimental data were established. The data of the virtual experiment confirmed the identity of the watt-ampere characteristics of the proposed one-port substitution circuit with the calculated parameters and the primary circuit in the whole range of load current changes, which indicates their power equivalence.
3. Based on the parameters of the proposed one-port substitution circuit, new analytical relations are derived to match the load resistance of the DCEN specified structure in order to obtain the maximum efficiency. Experimental data confirmed the complete coincidence of the coordinates of the maximum efficiency of the primary circuit with the calculated predicted values. The obtained results can be used for energy optimization of DCEN, amplifying and converting elements of which work in linear mode, modeled by dependent current sources.

REFERENCES

[1.] H. Johnson. Origins of the equivalent circuit concept: the voltage-source equivalent. Proceedings of the IEEE, vol. 91, no. 4, pp. 636–640, 2003. **https://doi.org/10.1109/JPROC.2003.811716.**

[2.] D. H. Johnson. Origins of the equivalent circuit concept: the current-source equivalent. Proceedings of the IEEE, vol. 91, no. 5, pp. 817–821, 2003. **https://doi.org/10.1109 /JPROC.2003.811795.**

[3.] L. Thévenin. Sur un Nouveaux Théoreme d'Electricité Dynamique. C. R. des Séances de l'Académie des Sciences, Tome: 97:159–161, 1883.

[4.] E. L. Norton. Design of finite networks for uniform frequency characteristic. Technical Report TM26–0–1860, Bell Laboratories, 1926.

[5.] V. D. Landon. The equivalent generator theorem. Proceedings of the Institute of Radio Engineers, Vol. 18, No. 2, 1930, pp. 294-297. https://doi.org/10.1109/JRPROC.1930.221995.

[6.] A. T. Starr. A new theorem for active networks. Journal of the Institution of Electrical Engineers, Volume: 73, Issue: 441, 1933, pp. 303-308**. https://doi.org/10.1049/jiee-1.1933.0129**.

[7.] Reza Hashemian. Hybrid Equivalent Circuit, an Alternative to Thevenin and Norton Equivalents, its Properties and Applications. 2009 52nd IEEE International Midwest Symposium on Circuits and Systems, **https://doi.org/ 10.1109/MWSCAS.2009.5235889.**

[8.] Alexander, Charles K. Fundamentals of electric circuits / Charles K. Alexander, Matthew N. O. Sadiku. – 5th ed. McGraw-Hill, New York, 2013. ISBN-13: 978-0073380575.

[9.] R.E. Collin. Limitations of the Thévenin and Norton Equivalent Circuit for a Receiving Antenna. IEEE Antennas and Propagation Magazine, vol. 45, issue: 2, April 2003, pp. 119 – 124. **https://doi.org/ 10.1109/MAP.2003.1203128.**

[10.] J. H. Bentley and K. M. Hess, A Programed Review for Electrical Engineering, 2nd ed. New York, NY, USA: Van Nostrand Reinhold Company, 2011, p. 19. **ISBN: 978**-94-009-4860-0.

[11.] I. Barbi. Power conservative equivalent circuit for dc networks, IEEE Access, vol. 8, pp. 113667–113674, 2020. **https://doi.org/10.1109/ACCESS.2020.3004026.**

[12.] L. Corradini. General Power-Equivalent Synthesis of Resistive DC Networks. IEEE Access, vol. 8, pp. 160711 - 160722, 2020. https://doi.org/10.1109/ACCESS.2020.3020652.

[13.] I. Barbi. Unified Power Conservative Equivalent Circuit for DC Networks," IEEE Access, vol. 8, pp. 178230 – 178237, 2020. **https://doi.org/10.1109/ACCESS.2020.3027489.**

[14.] Artemenko M.Yu., Petrov D.O., Kutafin Yu.V. One-Port Power Conservative Equivalent Circuit of DC Network with Dependent Voltage Sources. Visnyk NTUU KPI Seriia – Radiotekhnika Radioaparatobuduvannia, 2021, Iss. 67, pp. 5–13. **https://doi.org/**10.20535/RADAP.2021.85.5-13.

M. Artemenko. "One-Port Power Conservative Equivalent Circuit For Dc Networks With Dependent Current Sources."*American Journal of Engineering Research (AJER)*, vol. 10(12), 2021, pp. 194-201.