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Economic Order Quantity Models For Imperfect Items With Repair Under Conditions Of Permissible Delay In Payments (Case Study Ratna Elektronic)

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ABSTRACT: *Inventory is a product stored for future use. In the traditional EOQ model, there is a basic assumption that all products sent by the supplier are all in good condition without disability and in the absence of a policy of deferring payment. But in reality, in the production process not all products are produced in good condition and not all retailers are able to pay for the products ordered immediately so that the supplier can provide a policy of delay payment. Therefore, this study aims to produce EOQ inventory models for defective products with improvements under the policy of postponement of payments. There are two cases given, first when payments are made over a predetermined time cycle and second when payments are made less than the time cycle specified. The objective function is modeled to maximize expected returns. The optimal ordering cycle length and optimal order quantity are obtained by looking at the available theorems. Numerical simulation is used to illustrate the EOQ model applied to Electronic Ratna, Mojokerto. Sensitivity analysis of the optimal solution is shown to determine the effect of parameter change on the decision variable.*

Keywords: *EOQ, Delay of Payment, Improvement, Stock, Defective Product*

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I. INTRODUCTION

One way to improve supply chain performance is to coordinate between parties in the supply chain and implement good management in terms of inventory. According to (Bowersox, 2002)supply chain provides a network for business actors and suppliers who work together to deliver a product and services and information effectively and efficiently to consumers. The EOQ inventory model (Economic Order Quantity) is used to determine the exact purchase or order size. There is a basic assumption used in the EOQ model, ie all products shipped by the supplier are all in good condition without disability. In fact, in the production process not all products are produced in good condition. The existence of defective products is a natural thing, this is due to the production quality is not perfect or the process of displacement is not perfect. Defective products in inventory management systems can have a strong effect on the company's reputation. For that reason, some researchers develop models that consider defective products. One of them (Porteus, 1986)conducts research on the relationship between product quality with lot size indicating that the relationship between them is significant. This research is the basis of the emergence of research on the determination of lot size by considering the existence of defective products. According to (Zhang and Gerchak, 1990)an incoming order product may contain a defective product with a defective product number being a random variable. Furthermore, (Salameh and Jaber, 2000)develops an EPQ model for defective products and to avoid shortage they assume the production rate is greater than the demand level. This model considers the existence of a defective product that has a probability of ρ which is known in every lot of delivery. Zhang and Gerchak (1990) assumes that the detected defective product is withdrawn from inventory at the end of the screening period and is sold on the secondary market at a discount

Model Zhang and Gerchak (1990) has been widely developed by several researchers. Rezaei (2005)develops EOQ inventory models for defective products by allowing shortage. Chung and Huang (2006)developed an EOQ inventory model for defective products in the absence of a credit payment policy. Wee et al., (2006) develops EPQ inventory models for defective products by allowing shortage. Jabeer et al., (2014)developed an EOQ inventory model for defective products by performing repair options. Jabeer et al., (2014)assuming that the return of defective products to suppliers located far away is impossible to do as they require a higher cost. The detected defective product is withdrawn from inventory at the end of the screening period then sent to the local repair shop for repair. After the repair process is complete, the repaired product is returned to the inventory so that the product can be reused well and sold on the market.

According to the EOQ inventory model assumption, the retailer must pay for the product ordered immediately after the product is received. But in reality, not all retailers are able to pay for products that are ordered instantaneously so that suppliers can provide a policy of postponement of payment. The EOQ inventory model with the first payment deferment policy was developed by Goyal (1985) the model gives two different cases. First, when payments are made over a predetermined cycle time so that the retailer has to pay the fine supplied by the supplier. Secondly, when the payment is made less than the predetermined cycle time so that no fine is paid by the retailer to the supplier.In this research, we will develop EOQ inventory models for defective products with improvements based on research Jaber et al., (2014) and a policy of postponement of payments based on research Goyal (1985) the purpose of this EOQ inventory model development is to maximize expected returns and determine optimal ordering policies for products that have a probabilistic demand.

II. NOTATIONS AND ASSUMPTIONS

2.1. Notations

The notation used in this research is as follows:

D : Level of demand (unit / year)
t_i : Time of screening process in lot size (year)
t_R : Delivery time, repair, and return of defective products (years)
t_T : Total transportation time (year)
ρ : The product size is defective
$f(\rho)$: Probability of defective product
T : Cycle Time (year)
x : Rate of screening speed (unit / year)
S : Cost of repair setup (Rp)
A : Fixed transportation repair costs (Rp)
c_R : Repair cost paid by the retailer (Rp / unit)
c_1 : Material and work costs for the repair process (Rp / unit)
c_T : Shipping cost (Rp / unit)
h' : Storage costs during the repair process (Rp / unit / year)
h : Good product storage cost (Rp / unit / year)
c_i : The cost of screening (Rp / unit / year)
R : Rate of repair (unit / year)
h_R : Save cost for fixed product (Rp / unit / year)
m : Percentage of profit taken by repair shop (%)
K : Retail booking fee (Rp / message)
c_u : Purchase cost (Rp / unit)
P : Product sales price (Rp / unit)
I_e : Interest rate earned (Rp / unit / year)
I_k : Interest rate charged (Rp / unit / year)
M : Delayed payment delays (year)
$E[.]$: Expected value for a random variable
T^* : Optimized booking cycle length
y^* : The order quantity is optimal$=\frac{DT^*}{E(1-\rho)}$

2.2. Assumptions

The assumptions used in this study are as follows:

a. The demand level is known and constant.
b. No shortage allowed.
c. Each lot supplied by the supplier to the retailer contains a defective product of ρpercent with a probability distribution of the defective product known to be $f(\rho)$.
d. Items are shipped from suppliers located far away so the return of defective products is impossible to do as they require a higher cost.
e. Retailer performs repair process to repair company.
f. A repair company is a company outside the retailer with all the repair costs incurred by the retailer and all defective products can be repaired.
g. Supplier gives a discount on the purchase to the retailer in accordance with the agreement if there is a defective product.
h. 100% screening process of lot is done at x level of unit.
i. The screening time and total shipment of defective products from the retailer to the repair shop and back to the retailer is less than or equal to one cycle.
j. The retailer will complete the payment at the time of M and pay a fine on the product in stock at the rate of $I_k$during the $[M, T]$ interval during$T \geq M$.. Alternatively, the retailer completes the payment at time M and does not have to pay a fine for the product in stock during the cycle during $T \leq M$
k. Retailers may add to their sales revenue and earn interest during the $T = 0$to M period at$I_e$level under the payment deferment policy.
l. The interest earned is less than equal to the interest charged$(I_e \leq I_k)$.

III. RESULTS AND DISCUSSION

3.1. Model Formulation

For example $N(y,\rho))$ is a product notation either in every lot of delivery by [4], so it is obtained

$$N(y,\rho) = (1-\rho)y \tag{1}$$

To avoid shortages, then

$$N(y,\rho) \geq Dt \tag{2}$$

Substitute equation (1) into equation (2) and replace t value with $\frac{y}{x}$, then the value of ρ is limited to

$$\rho \leq 1 - \frac{D}{x} \tag{3}$$

Because in this study involves defective product, then replace y with $\frac{DT}{1-\rho}$, so equation (3) yields

$$t = \frac{y}{x} \leq T \tag{4}$$

This study has two cases, firstly when payments are made over a predetermined time cycle $(t \leq M \leq T)$ and second when payments are made less than the specified time cycle $(t \leq T \leq M)$. Both cases can be seen in Figure 1 and Figure 2.

The First Case: $t \leq M \leq T$

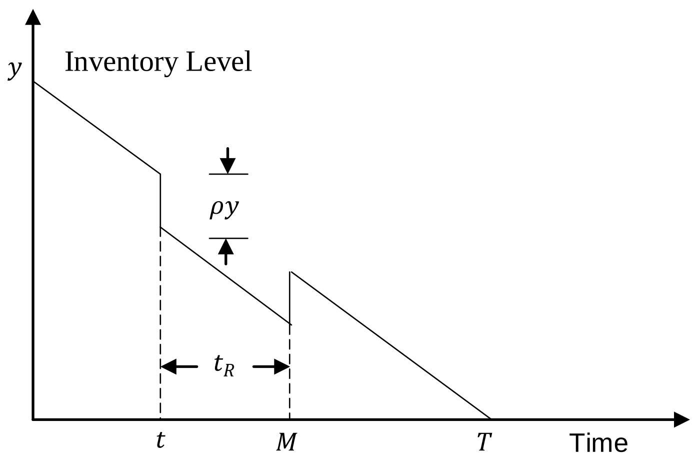

Figure 1. Graph of Inventory Level When $t \leq M \leq T$

When $t \leq M \leq T$, then let $TR_1(y)$ be the first case income total, $TC_1(y)$ represents the total cost of the first case inventory, and $TP_1(y)$ represents the total first case profit. Each is obtained

$TR_1(y)$ = Total product sales + interest earned during the payment deferment period

$$= Py + \frac{PI_e DM^2}{2} \tag{5}$$

$TC_1(y)$ = Good product storage fee + improved product storage cost + booking fee + purchase cost + cost of screening + product repair cost + interest charged for unsold inventory after time M

$$= h\left(\frac{y^2(1-\rho)^2}{2D} + \frac{\rho y^2}{x}\right) + h_R\left(\frac{\rho y^2}{D} - \rho y\left(\frac{y}{x} + \frac{\rho y}{R} + t_T\right) - \frac{\rho^2 y^2}{2D}\right) + K + c_u y + c_i y + \rho y(1+m)\left(\frac{S+2A}{\rho y} + c_1 + 2c_T + \frac{h'\rho y}{R} + h' t_T\right) + c_u I_k\left(\frac{y-DM+\rho y}{2}\right)(T-M) \tag{6}$$

$$TP_1(y) = TR_1(y) - TC_1(y)$$

$$= Py + \frac{PI_e DM^2}{2} - \left\{h\left(\frac{y^2(1-\rho)^2}{2D} + \frac{\rho y^2}{x}\right) + h_R\left(\frac{\rho y^2}{D} - \rho y\left(\frac{y}{x} + \frac{\rho y}{R} + t_T\right) - \frac{\rho^2 y^2}{2D}\right) + K + c_u y + c_i y + \rho y(1+m)\left(\frac{S+2A}{\rho y} + c_1 + 2c_T + \frac{h'\rho y}{R} + h' t_T\right) + c_u I_k\left(\frac{y-DM+\rho y}{2}\right)(T-M)\right\} \tag{7}$$

Substituting $y=\frac{DT}{(1-\rho)}$ and $t=\frac{y}{x}$ into equation (7), so the total profit is $TP_1(T)=TP_1\left(\frac{DT}{1-\rho}\right)$ and total annual profit is $TPU_1(T)=\frac{TP_1(T)}{T}$. Thus, obtained

$$\begin{aligned}TPU_1(T) = {} & \left[\frac{PI_eDM^2}{2T}-\frac{K}{T}-h\frac{DT}{2}-\frac{(1+m)(S+2A)}{T}+\frac{c_uI_k}{2}\left(DM-\frac{DM^2}{T}-\frac{DT}{(1-\rho)}-\frac{\rho DT}{(1-\rho)}+\right.\right.\\ & \left.\left.\frac{DM}{(1-\rho)}+\frac{\rho DM}{(1-\rho)}\right)\right]+(PD-c_uD-c_iD+h_R\rho Dt_T-\rho Dc_1(1+m)-\rho D2c_T(1+m)-\\ & h^{'}\rho Dt_T(1+m))\left(\frac{1}{(1-\rho)}\right)-\left(h\frac{\rho D^2}{x}+h_R\rho D-h_R\frac{\rho D^2}{x}-h_R\frac{\rho^2D^2}{R}-h_R\frac{\rho^2D}{2}+\right.\\ & \left.\frac{h^{'}\rho^2D^2(1+m)}{R}\right)T\left(\frac{1}{(1-\rho)^2}\right) \qquad (8)\end{aligned}$$

Since ρ is a random variable with a known probability density function of $f(\rho)$, then the value of $ETPU_1(T)$ is as follows.

$$\begin{aligned}ETPU_1(T) = {} & \left[\frac{PI_eDM^2}{2T}-\frac{K}{T}-h\frac{DT}{2}-\frac{(1+m)(S+2A)}{T}+\frac{c_uI_k}{2}\left(DM-\frac{DM^2}{T}-DTE\left(\frac{1}{1-\rho}\right)-\right.\right.\\ & \left.\left.DTE\left(\frac{\rho}{1-\rho}\right)+DME\left(\frac{1}{1-\rho}\right)+DME\left(\frac{\rho}{1-\rho}\right)\right)\right]+\left((PD-c_uD-c_iD)E\left(\frac{1}{(1-\rho)}\right)+\right.\\ & \left.(h_RDt_T-Dc_1(1+m)-D2c_T(1+m)-h^{'}Dt_T(1+m))E\left(\frac{\rho}{(1-\rho)}\right)\right)-\left(\left(h\frac{D^2}{x}-\right.\right.\\ & \left.\left.h_RD+h_R\frac{D^2}{x}\right)E\left(\frac{\rho}{(1-\rho)^2}\right)-\left(h_R\frac{D^2}{R}+h_R\frac{D}{2}-\frac{h^{'}D^2(1+m)}{R}\right)E\left(\frac{\rho^2}{(1-\rho)^2}\right)\right)T \qquad (9)\end{aligned}$$

This states that the domain of $ETPU_1(T)$ is the set of $[M,\infty)$.

Second case: $t\le T\le M$

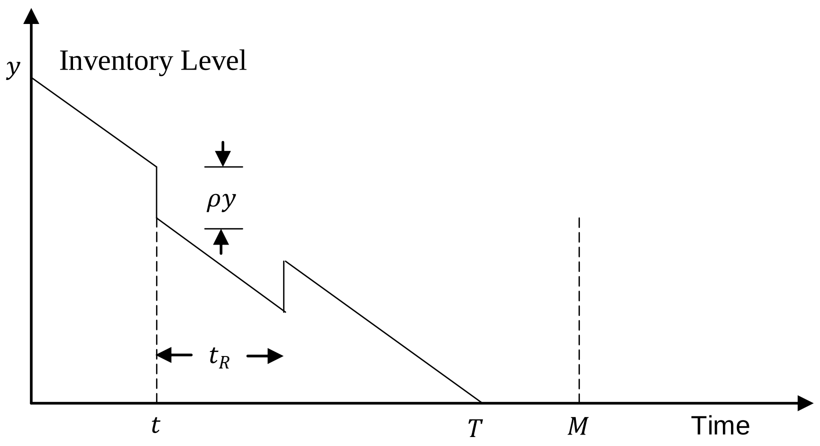

Figure 2. Graph of Inventory Rate When $t_1\le T\le M$

When $t\le M\le T$, then let $TR_2(y)$ be the total of the second case income, $TC_2(y)$ represents the total cost of the second case inventory, and $TP_2(y)$ represents the total profit of the second case. Each is obtained

$TR_2(y)$ = Total product sales + interest earned during the payment deferment period

$$= Py+\frac{PI_eDT^2}{2}+PI_eDT(M-T) \quad (10)$$

$TC_2(y)$ = Good product storage cost + improved product storage cost + booking fee + purchase cost + cost of screening + product repair cost

$$= h\left(\frac{y^2(1-\rho)^2}{2D}+\frac{\rho y^2}{x}\right)+h_R\left(\frac{\rho y^2}{D}-\rho y\left(\frac{y}{x}+\frac{\rho y}{R}+t_T\right)-\frac{\rho^2y^2}{2D}\right)+K+c_uy+c_iy+$$

$$\rho y(1+m)\left(\frac{S+2A}{\rho y}+c_1+2c_T+\frac{h'\rho y}{R}+h't_T\right) \quad (11)$$

$$\begin{aligned} TP_2(y) &= TR_2(y)-TC_2(y) \\ &= Py+\frac{PI_eDT^2}{2}+PI_eDT(M-T)-\left\{h\left(\frac{y^2(1-\rho)^2}{2D}+\frac{\rho y^2}{x}\right)+h_R\left(\frac{\rho y^2}{D}-\rho y\left(\frac{y}{x}+\frac{\rho y}{R}+t_T\right)\right.\right. \\ &\left.\left.-\frac{\rho^2y^2}{2D}\right)+K+c_uy+c_iy+\rho y(1+m)\left(\frac{S+2A}{\rho y}+c_1+2c_T+\frac{h'\rho y}{R}+h't_T\right)\right\} \quad (12)\end{aligned}$$

Substituting $y=\frac{DT}{(1-\rho)}$ and $t=\frac{y}{x}$ into equation (12), so total profit is $TP_2(T)=TP_2\left(\frac{DT}{1-\rho}\right)$ and total annual profit is $TPU_2(T)=\frac{TP_2(T)}{T}$. Thus, obtained

$$\begin{aligned} TPU_2(T) &= \left[PI_eDM-\frac{PI_eDT}{2}-\frac{K}{T}-h\frac{DT}{2}-\frac{(1+m)(S+2A)}{T}\right]+(PD-c_uD-c_iD+h_R\rho Dt_T- \\ &\rho Dc_1(1+m)-\rho D2c_T(1+m)-h'\rho Dt_T(1+m))\left(\frac{1}{(1-\rho)}\right)- \\ &\left(h\frac{\rho D^2}{x}+h_R\rho D-h_R\frac{\rho D^2}{x}-h_R\frac{\rho^2D^2}{R}-h_R\frac{\rho^2D}{2}+\frac{h'\rho^2D^2(1+m)}{R}\right)T\left(\frac{1}{(1-\rho)^2}\right) \quad (13)\end{aligned}$$

Since ρ is a random variable with a known probability density function of $f(\rho)$, then the value of $ETPU_2(T)$ is as follows.

$$\begin{aligned} ETPU_2(T) &= \left[PI_eDM-\frac{PI_eDT}{2}-\frac{K}{T}-h\frac{DT}{2}-\frac{(1+m)(S+2A)}{T}\right]+\left((PD-c_uD-c_iD)E\left(\frac{1}{(1-\rho)}\right)+\right. \\ &\left.(h_RDt_T-Dc_1(1+m)-D2c_T(1+m)-h'Dt_T(1+m))E\left(\frac{\rho}{(1-\rho)}\right)\right)- \\ &\left(\left(h\frac{D^2}{x}-h_RD+h_R\frac{D^2}{x}\right)E\left(\frac{\rho}{(1-\rho)^2}\right)-\left(h_R\frac{D^2}{R}+h_R\frac{D}{2}-\frac{h'D^2(1+m)}{R}\right)E\left(\frac{\rho^2}{(1-\rho)^2}\right)\right)T \quad (14)\end{aligned}$$

This states that the domain of $ETPU_2(T)$ is the set of $[t,M]$. $ETPU(T)$is the value of the total expected annual profit, so it is obtained

$$ETPU(T)=\begin{cases}ETPU_1(T) & if\ M\le T\\ ETPU_2(T) & if\ T\le M\end{cases} \quad (15)$$

Thus, the domain is the set of $[t,\infty)$. Because $ETPU_1(M)=ETPU_2(M)$, then $ETPU(T)$ can be defined. $T^*$is the maximum value of $ETPU(T)$.

3.2. Optimal Solution Model

The objectives of this study were to determine the optimal ordering cycle length (T^*), optimal order quantity (y^*), and total annual profit $(ETPU(T^*))$. Therefore

$$\begin{aligned} ETPU_1(T) &= \left[\frac{PI_eDM^2}{2T}-\frac{K}{T}-h\frac{DT}{2}-\frac{(1+m)(S+2A)}{T}+\frac{c_uI_k}{2}\left(DM-\frac{DM^2}{T}-DTE\left(\frac{1}{1-\rho}\right)-\right.\right. \\ &\left.\left.DTE\left(\frac{\rho}{1-\rho}\right)+DME\left(\frac{1}{1-\rho}\right)+DME\left(\frac{\rho}{1-\rho}\right)\right)\right]+\alpha_1-\alpha_2T \quad (16)\end{aligned}$$

and

$$ETPU_2(T) = \left[PI_eDM-\frac{PI_eDT}{2}-\frac{K}{T}-h\frac{DT}{2}-\frac{(1+m)(S+2A)}{T}\right]+\alpha_1-\alpha_2T \quad (17)$$

where

$$\alpha_1 = \left((PD-c_uD-c_iD)E\left(\frac{1}{(1-\rho)}\right)+(h_RDt_T-Dc_1(1+m)-D2c_T(1+m)-h'Dt_T(1+m))E\left(\frac{\rho}{(1-\rho)}\right)\right)$$

and

$$\alpha_2 = \left(\left(h\frac{D^2}{x}-h_RD+h_R\frac{D^2}{x}\right)E\left(\frac{\rho}{(1-\rho)^2}\right)-\left(h_R\frac{D^2}{R}+h_R\frac{D}{2}-\frac{h'D^2(1+m)}{R}\right)E\left(\frac{\rho^2}{(1-\rho)^2}\right)\right) \quad (18)$$

The values of $ETPU_1(T)$ and $ETPU_2(T)$ are defined on $T > 0$.
Next, the first decrease in equations (16) and (17) to the objective function T to obtain the optimal solution model in the first case (T_1^*) and the second case (T_2^*).

$$ETPU_1'(T) = \frac{K}{T^2} + \frac{(1+m)(S+2A)}{T^2} + \frac{DM^2}{2T^2}(C_u I_k - PI_e) - \frac{hD}{2} - \frac{C_u I_k}{2}\left(DE\left(\frac{1}{1-\rho}\right) + DE\left(\frac{\rho}{1-\rho}\right)\right) - \alpha_2 \quad (19)$$

$$T_1^* = \sqrt{\frac{2K+2(1+m)(S+2A)+DM^2(C_u I_k - PI_e)}{hD + C_u I_k\left(DE\left(\frac{1}{1-\rho}\right)+DE\left(\frac{\rho}{1-\rho}\right)\right)+2\alpha_2}} \quad (20)$$

$$ETPU_2'(T) = \frac{K}{T^2} + \frac{(1+m)(S+2A)}{T^2} - \frac{PI_e D}{2} - \frac{hD}{2} - \alpha_2$$

$$T_2^* = \sqrt{\frac{2K+2(1+m)(S+2A)}{PI_e D + hD + 2\alpha_2}} \quad (21)$$

The concavity test is performed by finding the second derivative of equation (16) and (17) to the purpose function T. The condition of the concavity is obtained if the second derivative of the decision variable is negative. The goal is to ensure that the resulting solution will give maximum results. Thus, obtained

$$ETPU_1''(T) = -\frac{[2K+2(1+m)(S+2A)+DM^2(C_u I_k - PI_e)]}{T^3} \quad (22)$$

$$ETPU_2''(T) = -\frac{[2K+2(1+m)(S+2A)]}{T^3} \quad (23)$$

Equation (23) states that $ETPU_2(T)$ is a conclave of $T > 0$. However, equation (22) states that $ETPU_1(\mathrm{T})$ is a conclave on $T > 0$ if $2K + 2(1+m)(S+2A) + DM^2(C_u I_k - PI_e) > 0$. Furthermore, $ETPU_1'(T = M) = ETPU_2'(T = M)$ where $ETPU_1'(T)$ and $ETPU_2'(T)$ are given in equations (18) and (20). Therefore, equation (15) states that the ETPU (T) conkaf on $[t, \infty)$ when $2K + 2(1+m)(S+2A) + DM^2(C_u I_k - PI_e) > 0$. Because $ETPU_2(T)$ is concurrent on $T > 0$, so it is obtained

$$ETPU_2'(T)\begin{cases} > 0 & jika \quad T < T_2^* \\ = 0 & jika \quad T = T_2^* \\ < 0 & jika \quad T > T_2^* \end{cases} \quad (24)$$

Lemma 1.
Assume that $2K + 2(1+m)(S+2A) + DM^2(C_u I_k - PI_e) \le 0$, then$T_2^* < M$.
Evidence. If$T_2^* \ge M$, then obtained

$$\frac{2K+2(1+M)(S+2A)}{PI_e D + hD + 2\alpha_2} \ge M^2 \quad (25)$$

Equation (25) above produces

$$2K + 2(1+m)(S+2A) \ge PI_e DM^2 + hDM^2 + 2M^2\alpha_2 \quad (26)$$

therefore obtained

$$2K + 2(1+m)(S+2A) + DM^2(C_u I_k - PI_e) \ge PI_e DM^2 + hDM^2 + 2M^2\alpha_2 + DM^2(C_u I_k - PI_e)$$
$$= hDM^2 + C_u I_k M^2\left(DE\left(\frac{1}{1-\rho}\right) + DE\left(\frac{\rho}{1-\rho}\right)\right) + 2M^2\alpha_2 > 0 \quad (27)$$

Equation (27) contradictions. Therefore $T_2^* < M$. Lemma 1 states the following theorem.

Teorema 1.
Assume that $2K + 2(1+m)(S+2A) + DM^2(C_u I_k - PI_e) \le 0$, then$ETPU(T)$has the maximum value on$T_2^*$ dan $T^* = T_2^*$.
Evidence.
Lemma 1 state that$t \le T_2^* < M$. When$ETPU_2(T)$konkaf on$T > 0$, then obtained

$$ETPU_2'(T)\begin{cases} > 0 & jika \quad T < T_2^* \\ = 0 & jika \quad T = T_2^* \\ < 0 & jika \quad T > T_2^* \end{cases} \quad (28)$$

Therefore, $ETPU_2(T)$ increases at $(0, T_2^*]$ and decreases in $[T_2^*, \infty)$ Due to $ETPU_2(T)$if $T \in [t, M]$, then $ETPU_2(T)$ increases on $[t, T_2^*]$ and decreases on $[T_2^*, M]$. Thus, $ETPU_2(T)$ has the maximum value in T_2^*

on $[t, M]$. In addition, if$2K + 2(1+m)(S+2A) + DM^2(C_u I_k - PI_e) \le 0$, then equation (18) states that $ETPU_1'(T) < 0$ and $ETPU_1(T)ETPU_1(T)$ decreases on $T > 0$. Due to $ETPU_1(T)$if $T \in [M, \infty)$, then $ETPU_1(T)$ decreases on $[M, \infty)$.Thus, $ETPU_1(T)$ has a maximum value in M at $[M, \infty)$. When $ETPU_1(M) = ETPU_2(M)$, then equation (15) states that $ETPU(T)$ has a maximum value in T_2^* on $[t, \infty)$. Therefore $T^* = T_2^*$.

When $2K + 2(1+m)(S+2A) + DM^2(c_u I_k - PI_e) > 0$, equation (22) states that $ETPU_1(T)$ is concurrent at $T > 0$. Therefore, it is obtained

$$ETPU_1{}'(T) \begin{cases} > 0 & if \quad T < T_1^* \\ = 0 & if \quad T = T_1^* \\ < 0 & if \quad T > T_1^* \end{cases} (29)$$

Example: $\Delta = 2K + 2(1+m)(S+2A) - M^2[D(PI_e + h) + 2\alpha_2]$

$$L = 2K + 2(1+m)(S+2A) + DM^2(c_u I_k - PI_e) (30)$$

Noted that

$$ETPU_1'(M) = ETPU_2'(M) = \frac{1}{2M^2}\{2K + 2(1+m)(S+2A) - M^2[D(PI_e + h) + 2\alpha_2]\}$$

Therefore, $\Delta > 0$if and only if$ETPU_1'(M) = ETPU_2'(M) > 0$.

Lemma 2.

i. If$\Delta > 0$, then$T_1^* > M$ dan $T_2^* > M$
ii. If$\Delta < 0$, then$T_1^* < M$ dan $T_2^* < M$
iii. Ifa $\Delta = 0$, thena $T_1^* = T_2^* = M$

Evidence.

Equations (24) and (29) state that Lemma 2 is retained. Lemma 2 produces Theorem 2 as follows.

Teorema 2.

Assume that$2K + 2(1+m)(S+2A) + DM^2(c_u I_k - PI_e) > 0$, then obtained

i. If$\Delta > 0$, maka $T^* = T_1^*$ dan $y^* = y_1^* = \frac{DT_1^*}{E(1-\rho)}$
ii. If$\Delta < 0$, maka $T^* = T_2^*$ dan $y^* = y_1^* = \frac{DT_2^*}{E(1-\rho)}$
iii. If$\Delta = 0$, maka $T^* = T_1^* = T_2^* = M$ dan $y^* = \frac{DM}{E(1-\rho)}$

3.3. Numerical Simulation

The data used is the implementation of model from RatnaElektronik, Mojokerto. Parameters used can be seen in table 1, where the calculation is done with the help of Matlab R2012a software.

Table 1. Parameter Value

Description	Value
The product size is defective(ρ)	$U\sim(0, 0{,}04)$
Level of demand(D)	2500 unit
Screen speed(x)	20000 / unit
The cost of screening(c_i)	Rp 8.400 / unit
The cost of booking(K)	Rp 2.800.000,00
Purchase fee(c_u)	Rp 1.793.000,00 / unit
Product sales price(P)	Rp 2.353.000,00 / unit
The cost of storing the product is good(h)	Rp 84.000,00 / unit
Cost of repair setup(S)	Rp 1.050.000,00
Fixed transportation repair costs(A)	Rp 1.400.000,00
Material and work costs for the repair process(c_1)	Rp 70.000,00 / unit
Shipping costs per unit(c_T)	Rp 28.000,00 / unit
Storage costs during the repair process(h')	Rp 56.000,00 / unit
The rate of repair speed(R)	30000 / unit
Total transportation time(t_T)	$\frac{2}{220}$
The cost of storing the repaired product(h_R)	Rp 98.000,00 / unit
Percentage of profit taken by repair shop(m)	20%
Delayed payments allowed(M)	0,1
Interest rate earned(I_e)	0,1
Interest rate charged(I_k)	0,15

It is assumed that ρ is a random variable with a probability density function of a defective product of

$$f(\rho) = \begin{cases} 25 & if\ \ 0 \le \rho \le 0.04, \\ 0 & Others. \end{cases}$$

Then obtained

$$E(\rho) = 0.02$$
$$E\left(\frac{1}{1-\rho}\right) = 1.02055$$
$$E\left(\frac{\rho}{1-\rho}\right) = 0.02055$$
$$E\left[\frac{\rho}{(1-\rho)^2}\right] = 0.0211168$$
$$E\left[\frac{\rho^2}{(1-\rho)^2}\right] = 0.000566949$$

The results of the calculation of the optimal solution model can be seen in Table 2, which is obtained

Table 2. Details of Optimal Solutions

Variables	Values	Optimal
T_1^* (year)	0,131858082	–
T_2^* (year)	0,137043931	–
L	15.681.250	> 0
Δ	6.938.413,743	> 0
T^*	0,131858082	$= T_1^*$
y^*	336 unit	$= y_1^*$
$c_R(y^*)$	Rp 838.564,3158	$c_R(y_1^*)$
$ETPU(T^*)$	Rp1.349.278.956	$ETPU(T_1^*)$

Numerical simulations are performed at intervals $D \in [500, 5000]$ and $\rho \in [0.02, 0.32]$, so the optimal ordering cycle length of Figure 3, the optimal order quantity in Fig. 4, the cost of improvement in Fig. 5, and the total annual profit in picture 6.

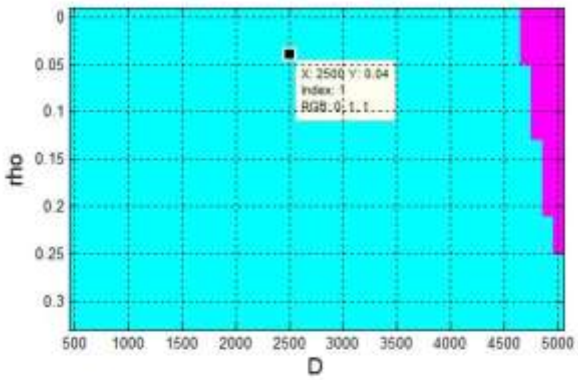

Figure 3. Length of Optimal Booking Cycle

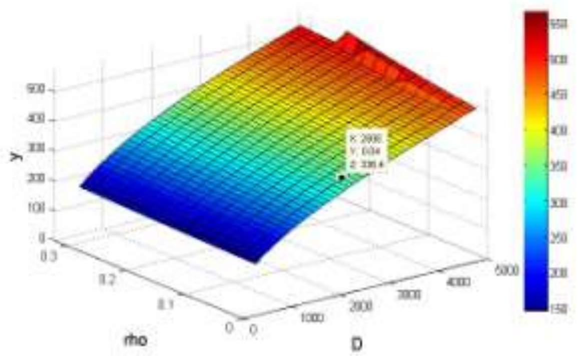

Figure 4. Quantity of Optimal Booking

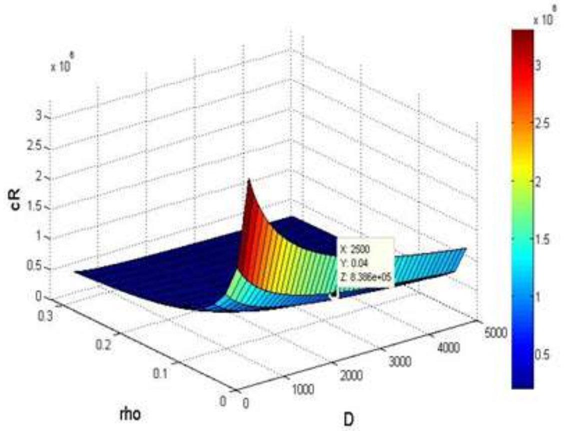

Figure 5. Repair Cost

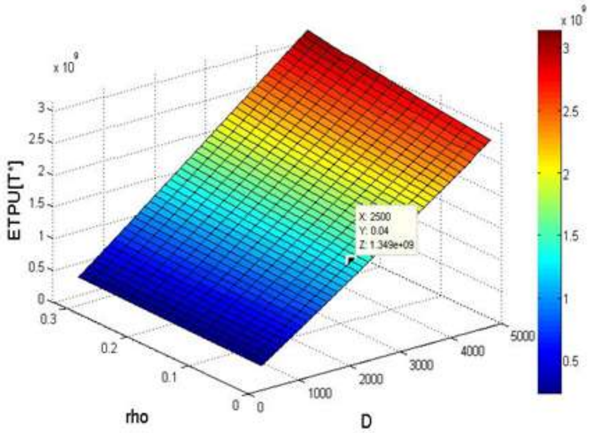

Figure 6. Annual Benefits

Figure 3 shows the length of the booking assignment at $\boldsymbol{T_1^*}$ie 48 days. Figure 4 shows the optimal number of 336 units. Figure 5 shows the repair cost of Rp 838,564. Figure 6 shows a profit of Rp 1,349,278,956. If at the required time, the length of the ordering cycle increases at the rate of $\Delta > 0$. The value of the expected total annual profit at all stages of when. So retailers have to ask for the time it takes. Retailers can benefit most from this condition. As ρ improves, optimum downtime improves, optimum ordering intensity increases, and repair costs decreases. Resellers should charge more good quality products to curious when $\boldsymbol{\rho}$ increases. Total annual profit increases as $\boldsymbol{\rho}$ increases. This happens when the percentage of $\boldsymbol{\rho}$ increases, the supplier provides a discounted purchase price to the retailer in accordance with the agreement and the cost of improvement when ρ increases also decreases.

3.4. Comparison of EOQ Model Results with and Without Delay Payments

The simulation for repair cost is done with interval $\rho \, \epsilon \, [0.02, 0.32]$ and simulation for annual gain is done with interval $D \, \epsilon \, [500, 5000]$. Based on Figure 7, it is found that the cost of repairs with the delay of payment is greater than the cost of repairs without delay of payment. While in Figure 8, it is found that the annual profit with the delay of payment is greater than the annual profit without delay of payment. Thus, this study suggests using the EOQ model with delayed payments.

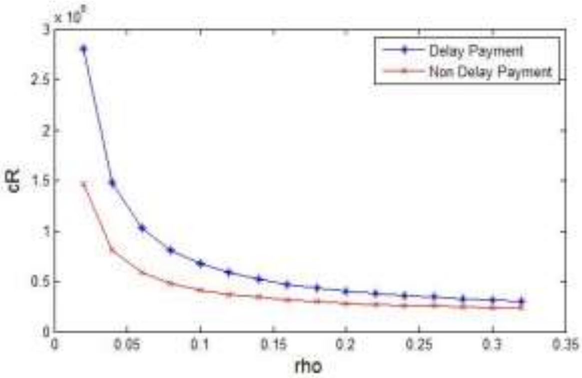

Figure 7. Cost of repairs

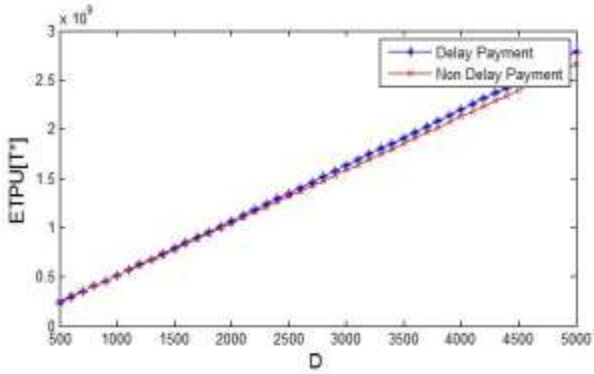

Figure 8. Annual Profits

3.5. Sensitivity Analysis

Sensitivity analysis is done by making changes to some parameter values in the model implementation. These parameter value changes are:

a. *Changes of Setup Fixing Cost*(S)

From the calculations in Table 3, it is concluded that the change in setup cost (S) increases, the value of T^*, y^* and $c_R(y^*)$ increases, while the value of $ETPU(T)$ decreases.

Table 3. Changes of Setup Fixes Cost(S)

Percentage	S	T^*	y^*	$c_R(y^*)$	$ETPU(T^*)$
- 50%	525.000	0,126449719	323	770.285	1.354.156.858
- 30%	735.000	0,128640353	328	798.148	1.352.181.085
- 15%	892.500	0,130259154	332	818.555	1.350.721.059
0%	1.050.000	0,131858082	336	838.564	1.349.278.956
+ 15%	1.207.500	0,133437853	340	858.195	1.347.854.133
+ 30%	1.365.000	0,134999138	344	877.467	1.346.445.981
+ 50%	1.575.000	0,137053188	350	902.630	1.344.593.396

b. Change of Product Disability Rate(ρ)

From the calculation in Table 4, it is concluded that the increasing parameter changes mengakibatkan result in the decreasing value of T^* and $c_R(y^*)$, while the values of y^* and $ETPU(T)$ are increasing.

Table4. Changes in Product Disability Rate(ρ)

Percentageρ	T^*	y^*	$c_R(y^*)$	$ETPU(T^*)$
2%	0,132581401	335	1.531.740	1.305.176.435
4%	0,131858082	336	838.564	1.349.278.956
8%	0,130429657	340	491.887	1.371.193.964
16%	0,127657803	347	318.349	1.419.292.299
32%	0,122620043	365	231.064	1.537.693.513

c. Change of Payment Delay Period(M)

From the calculations in Table 5, it is concluded that the change in the parameter of M increases the value of T^*, y^* and $ETPU(T)$ increases, while the value of $c_R(y^*)$ decreases further.

Table 5. Changes to Payment Delay Period(M)

Time period$M$	T^*	y^*	$c_R(y^*)$	$ETPU(T^*)$
0,05	0,129178184	330	852811	1317386390
0,1	0,131858082	336	838564	1349278956
0,15	0,136207458	347	816636	1379665790
0,2	0,142073067	362	789189	1408685106
0,25	0,149276282	381	758435	1436498010
0,3	0,15763384	402	726274	1463269792
0,35	0,166972494	426	694147	1489156705

IV. CONCLUSIONS

This study changed the two realistic assumptions of the emerging EOQ model for discussion between RatnaElectronics's production and financial sectors to describe a practical business situation. This journal is a combination of research from Goyal (1985) and Jaber, et al (2014) to develop EOQ inventory models for defective products with improvements under payment deferral policies. The decision variables sought in this research are optimal ordering cycle length, optimal order quantity, and total expectation of annual profit. Numerical simulations are given to illustrate all the results obtained in this study. Sensitivity analysis of the optimal solution is shown to determine the effect of parameter change on the decision variable. For further research, the model can be developed for multi-product and accommodate backorder.

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