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Optimal Tuning of Dynamic Controller via LQR in a Powered Wheelchair

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ABSTRACT: *The development of an optimal tuning design for Proportional and Integral (PI) controller, applied to a powered wheelchair, is presented in this paper. The aim is to control the left and right wheels velocities to follow a given reference trajectory by means of a kinematic controller. The optimal tuning is performed through the Linear Quadratic Regulator (LQR) theory, the main idea is to insert the integral action of the PI controller to the system. In this way, the classical mathematical model is developed in state space description to adjust the proportional and integral gains of the PI controller to guarantee an optimal operation of wheelchair. For two case studies, the tuning proposal is evaluated in mathematical model of the wheelchair. The first case is linear trajectory and the second case is a circular trajectory, the evaluation test presented satisfactory results.*

Keywords: *Dynamic Control, LQR, PI Controller, Powered Wheelchair, Optimal tuning.*

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I. INTRODUCTION

A powered wheelchair provides a certain independence to the disabled people, providing more mobility through control usually by joystick. However, there are still many users who have the most varied types of physical disabilities that make it impossible to use this type of control safely [1]. In order to provide more comfort and safety to wheelchair users, it is necessary to implement a more improved motion control system through the dynamic control.

To design a controller that meets the demands of the most diverse types of users, a mechanical model of the wheelchair is required. This model is divided into two parts, the first one represents the dynamics and the second the kinematics. These two models become possible a design of the controller based on the simulated response.

The main alternative to control system design is Proportional Integrative Derivative (PID) Controller. This controller is commonly used due to their simple structure and operation, which motivates ongoing research efforts to find alternative approaches to the project and new tuning rules to improve control performance in closed loop based on the PID [2].

In this paper is presented the development and implementation of a PI controller tuned through the Linear Quadratic Regulator theory applied to a powered wheelchair with trajectory tracking system. The tuning of the dynamic controller is accomplished by varying the values of the Q and R matrices, which gives optimal K_p and K_i gains. That are associated with the robustness and simplicity of the PI controller.

The rest of the paper is organized into five sections. Section II presents the kinematic and dynamic model of mobile robots, which are very similar to that of a powered wheelchair. In Section III, the kinematic controller used is briefly described and the proposed dynamic controller is presented through its mathematical formulation. The computational results are presented in Section IV obtained through the application of the proposed methodology. Finally, the conclusions are presented on the section V.

II. POWERED WHEELCHAIR

A powered wheelchair is modelled as differential traction mobile robot it has two front and rear wheels. The rear wheels are controllable. Therefore, there is control applicability for this type of system [1] [3].

The acceleration components are inserted for dynamic system representation, so as to result in the variation of wheel rotation. This mechanical energy is a result of the electric power supply to the motors. The kinematic model, the wheelchair velocities (linear and angular), the position and direction of the wheelchair are input variables, and the wheels diameter and mass center are constants. These models are represented in Fig. 1.

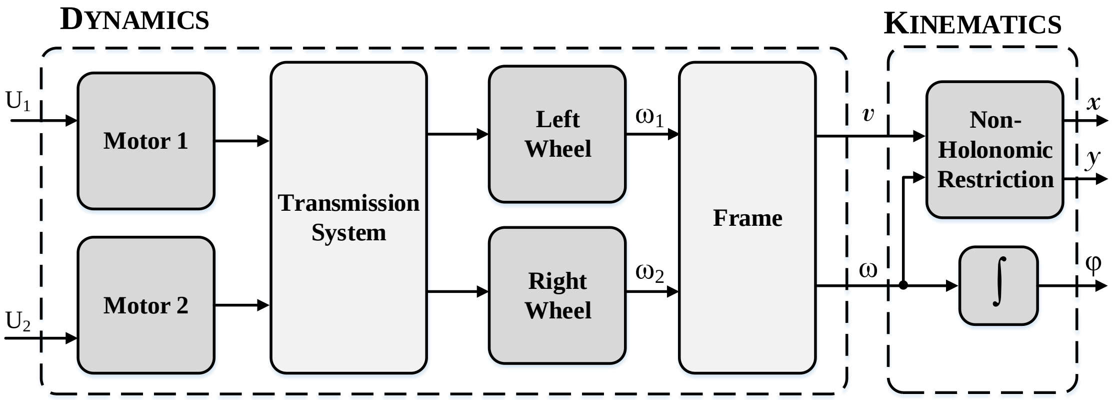

Figure 1: Powered wheelchair dynamic and kinematic system block diagram.

In the block diagram of figure, the input U_1 and U_2 are the left and right motors voltage. The v and ω are the wheelchair linear and angular velocity. The ω_1 and ω_2 is the left and right wheels angular velocities. The x e y are the positions of the wheelchair relative to X and Y axis and φ is the angle of the wheelchair in relation to X axis.

2.1. Kinematics of Differential Traction Robots

As previously mentioned a powered wheelchair is considered as a four-wheel robot with two free front wheels and two motor wheels at its rear, and its position is described by two coordinates X and Y, and φ is the steering angle, that are represented in Fig. 2.

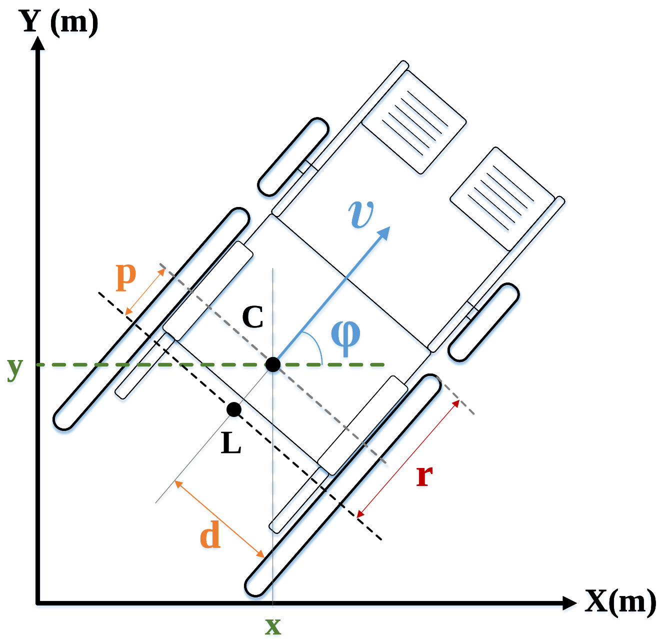

Figure 2: Wheelchair geometry schematic.

The distance between point C and point L is represented by$p$, r is the radius of the wheel, d is the distance from the point L to each wheel, C is the mass center of the system, v and ω are the wheelchair linear and angular velocities, as shown in Fig. 2.

This system is represented by a coordinate vector, as shown in figure, and given by

$$\boldsymbol{q} = [x \quad y \quad \varphi]^T \ . \tag{1}$$

The motion vector $\boldsymbol{q}$ is subject to a non-holonomic restriction, which means that kinematic model of the system is not able to move instantaneously in any direction and its motion according with [4], is given by

$$\dot{x} \cdot \sin\varphi = \dot{y} \cdot \cos\varphi \ . \tag{2}$$

The kinematics of the differential drive mobile robots relates the linear and angular velocities, respectively $[v \quad \omega]^T$ in cartesian velocities $[\dot{x} \quad \dot{y} \quad \dot{\varphi}]^T$, where the x-component of the linear velocity is expressed as $\dot{x} = v \cdot \cos\varphi$and the component of y as $\dot{y} = v \cdot \sin\varphi$, from this the system is rewritten as

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\varphi} \end{bmatrix} = \begin{bmatrix} \cos\varphi & -p \cdot \sin\varphi \\ \sin\varphi & p \cdot \cos\varphi \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} . \quad (3)$$

According from (3), the position of the wheelchair can be represented by $\boldsymbol{h} = [x \quad y]^T$, thus

$$\dot{\boldsymbol{h}} = M \begin{bmatrix} v \\ \omega \end{bmatrix}, \quad (4)$$

where

$$M = \begin{bmatrix} cos\,\varphi & -p \cdot sin\,\varphi \\ sin\,\varphi & p \cdot cos\,\varphi \end{bmatrix} . \quad (5)$$

2.2. Kinematics of Differential Traction Robots

Given the kinematic model, it is necessary to insert the dynamics provided by the wheelchair wheels. According to Fig. 4, ω_1 and ω_2 are the angular velocities of the left and right wheels, respectively. Then the linear velocity of each of the wheels, considering that the wheels do not slipping when rotated [3] [5], isdescribed as

$$v_i = r \cdot \omega_i \,, \quad (6)$$

when $i = 1$ refers to the left and $i = 2$ the right. The linear and angular velocity of the wheelchair is defined respectively as

$$v = \frac{(v_1 + v_2)}{2} \quad (7)$$

and

$$\omega = \frac{(v_2 - v_1)}{2 \cdot d} \, . \quad (8)$$

The (7) and (8) are rewritten so that provides the velocity vector $[v \quad \omega]^T$, to determine the vector $\dot{\boldsymbol{q}}$in (3), is given by

$$\begin{bmatrix} v \\ \omega \end{bmatrix} = N \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix}, \quad (9)$$

where

$$N = \frac{r}{2} \begin{bmatrix} 1 & 1 \\ -1/_d & 1/_d \end{bmatrix}. \quad (10)$$

The left and right wheels angular velocities are provided by two electric motors, through a transmission system, represented by the Fig. 1.

The parameters of the transfer function of wheelchair actuators are obtaned from measure data in [6], it is given by

$$G_1 = \frac{\omega_1}{U_1} = \frac{2.6}{0.28s + 1} \quad (11)$$

and

$$G_2 = \frac{\omega_2}{U_2} = \frac{2.7}{0.3s + 1} \, . \quad (12)$$

The identified system has as input the reference voltage of the power circuit of the motors and the output is given by the angular velocity of the wheels, with the transmission system already coupled to the motor.

III. CONTROL SYSTEM DESIGN

The mobile robots control with differential drive is divided into two parts: kinematic and dynamic control. The aim is to control the powered wheelchair positioning and orientation in the cartesian space x, y and φ, through the tracking a reference trajectory, which is described in Cartesian coordinates. In order to obtain the gains of the dynamic controller, which provide a better system performance, an optimal tuning process is developed through the LQR theory.

3.1. Kinematic Controller

The kinematic controller generates the references for dynamic controller, on a given trajectory. Based on the kinematics of the system, given by (4). Then for a given reference point, it is necessary to generate the reference velocities of the wheels, in other words, the inverse of the kinematics of the system according to [5] is represented by

$$\begin{bmatrix} v \\ \omega \end{bmatrix} = M^{-1} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} . \quad (13)$$

According to [5] the kinematic controller is given by

$$\begin{bmatrix} v_{ref}^{c} \\ \omega_{ref}^{c} \end{bmatrix} = M^{-1} \begin{bmatrix} x_d + \tanh\left(\frac{k_x}{I_x} e_x\right) \\ y_d + \tanh\left(\frac{k_y}{I_y} e_y\right) \end{bmatrix}. \quad (14)$$

The desired linear velocity is v_{ref}^{c} , ω_{ref}^{c} is the desired angular velocity, e_x and e_y are the positioning errors in relation to the x and y coordinates, respectively, k_x and k_y are the controller gains, where $k_x > 0$ and $k_y > 0$, the constants I_x and I_y are saturation constants, and x_d and y_d are the points of desired reference.

Note that this model provides the desired linear and angular velocity for the system, to generate the desired angular velocities for the wheels it is necessary to obtain the inverse model of (9), and is written as

$$\begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix} = N^{-1} \begin{bmatrix} v \\ \omega \end{bmatrix}, \quad (15)$$

replacing in (13), and after in (14), the angular velocities are given by

$$\begin{bmatrix} \omega_{ref}^{1} \\ \omega_{ref}^{2} \end{bmatrix} = N^{-1} \cdot M^{-1} \begin{bmatrix} x_d + \tanh\left(\frac{k_x}{I_x} e_x\right) \\ y_d + \tanh\left(\frac{k_y}{I_y} e_y\right) \end{bmatrix}, \quad (16)$$

The desired values of the left and right angular velocity where ω_{ref}^{1} and ω_{ref}^{2} are,in this order.

3.2. Dynamic Controller

For the dynamic control for each wheel, a PI controller was adopted, to control the wheelchair angular motion, avoiding steady-state error due to non-modeled non-linearities of the system. This controller in time-domain is given by

$$u_i(t) = K_p^i \cdot e(t) + K_{int}^i \cdot \int_0^T e(t)dt, \quad (17)$$

applying the Laplace transform in PI controller of (17) for design purposes, then

$$L_i(s) = \left(K_p^i s + K_{int}^i\right)\frac{1}{s} . \quad (18)$$

The Fig. 1 is customized to represent the closed loop system and the control system topology in s-plane. This customization is showed in Fig. 3.

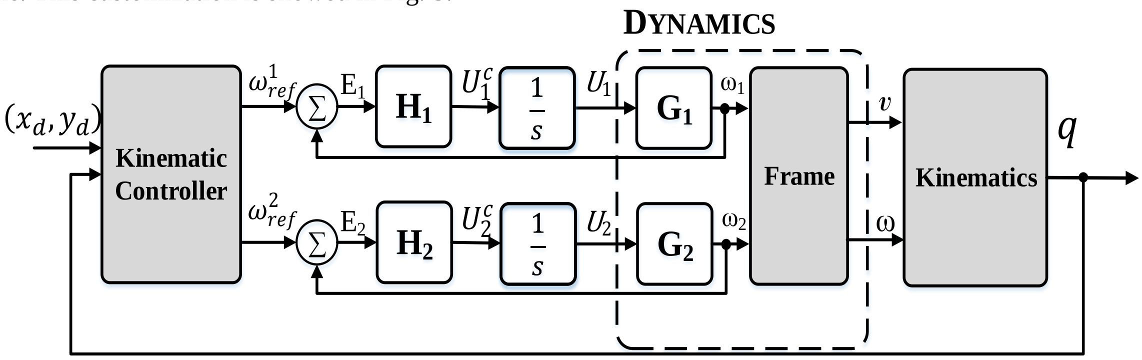

Figure 3: Powered wheelchair closed loop system block diagram.

Associating (18) with the control topology of Fig. 3 with the purpose of applying the LQR theory the controller is given by

$$H_i = \frac{U_i^c(s)}{E_i(s)} = K_p^i s + K_{int}^i , \quad (19)$$

where $E_i(s) = \omega_{ref}^i(s) - \omega_i(s)$, then

$$\frac{U_i^c(s)}{\left(\omega_{ref}^i(s) - \omega_i(s)\right)} = K_p^i s + K_{int}^i, \quad (20)$$

hence,

$$U_i^c(s) = K_p s\omega_{ref}^i(s) + K_i\omega_{ref}^i(s) - K_p s\omega_i(s) - K_{int}^i\omega_i(s). \quad (21)$$

Applying the inverse Laplace transform in (21), the new control law in time domain is given by

$$u_i^c(t) = K_p^i \frac{d\omega_{ref}^i}{dt} + K_{int}^i\omega_{ref}^i(t) - K_p^i\frac{d\omega_i}{dt} - K_{int}^i\omega_i(t). \quad (22)$$

3.3. LQR Tuning Method

The proposed LQR tuning method is an association of the wheelchair control implementation based on block diagrams in s plane and its control design method in state space. The implementation encompasses a high-level control realization of the classic PI controller in the closed of plant and sensors of Fig. 4. In terms of control design, the approaches lie on state space methods and optimal control.

The main steps of LQR tuning method are described in next paragraphs. The first step is the development closed loop of control system in state space description that is equivalent to the control system through the rearrangement of Fig. 3 and is showed in Fig. 4. The second step is computation of the proportional and integral controller gains of (19) that are PI gains of Fig. 4. And the last step defines the method associated with control specifications to provide operation specifications.

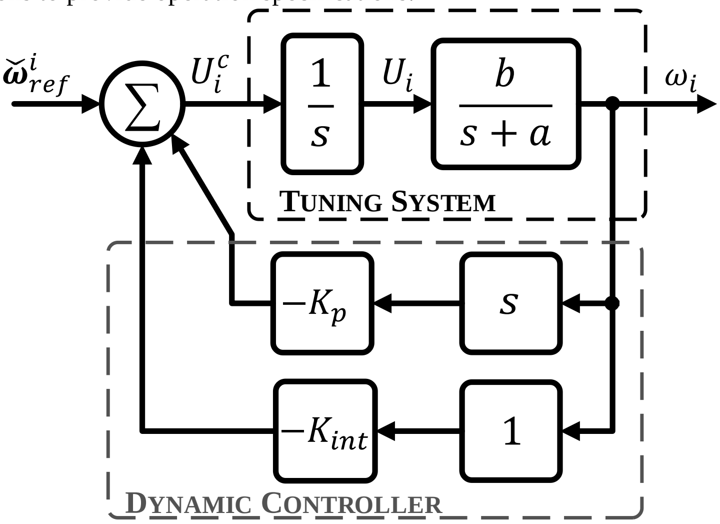

Figure 4: The block diagramdynamic control system with optimal PI.

The state space description of powered wheelchair model in closed loop control design system is given by

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -bK^i_{int} & -(a+bK^i_p)\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ b\end{bmatrix}\breve{\omega}^i_{ref}$$
$$\omega_i = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}, \tag{23}$$

where $x_1 = y$ and $\dot{x}_1 = x_2$ are angularvelocity and acceleration measured outputs. In the canonical state space form, the dynamic closed loop system is fully separated in two parts, the first one is the plant $G_i(s)$ with integral action, which is the tuning model to obtain the optimal state feedback gains which in turn is the second part that is given by (22).

The control law u_i^c in the state space form, according to (22), that presents the relations in the s-plane and state space descriptions. Consequently, the control law with state feedback is given by

$$u_i^c = -\boldsymbol{K}^i\boldsymbol{x} + \breve{\boldsymbol{\omega}}^i_{ref} \ , \tag{24}$$

with $\boldsymbol{K}^i = \begin{bmatrix}K^i_{int} & K^i_p\end{bmatrix}$, $\boldsymbol{x} = \begin{bmatrix}x_1 & x_2\end{bmatrix}^T$ and $\breve{\boldsymbol{\omega}}^i_{ref} = K^i_{int}\,\omega^i_{ref} + K^i_p\dot{\omega}^i_{ref}$.

The second step of proposed LQR tuning method is the optimal computation of optimal gains. According to [7] is demonstrated that to calculate the optimal control law, which will drive the states optimally, it is necessary to minimize the performance index that is given by

$$J(x,Q,u,R) = \frac{1}{2}\int_0^\infty (x^TQx + u^TRu)dt, \tag{25}$$

where $Q \geq 0$ is a semidefinite positive state weighting matrix, $R > 0$ is a positive definite input weighting matrix. From [8], the optimizing structure to minimize the performance index is presented as

$$\min_u J(x,Q,u,R)\ , \tag{26}$$

subject to

$$\dot{x} = Ax + Bu\ , \tag{27}$$

where, B is the system inputs matrix and A is the state transition matrix of the open-loop state space. Based on the control law presented in (24), the optimal gain vector K^* of the controller is expressed as

$$K^* = R^{-1}B^TP\ , \tag{28}$$

and P is the solution for the Algebraic Riccati Equation (ARE) that is given by

$$A^TP + PA - PBR^{-1}B^TP + Q = 0\ , \tag{29}$$

The third step is to establish a heuristic based on measurable metric to evaluate the method performance. This metrics are associated with control specifications. The design specifications must attend the technical metrics, such as: w_n , ζ and t_s , as well as, operation specifications, such: minimum energy consumption that is accomplished to the integrand of (25).

Manipulating(28) and (29), one can figure out that optimal gain matrix K^* has a strong interaction with Q and R matrices of the performance index that is given (26). In terms of Q and R matrices parametric dependency and design established numerical limits considerations, then optimal tuning model is given by

$$K^*(Q,R) \approx f_1(R) + f_2(Q,R), \tag{30}$$

where f_1 andf_2 represent the influence of these matrices on the gain values. The choice of Q and R will establish the main guidelines to determine the controller gains that satisfies designer specifications. This formulation is demonstrated in [9] in discrete time, after algebraic manipulations it is noted that for continuous time this approximation given in (30) is also obtained. Thus, the control law given in (24) is rewritten as

$$u_i(Q,R) = -\boldsymbol{K}_{QR}^i \boldsymbol{x} + \breve{\boldsymbol{\omega}}_{ref}^i . \tag{31}$$

IV. MODEL BASED SIMULATION RESULTS

For the controller tuning procedure, the values of the matrix Q and for $R = 1$ were varied to obtain state feedback gains through the LQR theory. Some Q matrix are chosen for the computation of the feedback gains K and to observe the system behavior. In Table 1, the values of K_{int} and K_p, the damping coefficient (ζ) and the poles of the closed-loop system are presented.

The LQR tuning method developed for LQR design tuning based on variations of weighting matrices. The method allows the selection of the weighting matrices Q and R. The presented tuning evaluation of the proposed method is based on variations of Q matrices and matrix R is fixed during the search process. According to relation (30), the main guide of the LQR tuning is approximated by

$$K \approx R^{-1}Q . \tag{32}$$

The purpose of the controller design is to reduce the positioning error to a given references trajectories by angular velocities of the left and right wheels. In this section two reference trajectory are used, the first is a point, located at the coordinates (35, 35) meters from the origin, and the second reference is a circular trajectory.

Table 1: Controller gains and tuning parameters.

Q	K_p	K_{int}	ζ	Poles
$Q_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 1 \end{bmatrix}$	0.71	0.31	1	-0.2950
				-9.9681
$Q_2 = \begin{bmatrix} 1 & 0 \\ 0 & 0.1 \end{bmatrix}$	0.29	1.00	1	-2.3032
				-4.0379
$Q_3 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	0.78	1.00	1	-0.9367
				-9.9284
$Q_4 = \begin{bmatrix} 10 & 0 \\ 0 & 1 \end{bmatrix}$	0.96	3.16	1	-3.1031
				-9.4774
$Q_5 = \begin{bmatrix} 1 & 0 \\ 0 & 10 \end{bmatrix}$	2.83	1.00	1	-0.3139
				-29.6270
Ziegler–Nichols method	0.404	4.36	0.58	$-3.68 + 5.2i$
				$-3.68 - 5.2i$

Based on the data provided by Table 1, it is noticed that for any value of Q the system remains stable, since all poles obtained have real negative part, but when using Q_4 the two poles of the system are further from the origin, providing a faster response and this is represented in Fig. 5, it presents the angular velocities of the left and right wheels with the application of the proposed controller for the first reference trajectory.

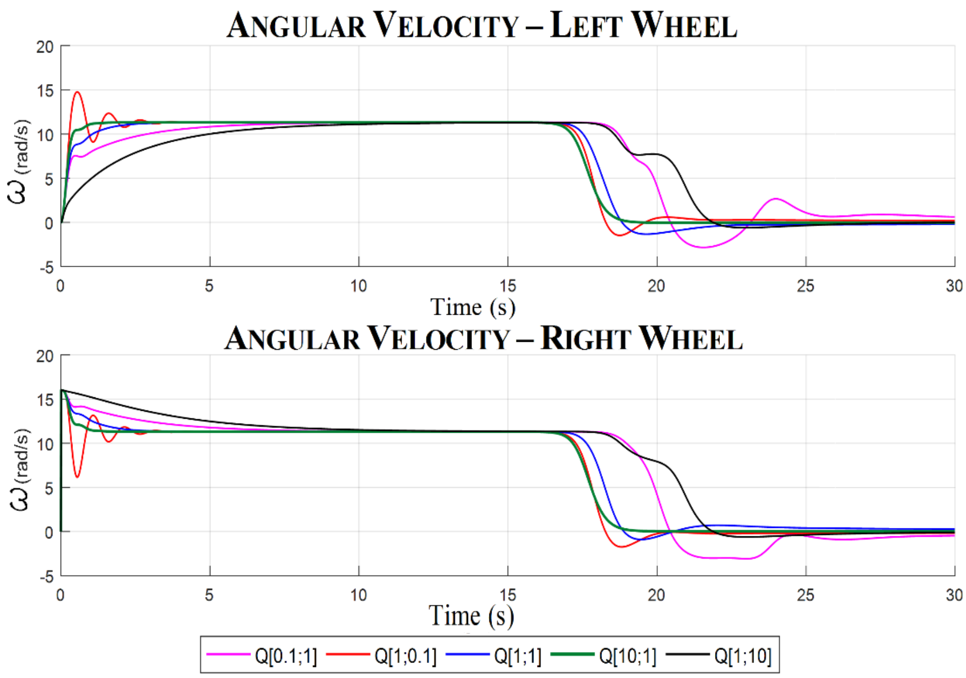

Figure 5: Left and right angular wheels velocities to reference trajectory for point.

The powered wheelchair is designed to follow a given desired trajectory smoothly, so it is necessary to determine the state feedback gains based on this premise. The analysis of proposed control effort indicates its importance in the system response on the Fig. 6. Based on the voltage applied to each of the motors, which makes it possible to evaluate the feasibility of the project.

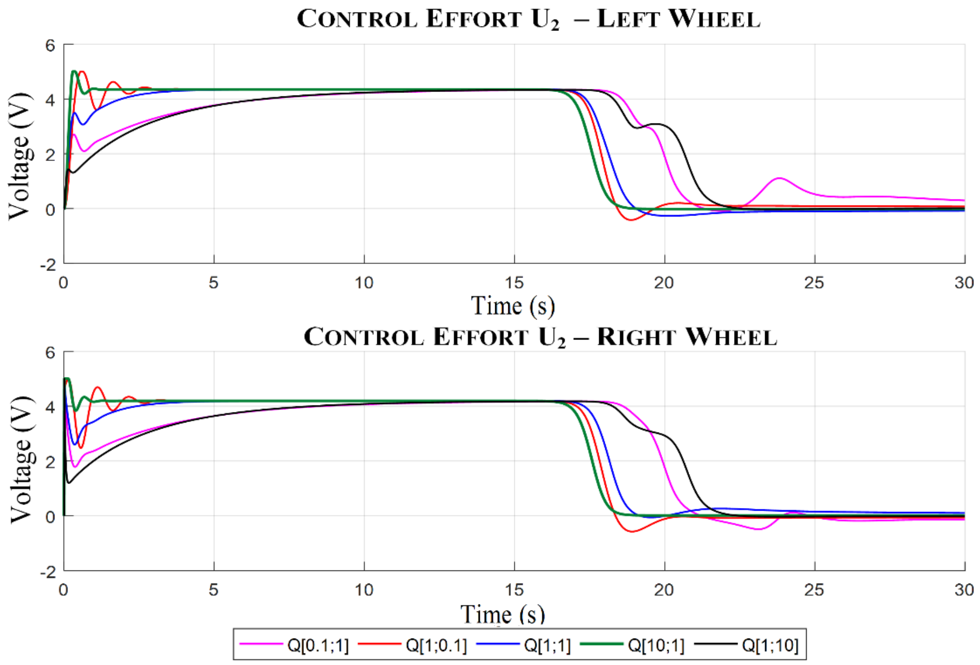

Figure 6: Control effort to reference trajectory for point.

The position of the wheelchair is given by rotations of its wheels can be illustrated by Fig. 6 in which it has a reference point 35 meters from the X-axis and 35 meters from the y-axis.

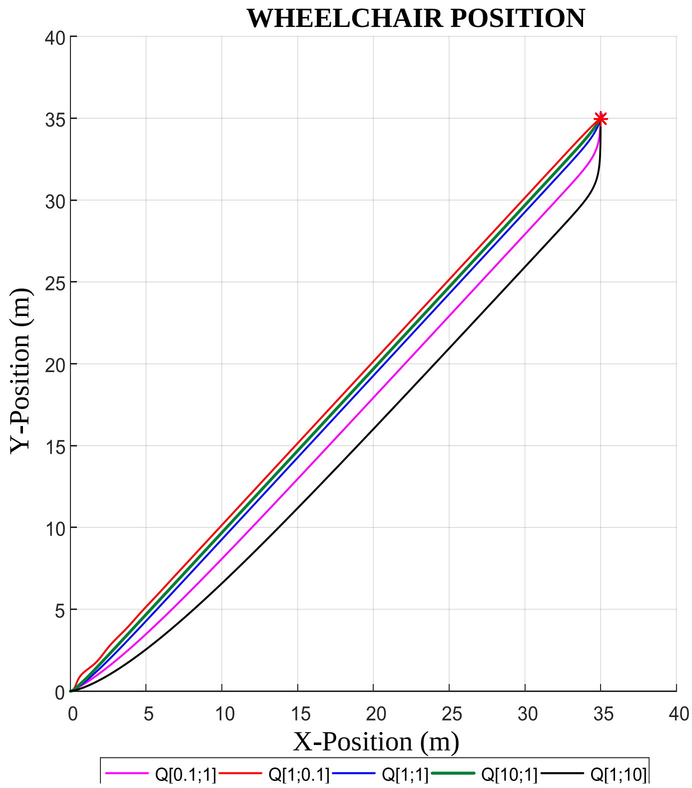

Figure 7: Wheelchair trajectory for point, the asterisk is the reference point.

Note that the response obtained for $Q_4 = diag(10,1)$showed in Fig. 6, 7 and 8 is the fastest and without oscillations in its trajectory, which guarantees a shorter distance traveled and consequently lower energy cost. Regarding the time, the system obtained a response of 18 seconds, as shown in the previous figures.

In the second phase of the simulations, the reference trajectory changed to a circular, where the kinematics of the system is observed with more details. This trajectory has a radius of 5 meters with angular velocity of 0.3 rad/s. The results showed on Fig. 8.

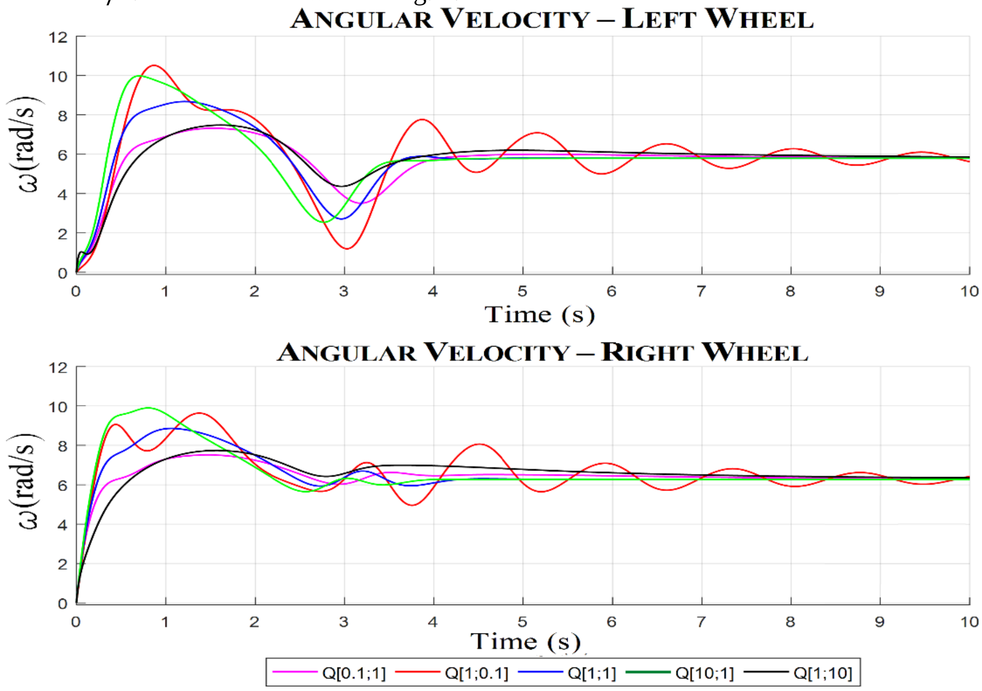

Figure 8: Left and right angular wheel velocity to reference circular trajectory.

The angular velocities for Q_4 stabilize in approximately 4 seconds, without oscillations. The right wheel has greater intensity in relation to the left wheel during the stabilization period, allowing the wheelchair to follow the circular path. The control effort shows this relation on Fig. 9.

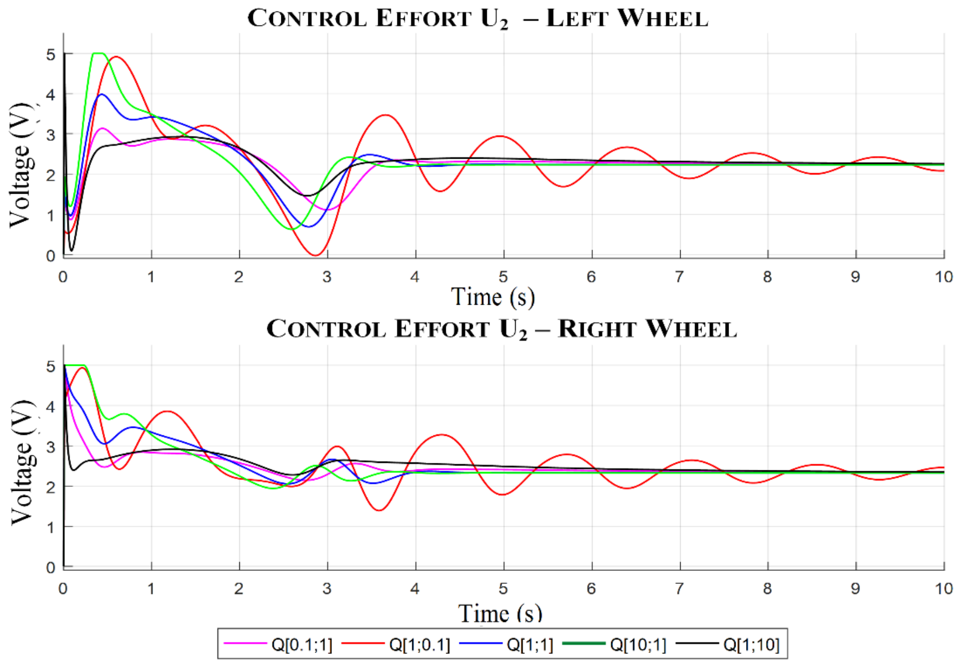

Figure 9: Control efforts to reference circular trajectory.

The control effort presented remains inside an acceptable limit, which turn the implementation of controller designs feasible and its response can be observed in Fig. 10, which shows the trajectory of the wheelchair.

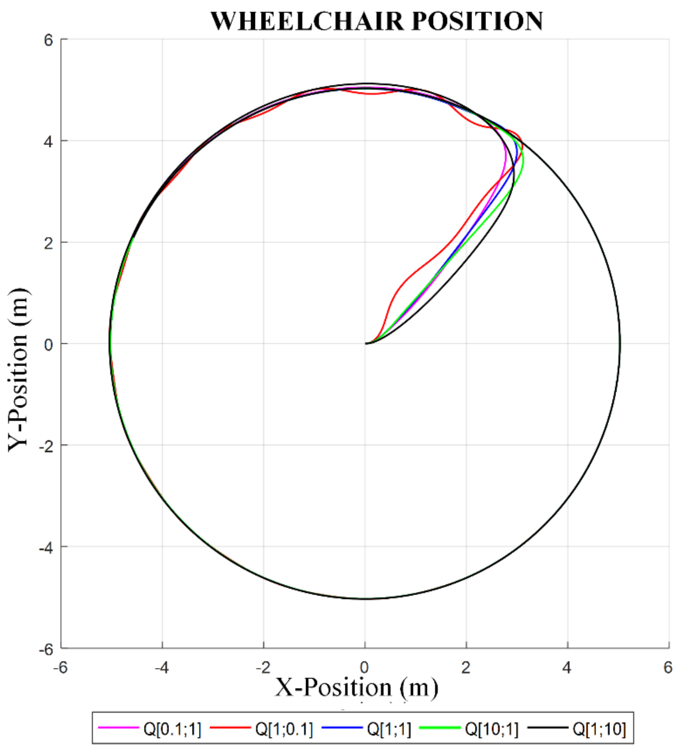

Figure 10: Wheelchair circular trajectory

V. CONCLUSION

In conclusion, the present work focused on the development of a powered wheelchair dynamic controller, which combines the simplicity and robustness of a PI controller with the optimality, gain and phase margin guaranteed by the LQR tuning. Firstly, the plant parameters of the transfer function were estimated by measuring data. The mathematical controller model was developed, transforming the gains K_p and K_i, of a PI controller, into state feedback gains. After that controller was adjusted by variation of the values of Q-weighting matrix and then applying the LQR theory. As the results of the performance evaluation, the simulations to reference trajectories showed the efficiency of the proposed tuning method, where the values of the control effort did not exceed the limit imposed by the real-world implementation, showing that the simulation results are satisfactory for an implementation in a powered wheelchair.

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